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IFUSP/P-826

THE STRUCTURE OF ^{11}Li: A "RYDBERG" NUCLEUS?

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Fevereiro/1990

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ABSTRACT

The effective long range R^{-2} interaction between ^{9}Li and the two neutrons (slightly bound) is taken here responsible for the loosely bound ^{11}Li nucleus. A large number of these loosely bound Rydberg states is expected to be generated. Similarities to Efimov states are pointed out.

*Supported in part by the CNPq.

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February/1990

Recently, several experimental groups have investigated and measured the interaction cross–section of secondary radioactive beams such as ^{11}Li and ^{14}Be[1] at $E = 0.8$ GeV/A. Among the several features of the experimental data a particularly interesting one is related to the momentum distribution of ^{9}Li fragments originating from the reaction $^{11}\text{Li} + \text{target} \longrightarrow {}^9\text{Li} + X$. These fragments originates from peripheral reactions and give information about the nuclear matter distribution near the surface of ^{11}Li.

The perpendicular momentum distribution of the ^{9}Li fragments shows a "two–peak" structure[2] with a narrow peak of a width $\Delta_N \simeq 20$ MeV/c sitting on top of a wide peak of a width of $\Delta \simeq 95$ MeV/c [3]. Whereas the wider momentum distribution can be well accounted for by the Goldhaber model[4], the narrow momentum distribution requires the existence of a neutron (2n) halo about of ^{9}Li core. The radius of this halo is ~ 6.5 fm, much larger than the radius of ^{9}Li (~ 2.5 fm). Related to the above is the fact that the separation energy of the "halo" neutrons is $S_{2n} \cong 0.2$ MeV, while that of only one neutron from the halo is $S_{1n} \cong 1$ MeV. This implies a pairing energy of the dineutron system of about 0.8 MeV.

It is obvious from the above facts that subtle three–body dynamics is involved in generating the halo structure of ^{11}Li. In particular, we remind the reader that a bound 2n system in free space does not exist, a bound ^{9}Li+n system does not exist, and yet the ^{9}Li+2n system is bound, albeit slightly. Migdal[5], back in 1973, addressed a question related to the above. It is of importance to reexamine the above, and eventually to understand the mechanism that generates the effective binding of the halo dineutron system in ^{11}Li. This is the purpose of the present paper.

Let us for the moment, take ^{9}Li to be structureless and consider all the three particles, ^{9}Li, n and a to be bosons. The Hamiltonian that describes ^{11}Li then is given by

$$H = H_0 + U_{n^9\text{Li}} + U_{n'^9\text{Li}} + V_{nn'} , \quad (1)$$

where U_{n^9Li} is the mean field felt by neutron n, and $V_{nn'}$ is the nn' interaction (we are calling the two halo neutrons by n and n'). The background Hamiltonian H describes the intrinsic structure of the three particles. In what follows, we use arguments given by Landau and Lifshitz[6] and Efimov[7].

If the nn' system forms a quasi–bound state, then the wave function of the bound ^{9}Li+2n system is given by

$$\psi_{^{11}Li(^9Li+2n)} \approx \varphi(R)\,\frac{e^{-\alpha r}}{r}\sqrt{\frac{\alpha}{2\pi}} \tag{2}$$

where R is the distance between the core and the center of mass of the dineutron and r is the relative coordinate of the 2n system. α is related to the binding energy (almost zero!) of the 2n system.

The effective Schrödinger equation of $\varphi(R)$ is then given by

$$-\frac{\hbar^2}{4m_N}\nabla_R^2 + \frac{\alpha}{4\pi}\int d^3r\,\frac{e^{-2\alpha r}}{r^2}\left\{U_{nA}\left[|\vec{R}+\frac{\vec{r}}{2}|\right] + U_{n'A}\left[|\vec{R}-\frac{\vec{r}}{2}|\right]\right\}\varphi =$$

$$= \left[E - \frac{\alpha^2\hbar^2}{m_N}\right]\varphi(R)\ . \tag{3}$$

Therefore an effective dineutron–core interaction can be defined from the above equation

$$U_{eff}(R) = \frac{\alpha}{4\pi}\int d^3r\,\frac{e^{-2\alpha r}}{r^2}\left[U_{nA}\left[|\vec{R}+\frac{\vec{r}}{2}|\right] + U_{n'A}\left[|\vec{R}-\frac{\vec{r}}{2}|\right]\right] \approx \tag{4}$$

$$\approx U_n(R) + \frac{1}{16\alpha^2}\,\frac{U_n'(R)}{R}\ . \tag{5}$$

Equation (5) was obtained by taking the limit $\frac{R}{r} \gg 1$ and keeping the lowest–order term. No restriction on the value of α was assumed. In the limit $\alpha \to 0$, however, a different effective interaction is obtained,

$$U_{eff}(R) \approx \frac{\alpha}{4\pi}\,\frac{\int d^3r[U_{nA}(r)+U_{n'A}(r)]}{R^2}\ . \tag{6}$$

The radial Schrödinger equation for $\varphi(R)$ then takes the form

$$\left[-\frac{d^2}{dR^2} + U_{eff}(R)\right]\frac{\varphi(R)}{R} = \varepsilon\,\frac{\varphi(R)}{R} \tag{7}$$

where ε is the binding energy of the $2n - {}^9Li$ system, which we drop.

With Eq. (6), the effective potential, $U_{eff}(R)$, becomes

$$U_{eff}(R) \cong \frac{\lambda}{R^2}\ , \qquad \text{large enough } R \tag{8}$$

$$\lambda = \frac{m_N}{2\hbar^2}\,\frac{\alpha}{\pi}\int d^3r\left[U_{nA}(r) + U_{n'A}(r)\right] - \ell(\ell+1)\ . \tag{9}$$

The solution to Eq. (7) is given in Landau and Lifshitz and we give here the results. If $\lambda > \frac{1}{4}$, the potential U_{eff} produces an infinite number of bound states condensed at zero energy. This condition on the value of λ, supplies us with the minimum value of the energy of the dineutron

$$\alpha = \frac{\hbar^2\pi[1+4\ell(\ell+1)]}{m_N\ 2\int d^3r[U_{nA}(r)+U_{n'A}(r)]}\ . \tag{10}$$

Using (10) we can estimate the value of the binding energy of the dineutron in the ^{11}Li nucleus. We take for $U_{nA} \cong U_0 \rho_{nA}$ with $U_0 = 50$ MeV and $\rho_n = \rho_p \cong 0.1$ fm^{-3} as given by the Hartree–Fock calculation of Bertsch et al.[8]. With these numbers we obtain

$$\begin{aligned} \varepsilon_{2n} &= 1.4 \text{ keV} & \ell &= 0 \\ &= 111.0 \text{ keV} & \ell &= 1 \\ &= 800.0 \text{ keV} & \ell &= 2 \ . \end{aligned} \tag{11}$$

Note that the orbital angular momentum refer to that of the Rydberg ^{9}Li+2n states. We remind the reader that the experimental value of two–neutron pairing energy is about 870 keV.

To recapitulate, we see from the above discussion that just a slight change in the binding energy of the dineutron is enough to produce these Rydberg states in ^{11}Li .

It would also seem possible that other Rydberg states with lower binding energies would appear in the ^{11}Li nucleus due the long–range nature of the effective potential.

Finally one wonders how does the dineutron acquires binding. This can presumable be related to the modification of the N–N potential at the surface of the nucleus and slight increase in the N–N interaction in the singlet state can provide the binding to the dineutron and gives origin to the Rydberg states of exotic nuclei.

ACKNOWLEDGMENTS

We thank G. Bertsch and B.A. Brown for providing us with the Hartree–Fock densities.

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