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Abstract:– It is shown that the three–body model of inclusive inelastic break–up ("break–up fusion", BF), reactions advanced recently by Austern et al., leads to a serious ambiguity. The use of two formally equivalent representation of the break–up transition amplitude, is shown to lead to manifestly different expressions for the BF cross–section. The cause of the ambiguity is traced to the many–body effects which are ignored in the three–body model.

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Recently, Austern et al.[1)] have developed a three–body model to describe the nuclear inclusive break–up reaction,

$$a+A \equiv (b+x) + A \longrightarrow b + \sum_{\text{all states}} (x+A)$$

where a and b are taken to be structureless. The cross–section derived by 1) has the following general structure

$$\frac{d^2\sigma}{d\Omega_b dE_b} = \frac{d^2\sigma EB}{d\Omega_b dE_b} + \frac{d^2\sigma BF}{d\Omega_b dE_b} \tag{1}$$

where the elastic break–up piece, $\frac{d^2\sigma EB}{d\Omega_b dE_b}$ is given by

$$\frac{d^2\sigma EB}{d\Omega_b dE_b} = \frac{2\pi\rho(E_b)}{\hbar v_a} \sum_{\vec{k}_x} |<\chi_x^{(-)}\, \chi_b^{(-)}|V_{bx}|\Psi_{3b}^{(+)}>|^2 \tag{2}$$

and the inclusive inelastic break–up, or break–up–fusion (BF) piece $\frac{d^2\sigma BF}{d\Omega_b dE_b}$ is represented by

$$\frac{d^2\sigma BF}{d\Omega_b dE_b} = -\frac{2\rho(E_b)}{\hbar v_a} \int <\Psi_{3b}^{(+)}|\chi_b^{(-)})\, W_{xA}\, (\chi_b^{(-)}|\Psi_{3b}^{(+)}>\, d^3r_x \tag{3}$$

In the above formulae, $\rho(E_b)$ is the density of final state of the spectator particle b, v_a the velocity of the projectile. The χ,s are distorted wave, V_{bx} is the real x–b interaction potential, W_{xA} the imaginary part of the x–A optical potential and $\Psi_{3b}^{(+)}$ is the b–x–A three–body wave–function calculated with appropriate optical potentials U_{xA}, U_{bA} and with V_{bx}.

Recently, Frederico et al.[2] and Hussein et al.[3] have analyzed the Austern cross–sections, Eqs. (1), (2) and (3) with the Faddeev equations. Related works were done by Ichimura[4] and Austern et al.[5]. The three–body model is an important advance in the development of a practical theory for inclusive break–up reactions. Further, from 2) and 3), it was possible through the use of Ψ_{3b} and the Faddeev equation, to establish interrelationship among several different theories of IB such an that of Udagawa et al.[6] and Hussein and McVoy[7].

It is the purpose of this paper to supply further analysis of the Austern formula and in particular, to point out an important ambiguity inherent in it. This ambiguity, which is derived in details below, is connected with Eq. (3) and asserts that, with no other assumptions other than those used in deriving the Austern formula, it is possible to re–write Eq. (3), with the overlap $<\Psi_{3b}^{(+)}|\chi_b^{(-)})$ replaced by $<\Psi_{3b}^{(+)}|\vec{k}_b)$. This is clearly a serious problem since $|\chi_b^{(-)}>$ is very different from $|\vec{k}_p>$. The ambiguity is traced to explicit many–body effects completely ignored in the three–body model.

To begin, we write below the starting point for the derivation of Eq. (1),

$$\frac{d^2\sigma}{d\Omega_b dE_b} = \frac{2\pi\rho(E_b)}{\hbar v_a} \sum_c |<\chi_b^{(-)}\,\Psi_{xA}^c|V_{xb}|\Xi\,(\vec{r}_x,\vec{r}_b,A)>|^2\,\delta(E-E_x-E^c) \qquad (4)$$

where Ψ_{xA}^c is the exact wave function of the xA system, and Ξ is the exact 2+A body wave function describing the x–b–A microscopic interaction.

Another, equivalent form, for $\frac{d^2\sigma}{d\Omega_b dE_b}$ can be derived easily

$$\frac{d^2\sigma}{d\Omega_b dE_b} = \frac{2\pi\rho(E_b)}{\hbar v_a} \sum_c |<\vec{k}_b\,\Psi_{xA}^c|(V_{xb}+U_b)|\Xi\,(r_x,r_b,A)>|^2\,\delta(E-E_x-E^c) \qquad (5)$$

where U_b is the optical potential of the spectator particle b.

To derive the Austern formula from Eq. (4), we proceed as follows. We write the following decomposition for the many–body wave function Ξ

$$\Xi = P\,\Xi + Q\,\Xi\ , \qquad (6)$$

where

$$P\,\Xi = \Psi_{3b}^{(+)}\,\Phi_A^0 \qquad (7)$$

where Φ_A^0 is the ground state wave function of the targe nucleus. $P\,\Xi$ represents the Austern approximation for Ξ. With this approximation and several manipulations including an exact optical reduction, one obtains Eqs. (1)–(3). For details see Austern et al.[1] and Hussein et al.[3]. It is important to note that with the optical reduction, all reference to the target in Eqs. (2)–(3) is contained implicitly in the optical potentials that enter in the equation that determines $\Psi_{3b}^{(+)}$.

We turn now to Eq. (5), and evaluate the cross–section with the Austern approximation $\Xi \simeq P\,\Xi$, Eq. (7). We rewrite then, Eq. (5) as

$$\frac{d^2\sigma}{d\Omega_b dE_b} = \frac{2\pi\rho(E_b)}{\hbar v_a} \sum_c <\Psi_{3b}^{(+)}\,\Phi_A^0\,|(V_{xb}+U_b^{\dagger})|\vec{k}_b\,\Psi_{xA}^c><\vec{k}_b\,\Psi_{xA}^c|\,\cdot$$

$$\cdot\,(V_{xb}+U_b)|\Psi_{3b}^{(+)}\,\Phi_A^0>\,\delta(E-E_x-E^c)\ . \qquad (8)$$

Upon using the identity $\delta(E-E_x-E^c) = -\,\mathrm{Im}\,\frac{1/\pi}{E-E_x-E^c+i\epsilon}$ and $<\Psi^c_{xA}|\,\frac{1}{E-E_x-E^c+i\epsilon} = \frac{1}{E-E_b-K_x-V_{xA}-H_A+i\epsilon}<\Psi^c_{xA}| \equiv G^{(+)}_{xA}<\Psi^c_{xA}|$, we have

$$\frac{d^2\sigma}{d\Omega_b dE_b} = -\frac{2}{\hbar v_a}\rho(E_b)<\Psi^{(+)}_{3b}|(V_{xb}+U^{\dagger}_b)|\vec{k}_b>$$

$$\times\, G^{(+)^{\dagger}}_x(E_x)\, W_{xA}\, G^{(+)}_x(E_x)<\vec{k}_b|(V_{xb}+U_b)|\Psi^{(+)}_{3b}> \qquad (9)$$

where the relation $\sum_c |\Psi^c_{xA}><\Psi^c_{xA}| = 1$ has been used and the optical x–particle Green's function $G^{(+)}_x(E_x) = \frac{1}{E_x-K_x-U_x+i\epsilon} \equiv <\Phi^0_A|G^{(+)}_{xA}|\Phi^0_A>$. Further, the identity, $\mathrm{Im}\,G^{(+)}_x = -\,G^{(+)^{\dagger}}_x W_{xA}\, G^{(+)}_x - \Omega^{(+)^{\dagger}}_x\, \mathrm{Im}\, G^{(+)}_0\, \Omega^{(+)}_x$ has been used. We now employ the identities[8]

$$G^{(+)}_x(\vec{k}_b|(V_{xb}+U_b)|\Psi^{(+)}_{3b}> = G^{(+)}_x G^{(+)^{-1}}_x(\vec{k}_b|\Psi^{(+)}_{3b}> = (\vec{k}_b|\Psi^{(+)}_{3b}> \qquad (10)$$

and[9]

$$<\chi^{(-)}_x \vec{k}_b|(V_{xb}+U_b)|\Psi^{(+)}_{3b}> = <\chi^{(-)}_x\, \chi^{(-)}_b|V_{xb}|\Psi^{(+)}_{3b}> \quad , \qquad (11)$$

to obtain finally for the cross–section the form of Eq. (1), with the EB one being the same as before, except that the BF component is now given by

$$\frac{d^2\sigma^{BF}}{d\Omega_b dE_b} = -\frac{2\rho(E_b)}{\hbar v_a}\int <\Psi^{(+)}_{3b}|\vec{k}_b)\, W_{xA}(\vec{k}_b|\Psi^{(+)}_{3b}>\, d^3\vec{r}_x \qquad (12)$$

To summarize, we have started with two alternative representations of the same transition matrix element $<\chi^{(-)}_b\, \Psi^c_{xA}|V_{xb}|\Psi^{(+)}_{3b}\, \Phi^0_A>$ and $<\vec{k}_b\, \Psi^c_{xA}|(V_{xb}+U_b)|\Psi^{(+)}_{3b}\, \Phi^0_A>$ and obtained two entirely different forms for the break–up–fusion cross–section, Eqs. (3) and (12). The elastic break–up cross–section is invariant with respect to the particular representation employed for the transition matrix element.

One particularly important difference between the two representations of $\frac{d^2\sigma^{BF}}{d\Omega_b dE_b}$ is that of unitarity. Integrating Eq. (12) over $\vec{k}_b$ immediately yields

$$\sigma^{BF}_{PW} = -\frac{2\rho(E_b)}{\hbar v_a}<\Psi^{(+)}_{3b}|W_{xA}|\Psi^{(+)}_{3b}> \qquad (13)$$

which represents the total reaction cross–section associated with the flux lost to the break–up channels. On the other hand the integral $\int \frac{d^2\sigma^{BF}}{d\Omega_b dE_b}\frac{d\vec{k}_b}{(2\pi)^3}$ cannot be carried out easily since $|\chi^{(-)}_b><\chi^{(-)}_b|$ does not form a complete set even if no bound state in the bA optical potential is permitted. This is so owing to the non–Hermitian nature of U_b. An approximate closed expression may be obtained by the transformation[10]

$$\int \frac{d\vec{k}_b}{(2\pi)^3}|\chi^{(-)}_b(\vec{k}_b)><\chi^{(-)}_b(\vec{k}_b)| =$$

$$= \int \frac{d\vec{k}_b}{(2\pi)^3}\left[|\tilde{\chi}^{(-)}_b(\vec{k}_b)> + G^{(-)}_b(U_b-U^{\dagger}_b)|\tilde{\chi}^{(-)}_b(\vec{k}_b>\right]<\chi^{(-)}_b(\vec{k}_b)| \quad ,$$

$$= 1 + \int \frac{d\vec{k}_b}{(2\pi)^3}\, G^{(-)}_b(U_b-U^{\dagger}_b)|\tilde{\chi}^{(-)}_b(\vec{k}_b><\chi^{(-)}_b(\vec{k}_b)|$$

when $|\tilde{\chi}>$ denotes the dual state of $|\chi>$, which results in the following

$$\int \frac{d\vec{k}_b}{(2\pi)^3} \langle \Psi_{3b}^{(+)} | \chi_b^{(-)}) \, W_{xA} \, (\chi_b^{(-)} | \Psi_{3b}^{(+)} \rangle = \langle \Psi_{3b}^{(+)} | W_{xA} | \Psi_{3b}^{(+)} \rangle +$$

$$+ \int \frac{d\vec{k}_b}{(2\pi)^3} \langle \Psi_{3b}^{(+)} | G_b^{(-)}(E_k)(U_b - U_b^{\dagger}) | \tilde{\chi}_b^{(-)}(\vec{k}_b)) \, W_{xA} (\chi_b^{(-)}(\vec{k}_b) | \Psi_{3b}^{(+)} \rangle \quad . \qquad (14)$$

Therefore the "plane–wave" total inclusive inelastic break–up cross–section obtained from Eq. (12)' is related to the Austern ("distorted–wave") total inclusive inelastic break–up cross–section, Eq. (3), through the following equation

$$\sigma_{PW}^{BF} = \sigma_{Austern}^{BF} - \frac{4i\rho(E_b)}{\hbar v_a} \int \langle \Psi_{3b}^{(+)} | G_b^{(-)}(E_k) \, W_{bA} | \tilde{\chi}_b^{(-)}(\vec{k}_b)) \times$$

$$\times \, W_{xA} (\chi_b^{(-)}(\vec{k}_b) | \Psi_{3b}^{(+)} \rangle \frac{d\vec{k}_b}{(2\pi)^3} \quad . \qquad (15)$$

Clearly, $\sigma_{PW}^{BF} \neq \sigma_{Austern}^{BF}$, although the starting point of the derivation of the cross–section is formally identical (see previous discussion). Note that the second term on the RHS of Eq. (15), although not manifestly real, must come out to be real, since the Austern cross–section, Eq. (3) is obviously real!

It is obvious that the origin of the above ambiguity is the neglect of $Q\,\Xi$ in Eq. (6). This component of the wave function is the one that carries information about the many–body nature of the inclusive break–up process. The explicit consideration of $Q\,\Xi$, should resolve the ambiguity. However, this clearly brings in a vicious circle since the resulting theory is then full–fledged many –body theory quite apparently unresolvable. A more modest approach such as that of Hussein and McVoy[6] or of Baur et al.[11] should be the alternative for a practical theory.

In conclusion, we have pointed out in this paper a serious problem with the three–body theory of inclusive break–up reactions advanced recently by Austern et al.[1]. The solution of this problem would necessarily take us to the realm of many–body complications. A viable alternative would be those theories based on simple DWBA description of the incident channel.

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8) In detail, Eq. (10) can be derived as follows

$$G_x^{(+)}(\vec{k}_b|(V_{xb}+U_b)|\Psi_{3b}^{(+)}> =$$

$$= G_x^{(+)}(\vec{k}_b|\,[(V_{xb}+U_b+K_b+K_x+U_x-E)-(K_b+K_x+U_x-E)]\,|\Psi_{3b}^{(+)}>$$

$$= G_x^{(+)}(\vec{k}_b|-(K_b+K_x+U_x-E)\,|\Psi_{3b}^{(+)}>$$

$$= G_x^{(+)}\,G_x^{(+)^{-1}}\,(\vec{k}_b|\Psi_{3b}^{(+)}> + G_x^{(+)}\,(\vec{k}_b|(\overleftarrow{K}_b-\overrightarrow{K}_b)|\Psi_{3b}^{(+)}> \;=\; (\vec{k}_b|\Psi_{3b}^{(+)}> \;.$$

In the above development, K refers to the kinetic energy operator, with respect to the target A. The term $G_x^{(+)}\,(\vec{k}_b|(\overleftarrow{K}_b-\overrightarrow{K}_b)|\Psi_{3b}^{(+)}>$ can be shown to be vanishingly small using the convergence factor argument of M. Ichimura et al., *Phys. Rev.* **C32**, 431 (1985). In fact, Austern et al.[1] derived the cross–section, Eq. (3), using a similar argument for $G_x^{(+)}\,(\chi_b^{(-)}|(\overleftarrow{K}_b-\overrightarrow{K}_b)|\Psi_{3b}^{(+)}>$.
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