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THE HEAVY ION PRE-BUNCHER FOR THE USP PELLETRON

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Part I - INTRODUCTION

The recent interest in boosters for tandem accelerators has led to an abundance of reports on the theory and techniques involved in heavy ion room temperature bunching[1,2,3,4,5,6]. Two types, the double drift and the single gap (gridded) harmonic bunchers, evolved as being the most satisfactory from the point of view of time resolution (~1ns), efficiency of bunching (60-75%) and range of ion mass which may be used for beams with these compression characteristics.

A priori, a choice between these two types is difficult to make, since under optimum performance, they are almost equivalent. However, for a given accelerator (see Fig. 1), limitations in the space along the beamline, spatial focussing of the beam along the beamline and flexibility in ion injection energy all contribute in favoring one or the other buncher type.

We have pursued extensive studies and calculations on both these types of buncher for use in the USP Pelletron. Before outlining these we will first summarize our conclusions regarding the virtues of each type for installation in our system.

THE DOUBLE DRIFT BUNCHER (DD)

a) The ungridded nature of this buncher demands that it be placed downstream from the electrostatic triplet lens in a region where the beam is spatially well focused in order that the difference in energy modulation of an off-axis particle relative to an axis particle remains insufficient to appreciably effect the time resolution. Calculations show that an equivalent buncher target distance of about 60 cm is indicated which, for the USP Pelletron, means mounting the buncher within the high pressure tank, a complicated construction and installation problem.

b) This short effective distance also means operating with rather high rf voltages (6000-7000 v at 5MHz). Furthermore, in order to accommodate ions of different mass, either the modulation frequency or the beam injection energy (or both) must be varied to compensate for the transit time variation. For the range of rf voltages required, changing the frequency means changing the parameters of the resonant circuits which necessarily follow the broad band rf amplifiers. For any extensive variations in ion mass, the complications of this procedure should be avoided if possible and certainly a much simpler alternative is to change the injection energy, keeping the frequency fixed.

c) It was calculated that the change in injection energy to bunch particles at 8MHz fundamental frequency with $10 \le A \le 60$ is from 60 keV to 300 keV. For the Pelletron this means that the existing bias source for the injector is inadequate and construction and installation of a new supply would need to be projected. Extension of the A mass region for a given injection energy range is possible using longer drift tubes but this would set at least part of the buncher in a region where the spatial extension of the beam is unsuitable for an ungridded buncher.

Thus we conclude that the DD buncher is not a convenient choice for us.

THE GRIDDED SINGLE GAP HARMONIC BUNCHER (GG)

a) Initially this type buncher, developed at Argonne National Laboratories for the ATLAS project operated at a relatively high frequency (48MHz) which satisfies the requirements for injection into a Linac but is unsuitable for time of flight measurements with the Pelletron. However, more recently the ATLAS people were able to achieve equivalent good perfomance with this type buncher pulsed at 12MHz by placing copper cone extensions on the original grid support to shape the electric fields in the regions before and

after the single gap so that ion transit time effects were minimized. At present they have pronounced good quality beams over a large mass range at this fundamental frequency plus just two harmonics, the 2f and the 3f waves.

b) In the Pelletron the gridded gap is to be positioned 1.2 meters upstream from the first accelerator electrode. This is a region of reasonably easy physical access. The buncher box (ATLAS DESIGN) will sit just above but outside the Pelletron tank entrace. Although the present beam optics is not the best at this location, the possibilities in repositioning the various optical elements to improve this feature are being calculated.

c) The voltages necessary for time focussing at the terminal stripper from this position are in the range of 2000-3000 volts (^{16}O, injected at 74 keV) at a fundamental frequency of 12MHz (see part V). This should be achieved without much difficulty.

d) The optimum injection energies for $10 \le A \le 100$ were calculated (part II) to be in the range 60 keV to 160 keV which is well within the capabilites of our present ion injection bias supply.

e) The time spread at the terminal stripper was calculated to be 0.8 ns and is aberration limited. This aberration in the single gap harmonic buncher is discussed in section V.

f) The time spread 30 meters downstream from the final accelerator electrode (which could be the position of the superbuncher or the scattering chamber for time of flight measurements) is 1.1 ns.

Thus the results of our calculations show that the prospects look very good for employing this type of buncher in the USP Pelletron.

Part II - CALCULATION OF THE OPTIMUM INJECTION ENERGY

The Equivalent Drift Length

(see ref. 2 and 5)

The availability of space along our beamline and the advantages of locating the buncher near the beam waist (ref. 3) led us to select a grid position 1.2 meters upstream from the first accelerating electrode of the Pelletron. This has the further advantage that the buncher will be entirely outside the tank and easily accessible. All the calculations that follow are for this position.

Choosing the terminal stripper as the time focus point we shall calculate the effective buncher length, the modulation constant for this region and the optimum injection energy as a function of ion mass.

The time a beam particle of mass m takes to travel a distance L in a uniform accelerating field with initial velocity v_i and final velocity v_f is called the transit time and is given by

$$t = \frac{L}{\frac{v_i + v_f}{2}} = \frac{2L}{\left(\frac{2}{m}\right)^{1/2}\left[E_i^{1/2} + E_f^{1/2}\right]}$$

or

$$t = \frac{2LA^{1/2}}{\gamma\left[E_i^{1/2} + E_f^{1/2}\right]} \qquad (1)$$

where A is the ion mass number and $\gamma = 1.38 \times 10^7$ m.s^{-1}. (MeV)$^{-1/2}$.

Since the action of the buncher produces a modulation in the beam injection energy, it is convenient to define a modulation constant, called β, as the change in transit time with injection energy, or the time compression.

Hence $\beta = {}^{\partial t}/_{\partial E_i}$ may be calculated from equation (1).

Let $x = \sqrt{E_i} + \sqrt{E_f}$ so that

$$\frac{\partial t}{\partial E_i} = \frac{2LA^{1/2}}{\gamma}\left[-x^{-2}\frac{\partial x}{\partial E_i}\right]$$

and

$$\frac{\partial x}{\partial E_i} = \frac{\partial\sqrt{E_i}}{\partial E_i} + \frac{\partial\sqrt{E_f}}{\partial E_i}$$

or

$$\frac{\partial x}{\partial E_i} = \frac{1}{2}E_i^{-1/2} + \frac{\partial\sqrt{E_f}}{\partial E_f}\cdot\frac{\partial E_f}{\partial E_i}$$

and

$$\frac{\partial x}{\partial E_i} = \frac{1}{2}E_i^{-1/2} + \frac{1}{2}E_f^{-1/2}$$

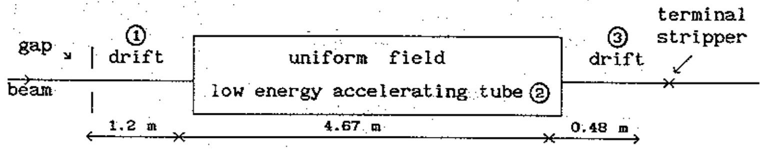
since $E_f = E_i$ + (some quantity independent of E_i)

finally, $$\beta = \frac{-2LA^{1/2}}{\gamma}\cdot\frac{\left[E_i^{-1/2} + E_f^{-1/2}\right]}{2\left[E_i^{1/2} + E_f^{1/2}\right]^2} \qquad (2)$$ for the uniform field region. For a field free region (drift space) $E_i = E_f$ and equation (2) gives $$\beta = \frac{LA^{1/2}}{2\gamma E_i^{3/2}} \qquad (3)$$

When β is known it is convenient to define as effective buncher length or equivalent drift region by

$$L_{eff} = \frac{2\gamma\beta E_i^{3/2}}{A^{1/2}} \qquad (4)$$

For the buncher position we are testing, the space up to the terminal stripper consists of three regions shown below.



The overall modulation constant is the sum of the modulation constants for each region:

$$\beta = \beta_1 + \beta_2 + \beta_3$$

and from equations (2), (3) and (4)

$$\beta = \frac{L_{eff}\,A^{1/2}}{2\gamma\,E_i^{3/2}} = \frac{L_1 A^{1/2}}{2\gamma\,E_i^{3/2}} + \frac{L_2 A^{1/2}\left[E_i^{-1/2} + E_f^{-1/2}\right]}{\gamma\left[E_i^{1/2} + E_T^{1/2}\right]^2} + \frac{L_3 A^{1/2}}{2\gamma\,E_T^{3/2}}$$

where E_i is the injection energy at the buncher and E_T is the ion energy at the terminal.

An equivalent drift length for the region between the buncher grid and the terminal stripper may be calculated from

$$L_{eff} = L_1 + 2L_2\left[\frac{E_i}{E_T}\right]\frac{1}{1 + \left[\frac{E_i}{E_T}\right]^{1/2}} + L_3\left[\frac{E_i}{E_T}\right]^{3/2} \qquad (5)$$

The optimum conditions for the beam transport are those that minimize the phase space product $\phi \equiv \Delta E\ \Delta\tau$. Calling the value of β necessary to insure this β_{opt} (see next section for its determination) equation (4) describes an optimum equivalent drift length i.e.

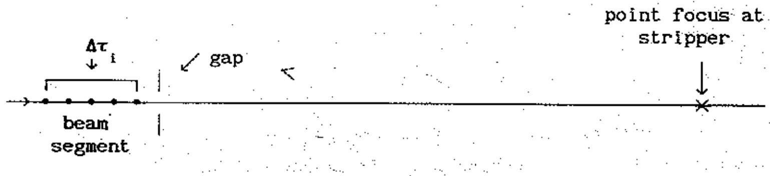
$$L_{eff} = \frac{2\gamma\beta_{opt}E_i^{3/2}}{A^{1/2}} \qquad (6)$$

Examination of equations (5) and (6) shows that for a given ion

mass A, a value of E_i may be found which equates the actual drift length (equation (5)) to the optimum drift length (equation (6)). This value of E_i is called E_i (opt) and is most easily obtained by a graphical solution of equations (5) and (6).

The Optimum Value of β

Before solving for E_i we must know the value β_{opt}. For this we examine a segment of beam in a time interval $\Delta\tau_i$ assuming, for the time being, that each particle out of the ion source and injected into the buncher has the same energy E_Q. Now if the buncher were ideal it would produce a point time focus at the terminal stripper as illustrated below.



To achieve a point focus in time the buncher would have to induce a maximum variation in transit time between the grid and stripper equal to $\Delta\tau_i$ itself by slowing down the leading particles and speeding up the trailing particles. Hence the energy modulation ΔE_i which must be placed on this time segment of particles must have the magnitude

$$\Delta E_i = \frac{\Delta\tau_i}{\beta}$$

and so these particles arrive at the stripper with an energy spread $\Delta E_T = \Delta E_i$. Thus at the stripper

$$\Delta E_T = \frac{\Delta\tau_i}{\beta} \qquad (7)$$

However, some energy spread ΔE_Q is always present in the beam from the ion source. Hence, even for an ideal buncher, a time spread exists at the focus equal to

$$\Delta\tau_T = \beta\Delta E_Q \qquad (8)$$

It is interesting to note that, for an ideal buncher whose object is described by ΔE_Q, $\Delta\tau_i$ and whose image is described by ΔE_T, $\Delta\tau_T$, the phase space product, $\phi = \Delta E\, \Delta\tau$ is conserved i.e.

$$\phi_{object} = \phi_{image}$$

as shown from equations (7) and (8).

A more realistic estimate of the phase space paramenters should include the energy straggling introduced by the terminal stripper $\Delta E_{1,s}$. Thus the total energy spread at this stripper is

$$\Delta E_T = \sqrt{\left[\frac{\Delta\tau_i}{\beta}\right]^2 + (\Delta E_{1,s})^2} \qquad (9a)$$

Another source of time spread comes from the spherical aberration $\Delta\tau_a$ of the buncher. Calculations of this are shown later on but it suffices now to say that the time spread due to spherical aberration for the single gap harmonic buncher is (see ref. 3 p. 256 and ref. 2 p. 131)

$$\Delta\tau_a = \frac{\alpha}{\omega_1} \qquad (9b)$$

where α = .06 and ω_1 is the fundamental frequency applied to the gap.

Thus the total time spread at the terminal stripper is

$$\Delta\tau_T = \sqrt{(\beta\Delta E_Q)^2 + (\Delta\tau_a)^2} \qquad (10)$$

The optimum value of β is that which gives a minimum in phase space product at the terminal stripper (ref. 2).

Hence, $\phi_T = \Delta E_T\ \Delta\tau_T$ should be a minimum and from equations (9) and (10)

$$\phi_T = \left[\frac{(\Delta\tau_i)^2}{\beta^2} + (\Delta E_{1,s})^2\right]^{1/2} \left[\beta^2(\Delta E_Q)^2 + (\Delta\tau_a)^2\right]^{1/2}$$

is a minimum when $\frac{\partial\phi_T}{\partial\beta} = 0$. This gives the value

$$\beta_{opt} = \left[\frac{\Delta\tau_i\ \Delta\tau_a}{\Delta E_Q\ \Delta E_{1,s}}\right]^{1/2} \qquad (11)$$

Caluculation of β_{opt} for the USP Pelletron

The single gap harmonic buncher is to operate at f_1 = 12 MHz. The efficiency foř this buncher is 75% (ref. 3).

Hence, 0.75T where T = 1/f = 83.3ns gives $\Delta\tau_i$ = 62.5ns and $\Delta\tau_a = \alpha/\omega_1$ = 0.796ns.

The ripple in the ion source supply gives ΔE_Q = 15 eV. The straggling and nonuniformity in the stripper carbon foils (5-10 $\mu g/cm^2$) gives $\Delta E_{1,s}$ = 35 keV (ref. 5). Thus from equation (11)

$$\beta_{opt} = 9.7 \times 10^{-6}\ \text{sec/MeV}$$

or

$$\beta_{opt} \simeq 1 \times 10^{-5}\ \text{sec/MeV}$$

Calculation of E_i (opt) for Various Ion Masses

We return to equation 5 and calculate effective drift length from grid to terminal for the Pelletron as a function of injection energy E_i assuming a terminal voltage 8MV, and recalling L_1 = 1.2 meters, L_2 = 4.67 meters and L_3 = 0.48 meters. The terms of the equation

$$L_{eff} = L_1 + 2L_2\left[\frac{E_i}{E_T}\right] \cdot \frac{1}{1 + \left[E_i/E_T\right]^{1/2}} + L_3\left[\frac{E_i}{E_T}\right]^{3/2}$$

are tabulated below, where E_i is in MeV.

E_i	$\left[\frac{E_i}{E_T}\right]$	$\left[\frac{E_i}{E_T}\right]^{3/2}$	$2L_2\left[\frac{E_i}{E_T}\right] \cdot \frac{1}{1+\left[E_i/E_T\right]^{1/2}}$	$L_3\left[\frac{E_i}{E_T}\right]^{3/2}$
.04	.005	.00035	.043 (3.5%)	.000168 (~0%)
.08	.010	.0010	.084 (6.5%)	.00048 (~0%)
.12	.015	.00184	.139 (10.3%)	.00083 (~0%)
.16	.020	.00282	.164 (12.0%)	.00135 (~0%)
.20	.025	.00395	.202 (14.4%)	.00190 (~0%)

The numbers in parentheses show the percent contribution of the term to the total effective drift length.

Hence for L_1 = 1.2 meters we have

E_i (MeV)	L_{eff} (meters)
.04	1.243
.08	1.284
.12	1.339
.16	1.364
.20	1.402

This is shown as the solid curve of figure 2.

Calculation of the Optimum Drift Length

Using

$$L_{eff} = 2\,\gamma E_i^{3/2}\,\frac{\beta_{opt}}{(A)^{1/2}}$$

where $\beta_{opt} = 9.7\times10^{-6}$ sec/MeV and $\gamma = 1.38\times10^{7}$ msec^{-1} (MeV)$^{-1/2}$, we get the following relations.

A	L (meters)
8	$94.6E_i^{3/2}$
16	$66.9E_i^{3/2}$
64	$33.5E_i^{3/2}$
144	$22.3E_i^{3/2}$
200	$18.9E_i^{3/2}$

which gives for L in meters,

E_i (MeV)	$E_i^{3/2}$	L(A=8)	L(A=16)	L(A=64)	L(A=144)	L(A=200)
.05	.011	1.04	0.735	.368	.245	.208
.10	.032	3.02	2.14	1.07	.714	.605
.15	.058	5.49	3.88	1.94	1.29	1.10
.20	.089	8.42	5.95	2.97	1.98	1.68
.25	.125	11.83	8.36	4.18	2.79	2.36
.30	.164	15.51	10.97	5.48	3.66	3.10

The dashed curves of figure 2 show L in meters versus E_i in MeV for the five A values above. The points of intersection of these curves with the curve $L = L_1 + xL_2 + yL_3$ where

$$x = 2\left[\frac{E_i}{E_T}\right]\left[\frac{1}{1 + \left[E_i/E_T\right]^{1/2}}\right] \text{ and}$$

$$y = \left[\frac{E_i}{E_T}\right]^{3/2}$$

give the optimum value of injection energy for our system for a given A. A graph of this is shown by the solid curve of figure 3.

Calculation of the Optimum Injection Energy Including Gap Effect

The nonuniformity of the electric field within the buncher gap results in a difference in modulation energy for an off-axis ion passing through the gap compared to an axis one. This is known as the gap effect. Skorka (ref. 2, p. 134) presents a set of equations for calculating gap effect influence on the optimum injection energy of the previous section. His results are

$$E_i^G(\text{opt}) = \lambda E_i(\text{opt})$$

where G means gap effect case and $E_i(\text{opt})$ is neglecting gap effect. The quantity λ is the root of the dimensionless equation

$$\lambda^6 + p_4\,\lambda^4 - p_1\,\lambda = 1 \qquad \text{where } \lambda \geq 1$$

The constants in this equation are determined from the relations

$$p_4 = \frac{K^2}{3E_i^2(\text{opt})}$$

where $K = gp\,\omega_1^2 \langle r^2\rangle \dfrac{A}{\alpha}$ and $p_1 = 2p_4 \dfrac{\phi_2}{\phi_1}$

where $\dfrac{\phi_2}{\phi_1} = \dfrac{2\pi p\Delta E_Q}{\alpha\Delta E_{i,s}}$

For the harmonic buncher pulsed at 12MHz we have:

$p=0.75$ $\alpha=.06$ $E_Q=15\text{eV}$ $E_{i,s}=35\text{keV}$

and

$g=16\times10^{-3}$ $\langle r^2\rangle^{1/2}=0.55\text{mm}$ $\omega_1=0.75$ rad/ns

where the value of the constant g determines the set of units used. Thus,

$K= 3.38\times10^{-4}$ A (MeV) and $\dfrac{\phi_2}{\phi_1} = 29$

The following table results where the values of E_i are taken from the solid curve in fig.3

A	$K^2\times10^4$	E_i (MeV)	$3E_i^2\times10^2$	$p_4\times10^2$	$p_1=58p_4$
16	.29	.072	1.55	.187	.108
40	1.82	.096	2.76	.659	.382
100	11.4	.136	5.54	2.06	1.19
200	45.7	.172	8.88	5.15	2.99

which gives the following equations for λ

A = 16 $\lambda^6 + .002\,\lambda^4 - .108\,\lambda = 1 \rightarrow \lambda = 1.03$ E_i^G (opt) = 74keV

A = 40 $\lambda^6 + .007\,\lambda^4 - .382\,\lambda = 1 \rightarrow \lambda = 1.06$ E_i^G (opt) = 101keV

A = 100 $\lambda^6 + .02\,\lambda^4 - 1.19\,\lambda = 1 \rightarrow \lambda = 1.15$ E_i^G (opt) = 156keV

A = 200 $\lambda^6 + .05\,\lambda^4 - 2.99\,\lambda = 1 \rightarrow \lambda = 1.29$ E_i^G (opt) = 223keV

The dotted curve in figure 3 shows the optimum injection energy as a function of ion mass A when the gap effect is included. It is also seen in fig.3 that for $10 \leq A \leq 100$ the optimum injection energies range from $60\text{keV} \leq E_i(\text{opt}) \leq 160$ keV which is within the capabilities of our present ion source bias supply.

PART III - THE TIME SPREAD AT THE TERMINAL STRIPPER AND SUPERBUNCHER

The time spread at the terminal stripper is calculated from equation (10) i.e.

$$\Delta\tau_T = \sqrt{(\beta\Delta E_Q)^2 + (\Delta\tau_a)^2}$$

where $\beta = \dfrac{L_{eff}\ A^{1/2}}{2\gamma E_i^{3/2}}$

From figure 2 for A=16, E_i(opt) = 74keV and L_{eff} is about 1.28 meters. This gives a value β = 1.00 x 10^{-5} sec/MeV. Therefore $\beta\Delta E_Q$ = 15 x 10^{-11} sec = 0.15 ns. From p.11 we have $\Delta\tau_a$ = 0.8ns so that

$$\Delta\tau_T = 0.8\text{ns} \quad \text{(aberration limited)}$$

Skorka (ref. 2, p.131) shows that for a system to be aberration limited the injection energy E_i must be larger than a characteristic injection energy E_i^A defined by

$$E_i^A = \left[\frac{L_{eff}\Delta E_Q}{2\gamma\Delta\tau_a}\right]^{2/3} A^{1/3}$$

For A = 16

$$E_i^A = \left[\frac{1.28\text{x}15\text{x}10^{-6}}{2\text{x}1.38\text{x}10^{7}\text{x}0.8\text{x}10^{-9}}\right]^{2/3} (16)^{1/3}$$

and E_i^A = 22 keV

so that the condition is indeed fulfilled.

Transport from Terminal to Superbuncher

In this phase of transport the beam traverses the drift region L_1 after the terminal stripper, the high energy part of the accelerating tube (uniform field) L_2 and finally another drift space L_3 from the accelerator exit to the suparbuncher. A reasonable value for L_3 would be about 30 meters.

Thus we have

L_1 = 0.49 meters $\qquad$ L_2 = 4.67 meters $\qquad$ L_3 = 30 meters

writing

$$\beta = \beta_1 + \beta_2 + \beta_3$$

gives

$$L = L_1 + 2L_2\left[\frac{E_T}{E_F}\right]\left[\frac{1}{1 + \left[E_T/E_F\right]^{1/2}}\right] + L_3\left[\frac{E_T}{E_F}\right]^{3/2}$$

where E_T is the terminal voltage, 8 MeV and E_F is about 56 MeV (charge state 6, A = 16).

Substitution gives

$$L = (0.49) + 2(4.67)\left[\frac{8}{56}\right]\left[\frac{1}{\left[1 + (8/56)^{1/2}\right]}\right] + 30\left[\frac{8}{56}\right]^{3/2}$$

or

L = 0.49 + 0.998 + 1.70

L = 3.20 meters equivalent drift length from terminal stripper to superbuncher.

The modulation constant for this transport is

$$\beta = \frac{L(A)^{1/2}}{2\gamma\ E_T^{3/2}} = \frac{3.20\text{x}4}{2\text{x}1.38\text{x}10^{7}\text{x}(8)^{3/2}}\ \text{sec/MeV}$$

so β = 2.1 x 10^{-8}sec/MeV

Calling ΔE_T = 35 keV (straggling in stripper) we have

$$\beta \Delta E_T = 0.74 \times 10^{-9} \quad \text{sec} = 0.74 \text{ ns}$$

Hence the time spread at the superbuncher is

$$\Delta\tau_{sb} = \sqrt{\left[\beta \Delta E_T\right]^2 + \left[\Delta\tau_T\right]^2}$$

$$= \sqrt{(0.74)^2 + (0.8)^2}$$

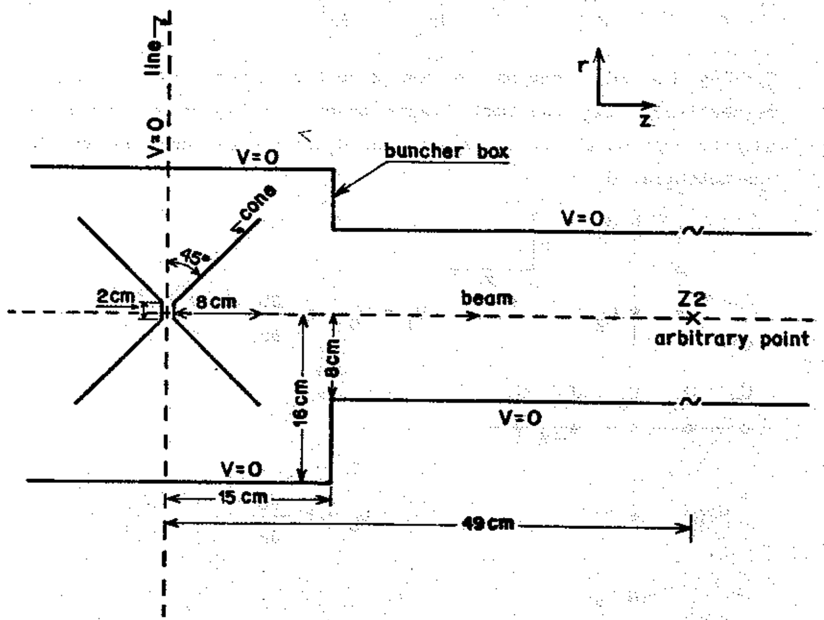
$$\Delta\tau_{sb} = 1.1 \text{ ns}$$

so that for A = 16 we may expect about 1 ns time spread at the entrance to the superbuncher or at a target to be used for time of flight work placed at this position.

PART IV – THE ELECTRIC FIELDS WITHIN THE BUNCHER BOX AND THE CALCULATION OF THE AXIAL TRANSIT TIME FACTOR

The Buncher Box Electric Fields

The buncher design of the ATLAS project which we have adopted is shown schematically below where the appropriate boundary conditions on the potential are also indicated.



In order to gain some intuition about the role of the cones, we decided to calculate the electric fields without the cones and with cones of different dimensions. To this end a computer program BUNEL was written which performs a numerical solution (ref.7) of Laplace's equation in cylindrical coordinates with theta symmetry for the potential U (or V) in the box shown above. At the point Z2 a circular contour of radius 8cm, maintained at the potential U=0 was supposed.

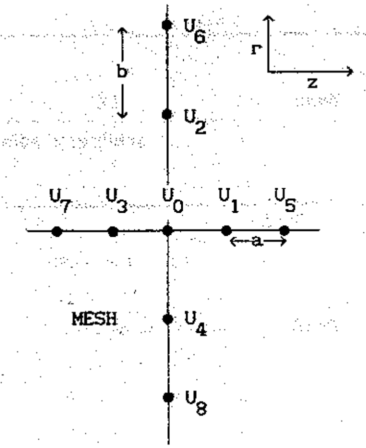
Following ref. 7 we write

$$\nabla^2 U = 0$$

or

$$\frac{\partial^2 U}{\partial r^2} - \frac{1}{r}\frac{\partial U}{\partial r} + \frac{\partial^2 U}{\partial z^2} = 0 \qquad (12)$$

Calling the bin lengths in the z and r directions a and b respectively (see the mesh figure below), a Taylor expansion is used to express the potentials U_1 and U_2 in the neighborhood of the the potential U_0.



$$U_1 = U_0 + a\frac{\partial U}{\partial z} + \frac{a^2}{2}\frac{\partial^2 U}{\partial z^2} + \cdots \qquad (13)$$

$$U_2 = U_0 + b\frac{\partial U}{\partial r} + \frac{b^2}{2}\frac{\partial^2 U}{\partial r^2} + \cdots \qquad (14)$$

and from the figure it is seen that $\dfrac{\partial U}{\partial z} = \dfrac{U_1 - U_3}{2a} + O(a^2)$

which when substituted into (13) and (14) gives

$$\frac{\partial^2 U}{\partial z^2} = \frac{2}{a^2}\left[U_1 - U_0 - a\frac{\partial U}{\partial z}\right] = \frac{2}{a^2}\left[U_1 - U_0 - \frac{(U_1 - U_3)}{2}\right] + O(a^3)$$

or,

$$\frac{\partial^2 U}{\partial z^2} = \frac{U_1 + U_3 - 2U_0}{a^2} + O(a^3) \qquad \text{for } z - \text{variable}$$

and,

$$\frac{\partial^2 U}{\partial r^2} = \frac{U_2 + U_4 - 2U_0}{b^2} + O(b^3) \qquad \text{for } r - \text{variable}$$

Substitution into the Laplace equation (12) gives for points not on the z-axis

$$2U_0\left[1 + \frac{a^2}{b^2}\right] = U_1 + U_3 + U_2\left[1 - \frac{b}{2r}\right]\left[\frac{a^2}{b^2}\right] + U_4\left[1 + \frac{b}{2r}\right]\left[\frac{a^2}{b^2}\right] \qquad (15)$$

and for points on the z-axis

$$2U_0\left[1 + \frac{a^2}{b^2}\right] = U_1 + U_3 + U_2\left[\frac{a^2}{b^2}\right] + U_4\left[\frac{a^2}{b^2}\right] \qquad (16)$$

The boundary conditions shown for the potentials are input data to the computer code for a prechosen mesh size (e.g. a=0.5cm, b=0.5cm, cone half angle 45°) and by linear interpolation, an initial set of potential values at each mesh point in the region within the box was calculated. Application of equations (15) and (16) was performed for each mesh point and the process then repeated until the value of U_0 for each mesh point was negligibly different from the anterior calculation. It was found that 20 iterations were sufficient for convergence.

Once the potential mapping was determined, the z-component of the electric field was calculated from

$$E_z = - \text{grad } U$$

along the lines of b = constant. Graphs of the E_z field along the axis are shown in figure 4 for a variety of cone angles (maintaining the distance from grid to cone base 8cm) and a grid without any cones at all (plane grid).

The Transit Time Factor for the Axial Particles

An ion velocity v_0 which enters an accelerating gap of width D will spend a time $t=\frac{V_o}{D}$ in crossing this gap. If this time is not negligible compared to the rf period, the transit time effects, which must be minimized for optimum performance of the buncher, must be considered. A transit time factor F is defined as the ratio of the modulation that the ion actually feels $\delta\varepsilon$ to the modulation it would feel if transit time effects were negligible, $\delta\varepsilon_m$.

As shown in ref. 3, for an ion which at $t=t_0$ enters a planar accelerating gap of width D with accelerating field $E_z = \frac{V_m}{D}\cos\omega t$, the modulation it would receive without transit time consideration would be $\delta\varepsilon_m = V_m\, q \cos\omega t_o$. When transit time is considered the energy gain in traversing the gap is

$$\delta\varepsilon = \int_0^D q\,\vec{E}\cdot d\vec{z} = \int_0^D \frac{q\,V_m}{D}\cos\omega t\,dz = \frac{q\,V_m}{D}\int_0^D \cos\omega t\,dz$$

since $z = v_0 t$,

$$\delta\varepsilon = \frac{q\,V_m\,v_o}{D}\int_0^{D/V_o}\cos\omega t\,dt = \frac{q\,V_m\,v_o}{\omega D}\left[\sin\frac{\omega D}{v_o}\right]$$

Hence,

$$F = \frac{\frac{qV_m v_0}{\omega D}\sin\left[\frac{\omega D}{v_o}\right]}{qV_m\cos\omega t} \quad \text{and since } t_0=0$$

$$F = \frac{\sin\left[\frac{\omega D}{v_o}\right]}{\frac{\omega D}{v_o}} = \frac{\sin\left[\frac{2\pi D}{\lambda}\right]}{\frac{2\pi D}{\lambda}}$$

where $v_0 = \lambda f$ defines λ as the distance the ion travels in one buncher period T=1/f.

Another definition for the transit time factor is

$$F = \frac{\int E(z)\cos\omega t\,dz}{\int E(z)dz} \qquad (17)$$

where E(z) is the maximum field in the gap. That this is equivalent to the previous definition is shown in what follows.

Writing $E(z) = -\frac{\delta V}{\delta z}$

$$F = \frac{\int -\frac{\delta V}{\delta z}\cos\omega t\,dz}{\int -\frac{\delta V}{\delta z}\,dz} = \frac{\int \frac{\delta V}{\delta z}\cos\omega t\,dz}{V_m}$$

which is easily integrated for a uniform field $E = V_m/D$.

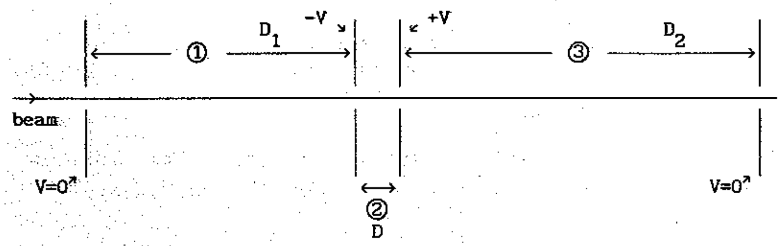
$$F = \frac{\int \frac{V_m}{D}\cos\omega t\,dz}{V_m} = \frac{1}{D}\int \cos\omega t\,dz$$

so that in crossing the gap the ion may be associated with a transit time factor

$$F = \frac{1}{D}\int_0^D \cos\omega t \, dz = \frac{1}{D}\int_0^D \cos\frac{\omega z}{v_0} \, dz = \frac{v_0}{\omega D} \sin\frac{\omega D}{v_0}$$

$$F = \frac{\sin\left(\frac{\omega D}{v_0}\right)}{\frac{\omega D}{v_0}} = \frac{\sin\left(\frac{2\pi D}{\lambda}\right)}{\frac{2\pi D}{\lambda}}$$

The behaviour of this function is shown in figure 5. Schematically the gaps are shown below where region 2 is the gridded harmonic gap.



Hence in region 2 we want F=1 so that the ion receives all the modulation available but in regions 1 and 3 we want F as small as possible so that the particle does not receive or lose energy in these regions. Inspection of figure 5 shows that the gap width D for region 2 should be very small and the distances D_1 and D_2 very large. It is, however, usually very impractical to have extended D_1 and D_2 regions since space is just not available. It is for these regions that the ATLAS people designed the single gridded gap with conducting cones extending into the regions D_1 and D_3 thereby shaping the electric field distribution (see fig. 4) in such a way as to reduce the transit time factor.

Below is a table of transit time factors calculated using equation (17) and the electric field distributions of figure 4 for various grid geometries (see sketch p. 19).

Configuration	Transit Time Factor, $F^{a)b)}$
plane grid	+ 13%
cone half angle 51.4°	- 3%
cone half angle 45°	- 2.3%
cone half angle 26.5°	- 1.0%

a) For all the cones the axial distance from the grid to the cone base was mantained at 8.0 cm.

b) Factors for ^{16}O, E_1 = 150keV, f = 12 MHz.

Thus the cones reduce the transit time factor considerably. Further calculations for rays 1.0 cm off axis but parallel to the axis showed that the transit time factors were only slightly different from the above results for the corresponding axial ray (see Fig. 6). Thus it appears that the more parallel the beam rays in passing through the gridded gap, the less the destruction in time resolution (modulation due to transit time effects). At present calculations are being carried out by J.C. Acquadro to determine the positioning of the different lenses in order that a beam of minimum divergence passes through the buncher which is positioned as close to the beam waist as is possible (about 1.2 meters upstream from the first accelerator electrode).

Finally we mention that the transit time factor is independent of the amplitudes of the maximum voltage applied to the gap. This has been verified computationally.

PART V - TIME RESOLUTION LIMIT DUE TO SPHERICAL ABERRATION IN THE HARMONIC BUNCHER

To calculate the effect of spherical aberration on the time resolution of the single gap harmonic buncher we first suppose that the modulation applied to the gap consists of one wave of amplitude V_0 and frequency f.

For a continuous beam of particles of mass M we consider a segment of total duration T (where T=1/f) entering the gap with energy E_0. At the gap exit different portions of this segment will have particle energies E_{Tot} determined by

$$E_{Tot} = E_0 + qV_0 \sin \omega t_B \qquad (18)$$

or

$$E_{Tot} = E_0 (1 + \alpha \sin \omega t_B)$$

where α represents the relative modulation $q \frac{V_0}{E_0}$ and t_B is the time associated with a particular portion of the segment relative to the time t_C associated with the center of the segment. Thus we define a phase $\phi_B = \omega t_B$ such that the entire segment is described by the condition $-\pi \le \phi_B \le \pi$.

The time at which a given portion of the segment of particles arrives at a time focus point situated an arbitrary distance L from the gap center is

$$t_L = t_C + t_B + \frac{L}{v(t_B)}$$

where $v(t_B) = \sqrt{\frac{2E_{Tot}}{M}} = KE_{Tot}^{1/2}$

and $K = \sqrt{\frac{2}{M}}$

From equation (18) we may write

$$t_L = t_C + t_B + \frac{L}{KE_{Tot}^{1/2}}$$

$$t_L = t_C + t_B + \frac{L}{KE_0^{1/2} (1 + \alpha \sin \phi_B)^{1/2}} \qquad (19)$$

Multiplying both sides of equation (19) by ω gives

$$\omega t_L = \omega t_C - \omega t_B + \frac{\omega L}{KE_0^{1/2} (1 + \alpha \sin \phi_B)^{1/2}} \qquad (20)$$

However, since the velocity v_0 of the unmodulated particles (center of segment) is given by $v_0 = KE_0^{1/2}$, by introducing $t_0 = \frac{L}{v_0}$ we get

$$\omega t_L = \omega t_C + \omega t_B + \frac{\omega t_0}{(1 + \alpha \sin \phi_B)^{1/2}}$$

or

$$\phi_L = \phi_C + \phi_B + \frac{\psi_{OT}}{(1 + \alpha \sin \phi_B)^{1/2}} \qquad (21)$$

By noting that $\phi_L - \phi_C = \Delta\phi$ where $\Delta\phi$ is the phase difference at the time focus of a given portion of the bunched segment with respect to the center of the bunch, we obtain a measure of the time resolution of the bunch at the time focus point through the relation $\Delta\phi = \omega(\Delta t)_{focus}$. Since the terminal stripper has been chosen as the focus point we write

$$\Delta\phi_{strip} = \phi_B + \frac{\psi_{OT}}{(1 + \alpha \sin \phi_B)^{1/2}} \qquad (22)$$

where we recall that $-\pi \le \phi_B \le \pi$ and $\psi_{OT} = \omega t_0 = \frac{\omega L}{KE_0^{1/2}}$.

A convenient expression for calculating ψ_{OT} is easily shown to be

$$\psi_{OT} = \frac{14.312\ fL}{\left(\frac{E_0}{A}\right)^{1/2}} \qquad (23)$$

if f is in MHz, L in meters, E_0 in keV and A in amu.

For a single harmonic gap to which three frequencies are applied (f, 2f, 3f with respective amplitude (V_0, V_1 and V_2) the expression for the phase spread in the bunch at the time focus is

$$\Delta\phi_{strip} = \phi_B + \frac{\psi_0}{\left[1 + \alpha \sin \phi_B - \beta \sin (2\phi_B) + \gamma \sin (3\phi_B)\right]^{1/2}} \qquad (24)$$

where

$$-\pi \le \phi_B \le \pi, \quad \alpha = \frac{V_0}{E_0}, \quad \beta = \frac{V_1}{E_0}, \quad \gamma = \frac{V_2}{E_0}$$

and ψ_{OT} is given by equation (23). The minus sign in the even harmonic is a result of the composition of the sine wave and its first two harmonics to form a sawtooth-like waveform at the gap.

Figure 7 is a sketch of a set of typical relative amplitudes of the three waves applied to the harmonic buncher. The solid parts of the curves would have a convergent action and the dashed parts a divergent action on the particles in the bunch at the time focus.

It is possible to extend the parametrization of equation (24) by allowing phase shifts Δ_2 and Δ_3 in the first two harmonics with respect to the fundamental wave. Thus we write equation (24)

$$\Delta\phi_{strip} = \phi_B + \frac{\psi_{OT}}{\left[1 + \alpha \sin \phi_B - \beta \sin (2\phi_B + \Delta_2) + \gamma \sin (3\phi_B + \Delta_3)\right]^{1/2}} \qquad (25)$$

A computer code (SPHERAB) was written to calculate the phases at the terminal stripper of an original beam segment of duration T entering the single harmonic buncher using the algorithm of equation (25).

Figure 8 shows a graph of the phase at the time focus $\Delta\phi_{strip}$ as a function of the particle phase in the original beam segment ϕ_B (relative to the segment center). The input data used to obtain this result is given in the table below for ^{16}O ions injected into the gap with the optimum energy 74 keV of figure 3.

A	= 16 amu
E_0	= 74 keV
L	= 1.28 meters
f	= 12.0 MHz

α = .0334	β = .0100	γ = .0022	Δ_3 = 0
V_0 = 2.472kV	V_1 = 0.740kV		V_2 = 0.163kV

As shown in equation (9b) the time resolution characteristics of the ATLAS prebuncher are described by

$$\Delta\tau_a = \alpha/\omega$$

Where ω is the fundamental frequency applied to the gap (12MHz) and the value of α obtained is α = .06 radians, defining a fwhm.

The area in figure 8 delineated by the dashed lines corresponds to $\Delta\tau$ = 0.8 nanoseconds or α = .06 radians = ± 1.7° (fwhm) for the acceptable phase variation at the stripper terminal.

The values of ϕ_B which fall within this region show that an efficiency, p, of 60% in the beam bunching may be expected for the modulation parameters used in this case.

The ATLAS group reports values of p up to 75% for this time resolution . More precise parametrization of equation (25) would give this value for the three wave harmonic gap.

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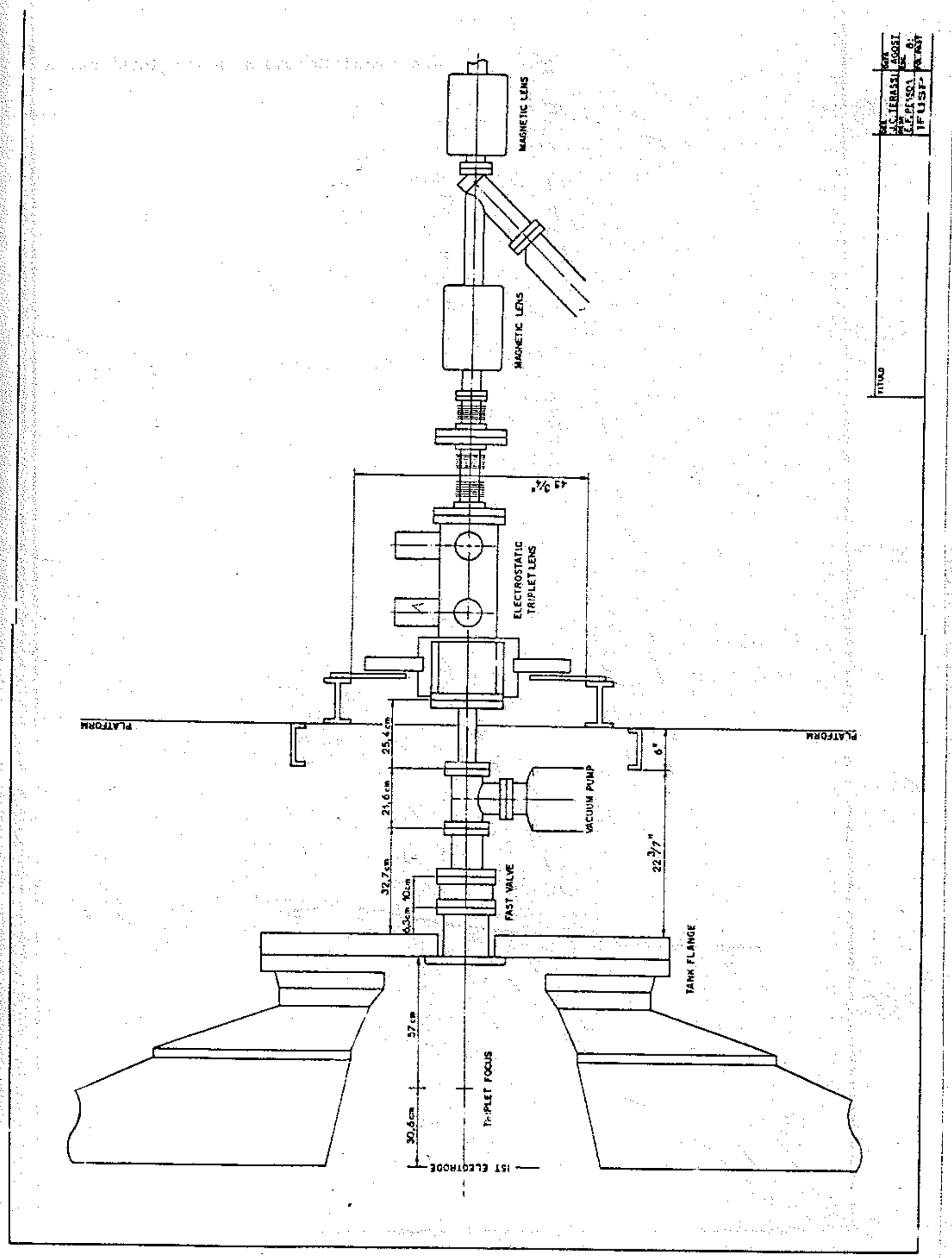


FIGURE 1

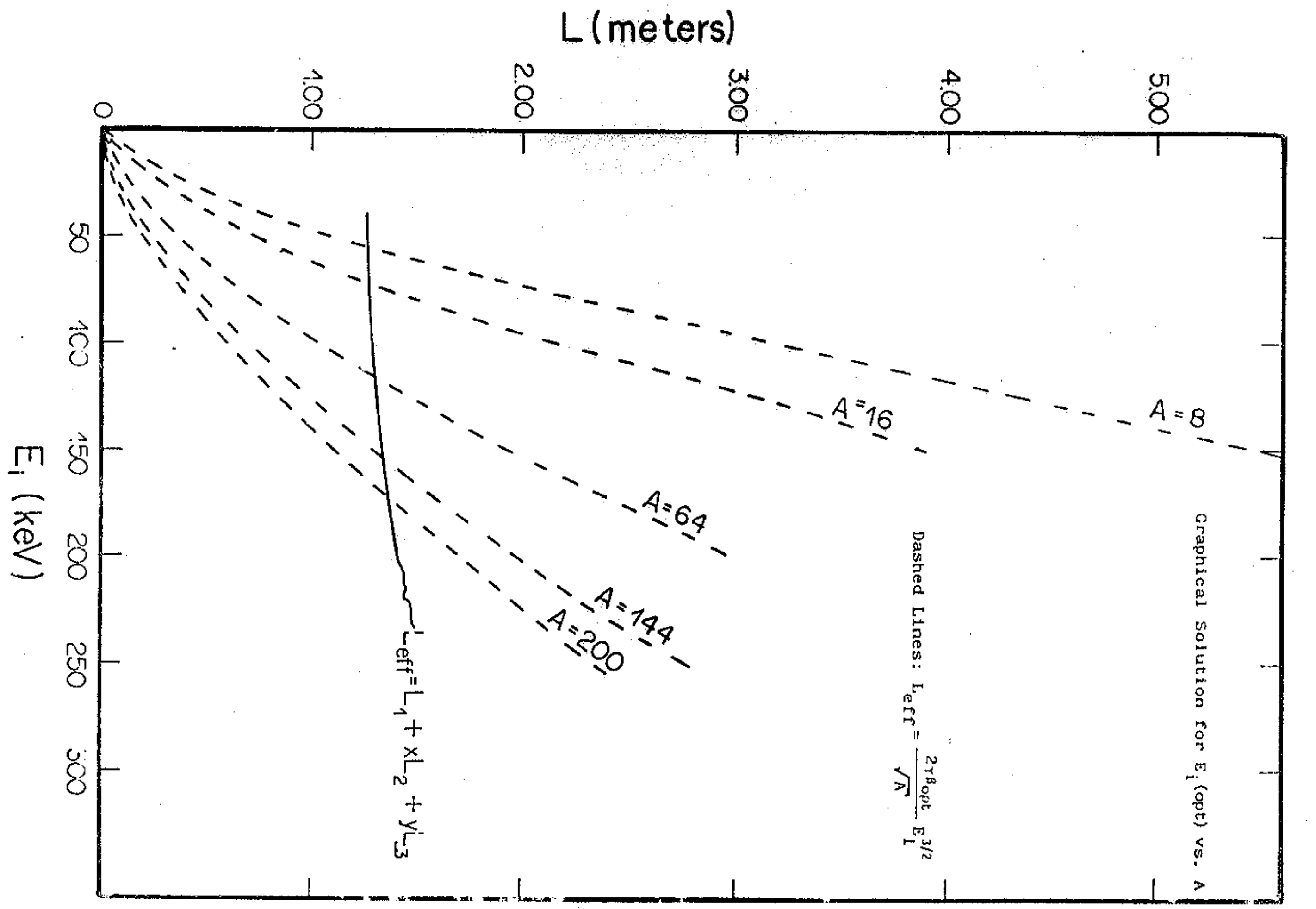


FIGURE 2

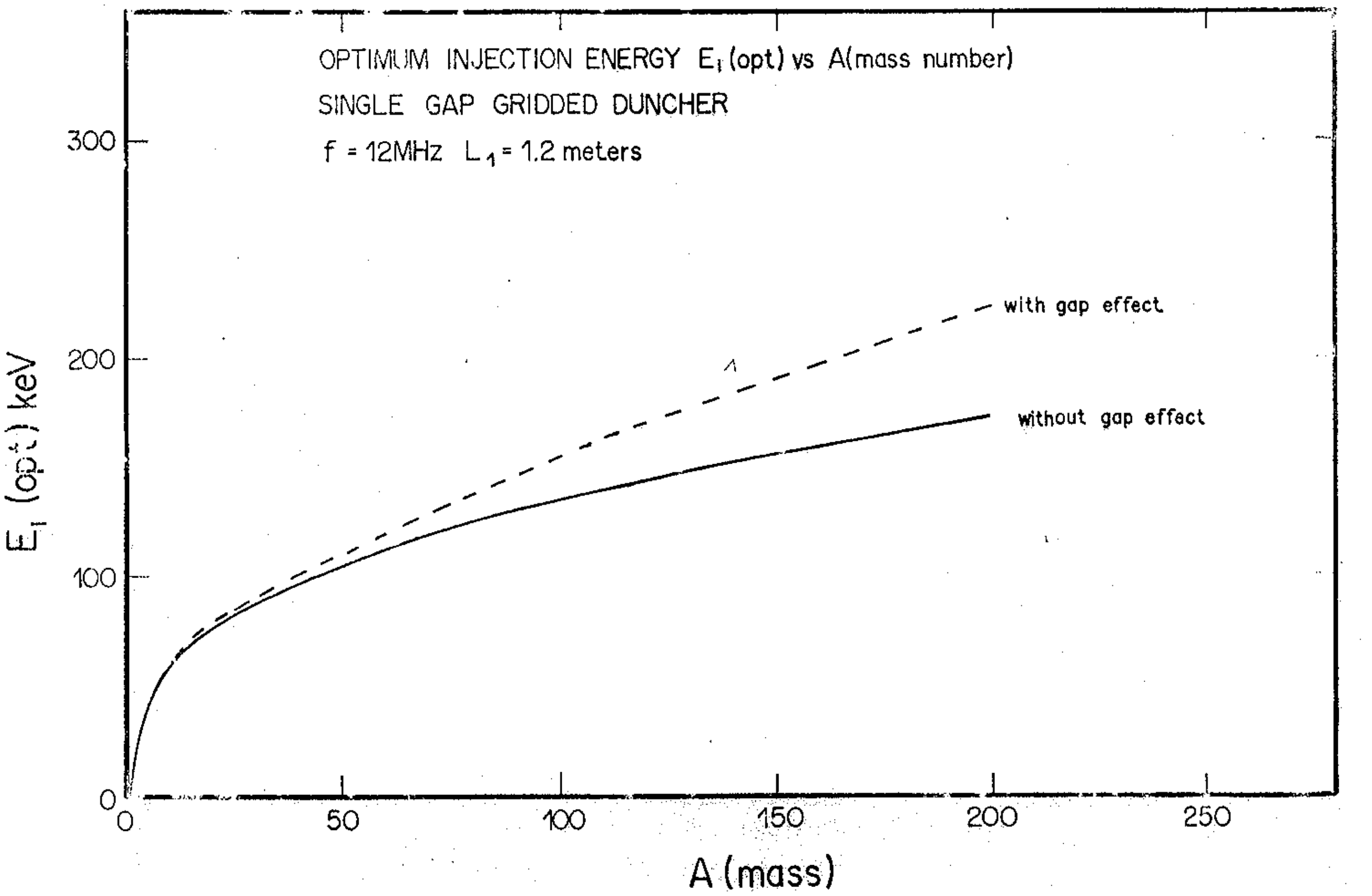


FIGURE 3

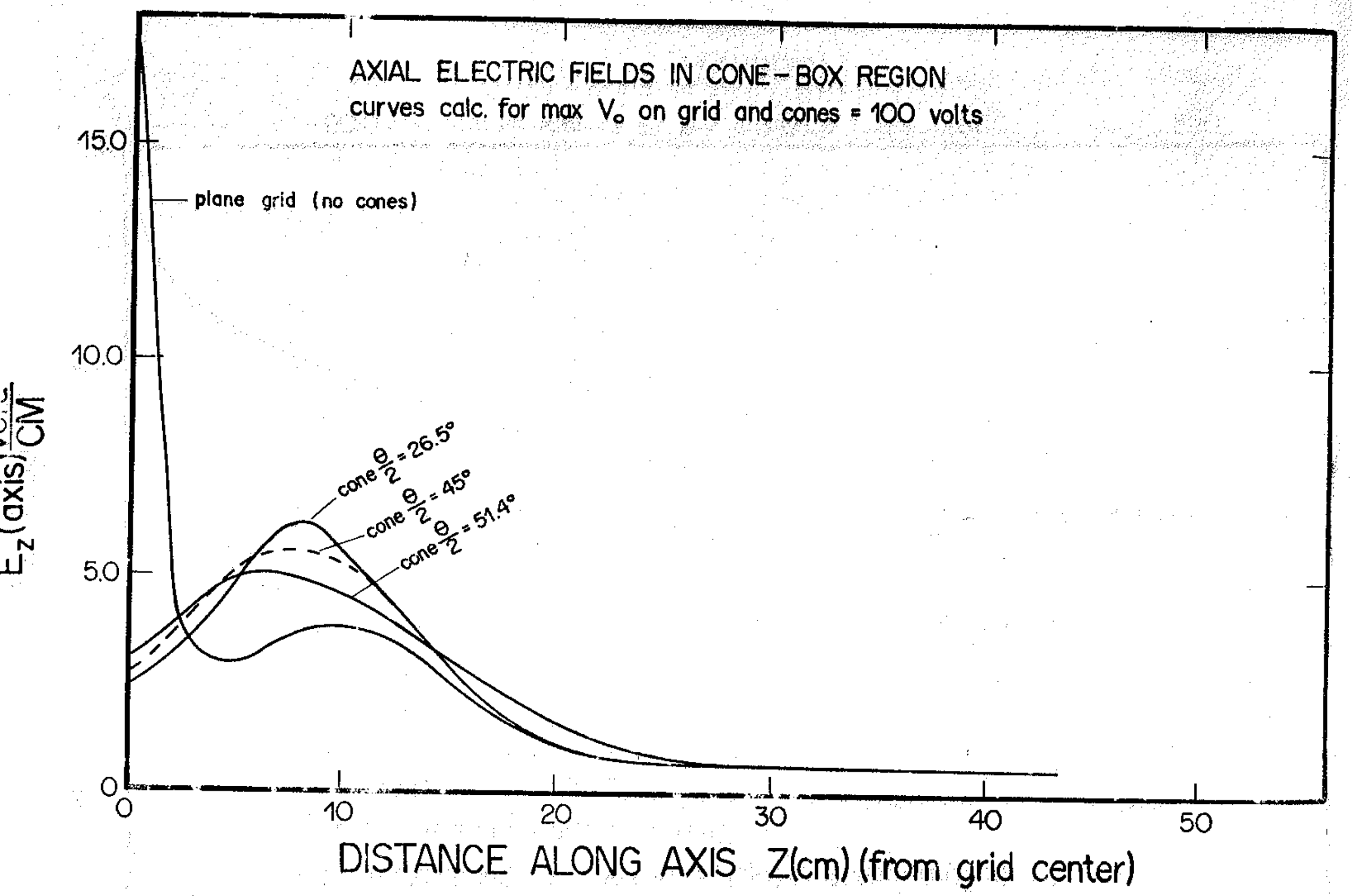


FIGURE 4

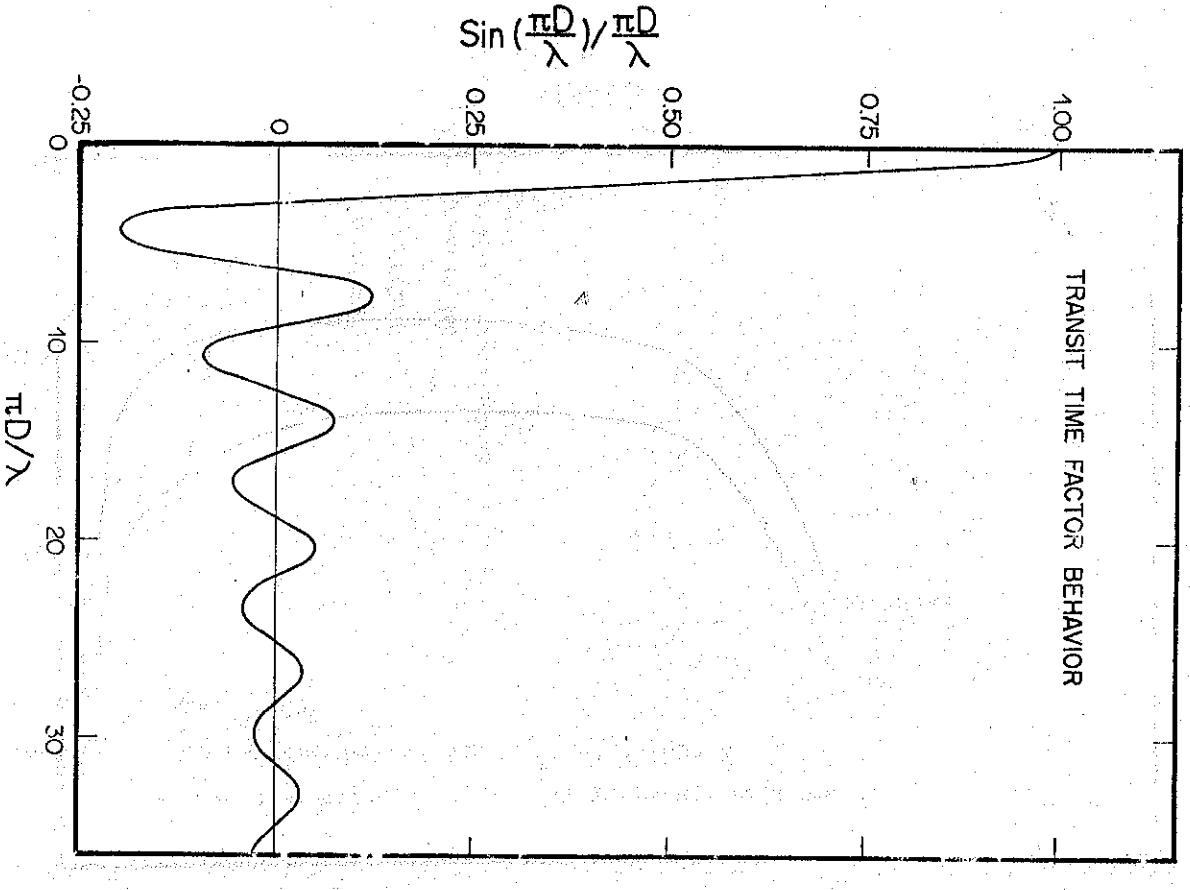


FIGURE 5

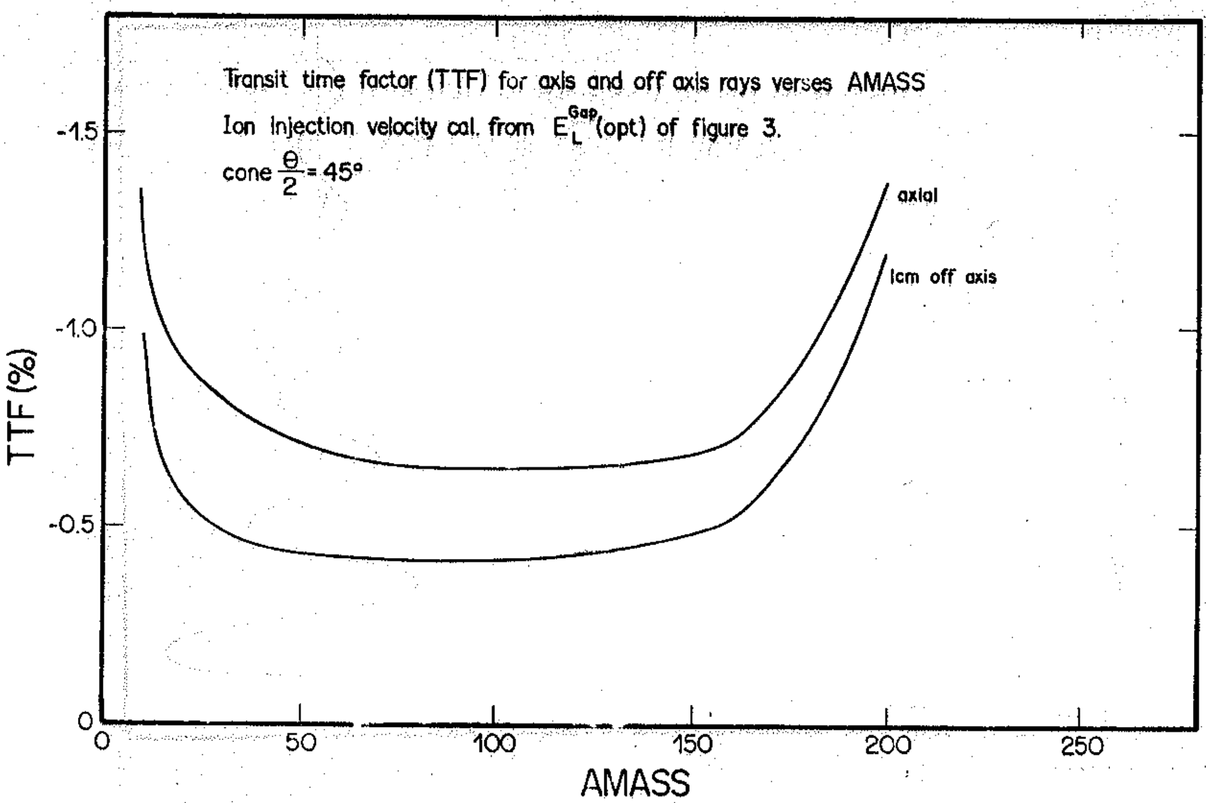


FIGURE 6

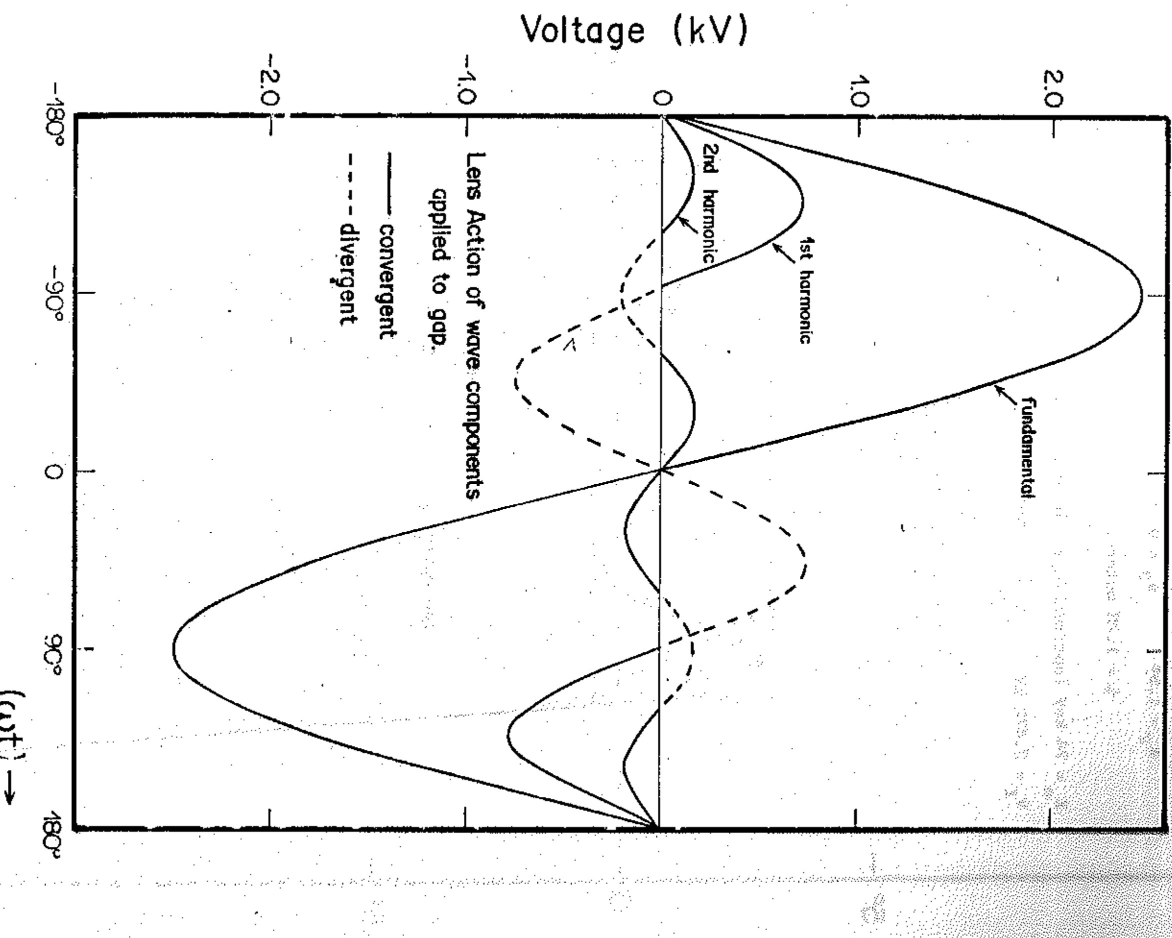
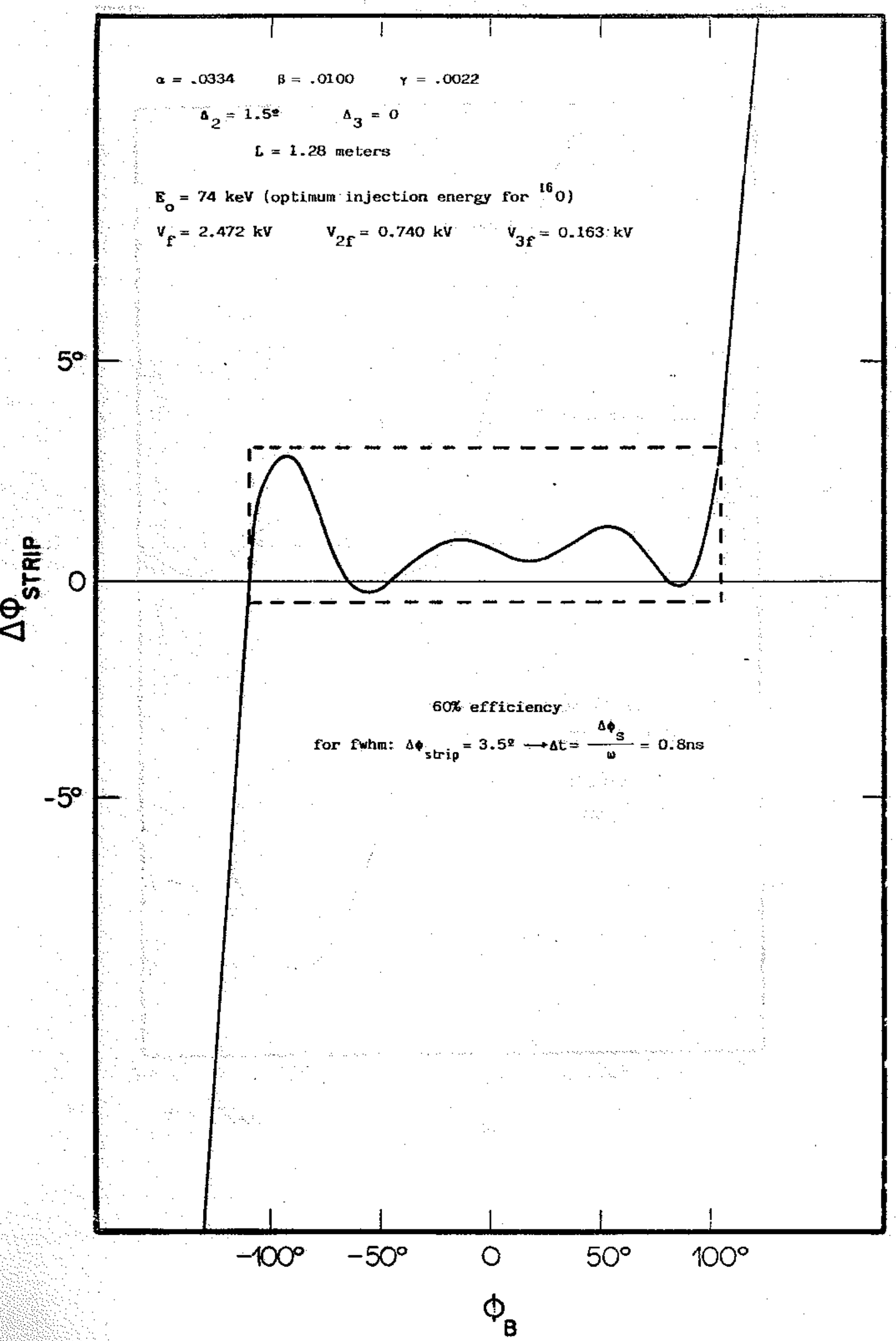


FIGURE 7



α = .0334 β = .0100 γ = .0022
Δ2 = 1.5° Δ3 = 0
L = 1.28 meters
Eo = 74 keV (optimum injection energy for 16O)
Vf = 2.472 kV V2f = 0.740 kV V3f = 0.163 kV
5°
0
-5°
ΔΦSTRIP
60% efficiency
for fwhm: Δφstrip = 3.5° → Δt = Δφs/ω = 0.8ns
-100° -50° 0 50° 100°
ΦB