UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE FÍSICA
CAIXA POSTAL 20516
01498 - SÃO PAULO - SP
BRASIL

PUBLICAÇÕES

IFUSP/P-838

BRST QUANTIZATION OF SPINNING RELATIVISTIC PARTICLES WITH EXTENDED SUPERSYMMETRIES

M. Pierri and **V.O. Rivelles**
Instituto de Física, Universidade de São Paulo

Abril/1990

Using the representation (26) for the γ_i and multiplying by the same combination of gamma matrices as in (27) we get precisely the propagator (25).

Therefore for theories of relativistic particles with N extended supersymmetries the transition amplitude should be identified with the propagator for the field strenght of the theory and not for their potentials. This can be understood from the Dirac quantization where upon second quantization the wave function turns out to be the field strenght [5] and not the potential and the equations of motion involve only the field strenghts. This is quite different from the cases $N = 0$ and 1 where we find the propagators for the fundamental fields.

In general a bosonic [fermionic] massless particle with spin s is described by a potential field which is a symmetric tensor [spinor-tensor] of order s $[s - 1/2]$. Its field strenght will have s $[s - 1/2]$ derivatives so that the propagator for the field strenght $\langle F_{\mu_1 \ldots}, F_{\nu_1 \ldots}\rangle$ will have $2s - 2$ powers of the momenta. Comparing this with (24) we conclude that it describes the propagator for the field strenght with spin $s = \frac{N}{2}$ after appropriate multipliplications by gamma matrices.

As we have shown the gauge symmetry of the field theory does not manifest itself in this formulation. Also in the Dirac quantization the gauge symmetry is hidden in the Bianchi identities (which are equations of motion there). Only when the Bianchi identities are solved in terms of the potentials is that the gauge symmetry becomes manifest. Although we have started with a particle theory with several local symmetries (diffeomorphisms, supersymmetries and internal $O(N)$ symmetry) neither of them gave origin to an explicity gauge symmetry for the field theory. This may well explain why we can not find the fundamental gauge symmetry for string theories since there too we start from a first quantized theory. Other approaches to this problem (based on string theory) starts with the particle coordinate x^μ and a set of bosonic or fermionic oscillators acting on a Fock space [12]. This approach is essentially operatorial and makes it difficult to interpret the mechanical system that it describes (except for the case of lower spin). Also it is not clear how to make the path integral quantization of this class of models. This shows that we are still in need of a satisfactory prescription to find the local symmetries of a second quantized theory starting from a particle theory.

BRST QUANTIZATION OF SPINNING RELATIVISTIC PARTICLES WITH EXTENDED SUPERSYMMETRIES

M.Pierri

and

V.O.Rivelles*

Instituto de Física, Universidade de São Paulo
Cx.Postal 20516, CEP 01498, S.Paulo, S.P., Brasil

Abstract

We perform the calculation of the transition amplitude for spinning relativistic particles with N-extended supersymmetries using the BRST-BFV technique. The resulting transition amplitude can be interpreted in a field theory context as the Feynman propagator for the field strenght of a field with spin $N/2$. We also show that there is a global anomaly for odd dimensions so that the model is consistent only for even dimensions. The gauge invariance of the respective field theory does not become manifest in this formulation.

* Partially supported by CNPq

The conection between the quantum mechanics of relativistic particles and quantum field theory has been recently revived by employing new technics of quantization with the aim of gain some insight into the construction of a field theory for strings . It seems that the best way to make this connection is to employ the machinary of constrained systems and the path integral formalism [1]. Since the theories for relativistic particles present local symmetries ghosts must be introduced and a systematic way to deal with them is the BRST-BFV formalism [2]. Then the connection with quantum field theory can be made by noticing that the transition amplitude for a relativistic particle is the Feynman propagator for the Klein-Gordon field and the transition amplitude for the spinning particle is the Feynman propagator for the Dirac field [3]. The same is true for the chiral and the supersymmetric chiral particles in two dimensions [4].

A natural extension of the spinning particle which has N-extended local supersymmetries as well as local $O(N)$ symmetry was formulated and it was shown through the Dirac quantization that it describes massless particles with spin $N/2$ [5]. It was also shown that the theory has a rigid conformal invariance [6]. When $N = 2$ it was found that there is a global $O(2)$ anomaly for odd dimensions [7] but it can be removed by the addition of a Chern-Simons like term and the resulting field theory describes antisymmetric tensors [8]. In fact the theory without the Chern-Simons term is non trivial only in even dimensions [8].

The path integral formulation of this theory was used to study the dimension and global structure of the supermoduli space of the extended spinning particle and as expected it was shown that they depend on the choice of boundary conditions of the parameters. Also a formal expression for the transition amplitude was calculated [9].

In this paper we will use the BFV version [2] of the path integral formalism to evaluate explicitly the transition amplitude for the extended spinning particle. We will show that as in the relativistic particle case a convenient gauge choice for the reparametrization invariance is the proper time gauge. For the local supersymmetries we find the same gauge choice as for the spinning particle while for the internal $O(N)$ symmetry a sort of axial gauge is used. We then show that there is a $O(N)$ global anomaly for odd dimensions. For even dimensions we show that for $N \geq 2$ the transition amplitude can be identified with the Feynman propagator of the field strenght and not with the potential fields of the theory. This is in contrast with the cases $N < 2$ where the transition amplitudes are identified with the Feynman propagators of the basic fields. The raison d'etre for this resides in the fact that in the particle theory the wave function is a gauge invariant quantity (regarding the gauge transformations of the field theory) and upon second quantization they should remain so and must be identified with the field strenghts. The gauge invariance of the potentials remain hidden in this formulation.

The extended supersymmetric spinning particle is described by the following action [5]

$$S = \int_{t_1}^{t_2} dt \left[\frac{1}{2V}\left(\dot{x}^\mu - i\chi_i\psi_i^\mu\right)\left(\dot{x}^\nu - i\chi_j\psi_j^\nu\right)\eta_{\mu\nu} - \frac{i}{2}\psi_i^\mu\dot{\psi}_j^\nu\eta_{\mu\nu} - \right.$$
$$\left. - \frac{i}{2}f_{ij}\psi_i^\mu\psi_j^\nu\eta_{\mu\nu}\right] - \psi_i^\mu(t_1)\psi_i^\nu(t_2)\eta_{\mu\nu} \qquad (1)$$

where $x_\mu(t)$ is the particle coordinate, $\mu = 0,\ldots,D-1$; $\psi_i^\mu(t)$ is the fermionic coordinate, $i = 1,\ldots,N$; $V(t)$ is the (one-dimensional) einbein, $\chi_j(t)$ are the (one-dimensional) gravitinos and f_{ij} are the connections for the internal $O(N)$ symmetry. A boundary term was added so that we can perform the variational principle on the fermionic coordinates assuming only one boundary condition [3].

The action is invariant by world-line reparametrizations

$$\begin{aligned} &\delta x^\mu = \epsilon\dot{x}^\mu && \delta V = (\epsilon N)^{\cdot} \\ &\delta\psi_i^\mu = \epsilon\dot{\psi}_i^\mu && \delta\chi_i = (\epsilon\chi_i)^{\cdot} \\ &\delta f_{ij} = (\epsilon f_{ij})^{\cdot} && \end{aligned} \qquad (2)$$

(where a dot represents differentiation with respect to the proper time t) if the parameter ϵ satisfies the boundary conditions $\epsilon(t_1) = \epsilon(t_2) = 0$. Then it follows that an appropriate gauge choice is the proper time gauge $\dot{V} = 0$.

The action (1) is also invariant by world-line local N-extended supersymmetry transformations:

$$\begin{aligned} &\delta x^\mu = i\alpha_i\psi_i^\mu && \delta V = 2i\chi_i\alpha_i \\ &\delta\psi_i^\mu = -\frac{1}{V}\alpha_i\left(\dot{x}^\mu - i\chi_j\psi_j^\mu\right) && \delta\chi_i = \dot{\alpha}_i - f_{ij}\alpha_j \\ &\delta f_{ij} = 0 && \end{aligned} \qquad (3)$$

if the parameters α_i obey the boundary conditions $\alpha_i(t_1) = \alpha_i(t_2) = 0$. Then it follows that an appropriate gauge choice is $\dot{\chi}_i = 0$.

Finally the action (1) is invariant by local $O(N)$ transformations:

$$\begin{aligned} &\delta x^\mu = 0 && \delta V = 0 \\ &\delta\psi_i^\mu = b_{ij}\psi_j^\mu && \delta\chi_i = b_{ij}\chi_j \\ &\delta f_{ij} = \dot{b}_{ij} + b_{ik}f_{kj} - b_{jk}f_{ki} && \end{aligned} \qquad (4)$$

with no boundary conditions on the parameters b_{ij}. Then an appropriate gauge choice is $f_{ij} = 0$.

To each local symmetry (1-3) is associated a first class constraint, respectively

$$\begin{aligned} \mathcal{H} &= p^2 \\ \phi_i &= p_\mu\psi_i^\mu \\ \phi_{ij} &= i\psi_i^\mu\psi_j^\nu\eta_{\mu\nu} \end{aligned} \qquad (5)$$

where p_μ is the momentum conjugated to x^μ and the fermionic coordinates have the following Poisson brackets

$$\{\psi_i^\mu, \psi_j^\nu\} = -i\delta_{ij}\eta^{\mu\nu} \qquad (6)$$

The constraints (5) close the Poisson bracket algebra

$$\begin{aligned} \{\phi_i, \phi_j\} &= \delta_{ij}\mathcal{H} \\ \{\phi_{ij}, \phi_k\} &= \delta_{ik}\phi_j - \delta_{jk}\phi_i \\ \{\phi_{ij}, \phi_{kl}\} &= \delta_{ik}\phi_{jl} - \delta_{il}\phi_{jk} - \delta_{jk}\phi_{il} + \delta_{jl}\phi_{ik} \end{aligned} \qquad (7)$$

We now extend the phase space introducing the canonical momenta for V, χ_i and f_{ij}, respectively p_v, π_i and p_{ij} and impose them as new constraints. Now for each constraint we associate a pair of cannonicaly conjugated ghosts: for $\mathcal{H}$, $\overline{\mathcal{P}}$ and η ; for p_v, $\mathcal{P}$ and $\overline{\eta}$; for ϕ_i, $\overline{P}_i$ and c_i; for π_i, P_i and $\overline{c}_i$; for ϕ_{ij}, $\overline{\mathcal{P}}_{ij}$ and η_{ij} and for p_{ij}, $\mathcal{P}_{ij}$ and $\overline{\eta}_{ij}$.

We can then build the BRST charge. Using the algebra (7) we find

$$Q = \eta\mathcal{H} + c_i\phi_i + \frac{1}{2}\eta_{ij}\phi_{ij} + \mathcal{P}p_v + P_i\pi_i + \frac{1}{2}\mathcal{P}_{ij}p_{ij} + \overline{P}_i\eta_{ij}c_j -$$
$$- \frac{i}{2}\mathcal{P}c_ic_i - \frac{1}{2}\overline{\mathcal{P}}_{ij}\eta_{jk}\eta_{ki} \qquad (8)$$

which is nilpotent $\{Q,Q\}=0$. The gauge fixing fermion Ψ which implements the above mentioned gauge choices $\dot{V}=\dot{\chi}_i=f_{ij}=0$ is

$$\Psi=\overline{\mathcal{P}}V+\overline{P}_i\chi_i+\frac{1}{2}\overline{\mathcal{P}}_{ij}f_{ij}+\frac{1}{2\epsilon}\overline{\eta}_{ij}f_{ij} \tag{9}$$

where ϵ is an infinitesimal parameter which will be set to zero at the end of calculations. Then the effective action is

$$\begin{aligned}S_{eff}=\int_{t_1}^{t_2}dt\Big[&p\dot{x}+p_v\dot{V}-\frac{i}{2}\dot{\psi}_i\psi_i+\dot{\chi}_i\pi_i+\frac{1}{2}p_{ij}\dot{f}_{ij}+\dot{\overline{\eta}}\mathcal{P}+\dot{\eta}\overline{\mathcal{P}}+\\&+\dot{\overline{c}}_iP_i+\dot{c}_i\overline{P}_i+\frac{1}{2}\dot{\overline{\eta}}_{ij}\mathcal{P}_{ij}+\frac{1}{2}\dot{\eta}_{ij}\overline{\mathcal{P}}_{ij}-\{Q,\Psi\}\Big]-\psi_i(t_1)\psi_i(t_2)\end{aligned} \tag{10}$$

After computing the Poisson bracket $\{Q,\Psi\}$ we perform the following change of variables (which has the Jacobian equals to one) in the action (10)

$$\begin{aligned}p_{ij}&\rightarrow\epsilon p_{ij}\\\overline{\eta}_{ij}&\rightarrow\epsilon\overline{\eta}_{ij}\end{aligned} \tag{11}$$

and take the limit of ϵ going to zero. Then the effective action (10) reduces to

$$\begin{aligned}S_{eff}=\int_{t_1}^{t_2}dt\Big(&p\dot{x}+p_v\dot{V}-\frac{i}{2}\dot{\psi}_i\psi_i+\dot{\chi}_i\pi_i++\dot{\overline{\eta}}\mathcal{P}+\dot{\eta}\overline{\mathcal{P}}+\dot{\overline{c}}_iP_i+\dot{c}_i\overline{P}_i+\\&+\frac{1}{2}\dot{\overline{\eta}}_{ij}\mathcal{P}_{ij}+\frac{1}{2}p_{ij}f_{ij}+V\mathcal{H}-\chi_i\phi_i+\frac{1}{2}f_{ij}\phi_{ij}+\mathcal{P}\overline{\mathcal{P}}+P_i\overline{P}_i+\\&+\frac{1}{2}\mathcal{P}_{ij}\overline{\mathcal{P}}_{ij}-\overline{P}_i\eta_{ij}\chi_j+\overline{P}_ic_jf_{ij}+i\overline{\mathcal{P}}c_i\chi_i-\frac{1}{2}\overline{\mathcal{P}}_{ij}\eta_{kj}f_{ki}+\\&+\frac{1}{2}\overline{\mathcal{P}}_{ij}\eta_{ik}f_{kj}+\frac{1}{2}\mathcal{P}_{ij}\overline{\eta}_{ij}\Big)\end{aligned} \tag{12}$$

We now take the following boundary conditions which are invariant by the BRST transformations generated by (8)

$$\begin{gathered}x^\mu(t_1)=x^\mu_1\qquad x^\mu(t_2)=x^\mu_2\qquad\frac{1}{2}\left(\psi^\mu_i(t_1)+\psi^\mu_i(t_2)\right)=\gamma^\mu_i\\p_v(t_1)=p_v(t_2)=0\qquad\pi_i(t_1)=\pi_i(t_2)=0\qquad p_{ij}(t_1)=p_{ij}(t_2)=0\\\eta(t_1)=\eta(t_2)=0\qquad\overline{\eta}(t_1)=\overline{\eta}(t_2)=0\\c_i(t_1)=c_i(t_2)=0\qquad\overline{c}_i(t_1)=\overline{c}_i(t_2)=0\\\eta_{ij}(t_1)=\eta_{ij}(t_2)=0\qquad\overline{\eta}_{ij}(t_1)=\overline{\eta}_{ij}(t_2)=0\end{gathered} \tag{13}$$

Then the transition amplitude is given by

$$Z(x_1,x_2,\gamma_i)=\int\mathcal{D}\mu\,\mathrm{e}^{-iS_{eff}} \tag{14}$$

where the measure is

$$\begin{aligned}\mathcal{D}\mu=\mathcal{D}x^\mu\mathcal{D}p_\mu\mathcal{D}\psi^\mu_i\mathcal{D}V\mathcal{D}p_v\mathcal{D}\chi_i\mathcal{D}\pi_i\mathcal{D}f_{ij}\mathcal{D}p_{ij}\mathcal{D}\overline{\eta}\mathcal{D}\mathcal{P}\mathcal{D}\eta\mathcal{D}\overline{\mathcal{P}}\\\mathcal{D}\overline{c}_i\mathcal{D}P_i\mathcal{D}c_i\mathcal{D}\overline{P}_i\mathcal{D}\eta_{ij}\mathcal{D}\overline{\mathcal{P}}_{ij}\mathcal{D}\overline{\eta}_{ij}\mathcal{D}\mathcal{P}_{ij}\end{aligned} \tag{15}$$

and the effective action is given by (12).

Since the theory is described in terms of fermionic variables there is the possibility that anomalies could arise. They appear as a failure of the classical symmetries being kept valid at the quantum level due to the regularization of some divergences. A complete analysis of global anomalies in one dimensional theories [10] [11] has been performed and indeed there is a potential anomaly in our theory. The best way to find it is to go back to the gauge invariant action (1) and perform the integration in ψ^μ_i with periodic boundary conditions. It gives a factor of $\det^{D/2}(i\partial_t\delta_{ij}-f_{ij})$ in the measure of the respective functional integral. This determinant can be evaluated and the result (after regularization) is, upon to a sign, [11]

$$\det{}^{1/2}(i\partial_t\delta_{ij}-f_{ij})=\prod_{m=1}^{N/2}\sin\frac{\theta_m}{2}\qquad 0\leq\theta_m\leq2\pi \tag{16}$$

where θ_m are the elements of the gauge field f after an appropriate rotation, which for N even is

$$f = \frac{1}{2\pi}\begin{pmatrix} 0 & \theta_1 & & & & & \\ -\theta_1 & 0 & & & & & \\ & & 0 & \theta_2 & & & \\ & & -\theta_2 & 0 & & & \\ & & & & \ddots & & \\ & & & & & 0 & \theta_{N/2} \\ & & & & & -\theta_{N/2} & 0 \end{pmatrix} \tag{17}$$

Under a global gauge transformations we can change one of the θ's, say θ_i, to $\theta_i + 2\pi$ keeping all other θ's the same. Then using (16) the determinant changes as

$$\det{}^{D/2}(i\partial_t\delta_{ij} - f_{ij}) \to (-1)^D \det{}^{D/2}(i\partial_t\delta_{ij} - f_{ij}) \tag{18}$$

This shows that we do not have global anomalies only for even dimensions. This result remains true for N odd [10]. This result can also be understood in the following way. The determinant in (16) is a mapping ϕ from $S^1 \times S^1$ to $O(N)$, the first S^1 being parametrized by t (recall that we have periodic boundary conditions) and the second S^1 by u, $0 \leq u \leq 1$ which is an auxiliary variable responsible for a gauge change. When u varies from 0 to 1 we are performing a global gauge transformation back to the original gauge. We can think of it as curves on the manifold of $O(N)$ parametrized by u and when u varies from 0 to 1 we have a closed curve on the manifold of $O(N)$. Since $\pi_1(O(N)) = Z_2$, $\phi(t,0) = -\phi(t,1)$ showing that the expression (16) changes sign under a global gauge transformation. So from now on we assume that D is even.

We now want to evaluate the transition amplitude (14). First we perform the functional integrals of the momenta of the Lagrange multipliers. The integral on p_{ij} gives a $\delta[f_{ij}]$ and the integral on f_{ij} allow us to set $f_{ij} = 0$ everywhere. The integral on p_v gives $\delta[\dot{V}]$ which gives an undetermined factor $\det \partial_t$ which can be absorbed in the overall normalization of Z; it also states that only the zero mode of $V(t)$ contributes to the integral. Hence it reduces the functional integral on $V(t)$ to an ordinary integral on $V(0)$ whose integration limits are from 0 to ∞ as required by causality [3].

Analogously the integration on π_i reduces the functional integration on χ_i to an ordinary (Berezin) integral on $\chi_i(0)$.

The integration over the fermion ghosts $\overline{\eta}$, $\mathcal{P}$, η, $\overline{\mathcal{P}}$ gives a factor $\Delta t = t_2 - t_1$. The integration over the bosonic ghosts $\overline{c}_i$, P_i, c_i and $\overline{P}_i$ gives a factor $(\Delta t)^{-N}$. The integration over the fermionic ghosts η_{ij}, $\overline{\mathcal{P}}_{ij}$, $\overline{\eta}_{ij}$ and $\mathcal{P}_{ij}$ gives just a constant independent of Δt.

With all these integrations performed the transition amplitude (14) reduces to

$$Z(x_1, x_2, \gamma_i) = \int_0^\infty dV(0) \int d\chi_i(0) \int \mathcal{D}x^\mu \mathcal{D}p_\mu \mathcal{D}\psi^\mu (\Delta t)^{-(N-1)}$$
$$\exp\left[i\int dt\,\left(p\dot{x} - \frac{i}{2}\dot{\psi}_i\psi_i + V(0)\mathcal{H} - \chi_i(0)\phi_i\right)\right] \tag{19}$$

We now perform the following change of variables (with the jacobian equals to one)

$$x^\mu(t) = x_1^\mu(t) + \frac{\Delta x^\mu}{\Delta t}(t - t_1) + y^\mu(t) \tag{20}$$
$$\psi_i^\mu(t) = \gamma_i^\mu + \theta_i^\mu(t) \tag{21}$$

where $\Delta x^\mu = x_2^\mu - x_1^\mu$ and with the following boundary conditions for y^μ and θ_i^μ

$$y^\mu(t_1) = y^\mu(t_2) = 0 \tag{22}$$
$$\theta_i^\mu(t_1) = -\theta_i^\mu(t_2) \tag{23}$$

The integration over y^μ reduces the functional integral over p_μ to an ordinary integral on its zero mode $p_\mu = p_\mu(0)$. The integration over θ_i^μ gives a factor $(\det \partial_t)^{N/2}$ which is just a constant (after regularization) and can be absorbed in the overall normalization constant of Z. The integration over $\chi_i(0)$ gives $\prod p{\cdot}\gamma_i \Delta t = (\Delta t)^N \prod p{\cdot}\gamma_i$ and finally the integration on $V(0)$ gives $(\Delta t)^{-1}/(p^2 + i\epsilon)$. The factor of $i\epsilon$ was introduced to guarantee the convergence of the integral .

Therefore the transition amplitude (19) takes the final form

$$Z(x_1, x_2, \gamma_i) = \int dp\, e^{ip\Delta x} \prod_{i=1}^N \frac{p\cdot\gamma_i}{p^2 + i\epsilon} \tag{24}$$

with all factors of Δt cancelling out.

We now wish to relate the transition amplitude (24) to some propagator in field theory. In order to do that we will particularize to $N = 2$ and $D = 4$ which means that we are dealing with a particle of helicity 1, the photon. Since the integrand in (24) is of the form $\frac{p^\mu p^\nu}{p^2}$ it can not be related to the propagator of the potential A_μ but of its field strenght $F_{\mu\nu}$. This propagator is known to be

$$\langle F_{\mu\rho}(x_1), F^\rho{}_\nu(x_2)\rangle = \int dp\, e^{ip\Delta x}\left(\frac{p_\mu p_\nu}{p^2} + \frac{1}{4}\eta_{\mu\nu}\right) \tag{25}$$

Since the last term of (25) is proportional to $\delta(\Delta x)$ and we are assuming that the two points x_1 and x_2 are not coincident the term in $\eta_{\mu\nu}$ does not contribute to the propagator (as far as $x_1 \neq x_2$). Now taking the representation for γ_i [5]

$$\begin{aligned} \gamma_1^\mu &= \gamma^\mu \otimes 1 \\ \gamma_2^\mu &= \gamma_5 \otimes \gamma^\mu \end{aligned} \tag{26}$$

we find from (24)

$$\begin{aligned} &\frac{1}{16}(\gamma_\mu\gamma_5)_{\alpha_1}{}^{\beta_1}(\gamma_\nu)_{\alpha_2}{}^{\beta_2} Z[x_1, x_2, \gamma_i]_{\beta_1\beta_2}{}^{\alpha_1\alpha_2} = \\ &\qquad = \int dp\, e^{ip\Delta x}\left(\frac{p_\mu p_\nu}{p^2}\right) = \langle F_{\mu\rho}(x_1), F^\rho{}_\nu(x_2)\rangle \end{aligned} \tag{27}$$

At this point we should remark that if we have chosen another gauge for the $O(2)$ symmetry the expression (24) would formally change and we could get the propagator (25). To do that we choose the gauge $f_{ij} = \frac{\chi_i\chi_j}{2\Gamma(\delta)V^{-\delta+1}}$ where δ is an infinitesimal parameter which will be set to zero at the end of the calculations and it is used to regularize the divergent integrals which we will meet. This gauge choice is implemented by the gauge fixing fermion

$$\Psi = \overline{\mathcal{P}}V + \overline{\mathcal{P}}_i\chi_i + \frac{1}{2}\overline{\mathcal{P}}_{ij}f_{ij} + \frac{1}{2\epsilon}\overline{\eta}_{ij}f_{ij} + \frac{1}{4}\frac{\overline{\mathcal{P}}_{ij}\chi_i\chi_j}{\Gamma(\delta)V^{-\delta+1}} \tag{28}$$

and we get for the transition amplitude (24)

$$Z(x_1, x_2, \gamma_i) = \int dp\, e^{ip\Delta x}\frac{1}{p^2 + i\epsilon}\left(\prod_{i=1}^{2} p\cdot\gamma_i + \frac{1}{4}\prod_{\substack{i,j=1\\ i\neq j}}^{2}\gamma_i\gamma_j p^2\right) \tag{29}$$

Using the representation (26) for the γ_i and multiplying by the same combination of gamma matrices as in (27) we get precisely the propagator (25).

Therefore for theories of relativistic particles with N extended supersymmetries the transition amplitude should be identified with the propagator for the field strenght of the theory and not for their potentials. This can be understood from the Dirac quantization where upon second quantization the wave function turns out to be the field strenght [5] and not the potential and the equations of motion involve only the field strenghts. This is quite different from the cases $N = 0$ and 1 where we find the propagators for the fundamental fields.

In general a massless particle with spin s is described by a potential field which is a symmetric tensor of order s. Its field strenght will have s derivatives so that the propagator for the field strenght $\langle F_{\mu_1\ldots\mu_{2s}}, F_{\nu_1\ldots\nu_{2s}}\rangle$ will have $2s-2$ powers of the momenta. Comparing this with (24) we conclude that it describes the propagator for the field strenght with spin $s = \frac{N}{2}$ after appropriate multipliplications by gamma matrices.

As we have shown the gauge symmetry of the field theory does not manifest itself in this formulation. Also in the Dirac quantization the gauge symmetry is hidden in the Bianchi identities (which are equations of motion there). Only when the Bianchi identities are solved in terms of the potentials is that the gauge symmetry becomes manifest. Although we have started with a particle theory with several local symmetries (diffeomorphisms, supersymmetries and internal $O(N)$ symmetry) neither of them gave origin to an explicity gauge symmetry for the field theory. This may well explain why we can not find the fundamental gauge symmetry for string theories since there too we start from a first quantized theory. Other approaches to this problem (based on string theory) starts with the particle coordinate x^μ and a set of bosonic or fermionic oscillators acting on a Fock space [12]. This approach is essentially operatorial and makes it difficult to interpret the mechanical system that it describes (except for the case of lower spin). Also it is not clear how to make the path integral quantization of this class of models. This shows that we are still in need of a satisfactory prescription to find the local symmetries of a second quantized theory starting from a particle theory.

Acknowledgements: M. Pierri acknowledges finantial help from CNPq.

References

[1] For a review see: M.Henneaux, Phys.Rep.126 (1985) 1
[2] E.S.Fradkin and G.Vilkovisky, Phys.Lett.55B (1975) 224
L.A.Batalin and G.Vilkovisky, Phys.Lett.69B (1977) 309
[3] M.Henneaux and C. Teitelboim, Ann.Phys. (NY) 143 (1982) 127
[4] M.Gomes, V.O.Rivelles and A.J. da Silva, Phys.Lett. 218B (1989) 63
J.Gamboa and V.O.Rivelles, Phys.Lett. B (to be published)
[5] P.S.Howe, S.Penati, M.Pernici and P.K.Townsend, Phys.Lett. 215B (1988) 555
[6] W.Siegel, Intl.J.Mod.Phys. A3 (1988) 2713; ibid A4 (1989) 2015
[7] P.S.Howe and P.K.Townsend, preprint CERN/TH-5519/89 (1989)
[8] P.Howe, S.Penati, M.Pernici and P.K.Townsend, Class. Quantum Grav. 6 (1989) 1125
[9] G.Papadopoulos, Class. Quantum Grav. 6 (1989) 1745
[10] S.Elitzur, E.Rabinovici, Y.Frishman and A.Schwimmer, Nucl.Phys. 273B (1986) 93
[11] E.Witten in "Symposium on Anomalies, Geometry and Topology" Eds. W.R.Bardeen and A.R.White (World Sc., 1985) 61
[12] S.Ouvry and J.H.Stern, Phys.Lett. 177B (1986) 335
A.K.H.Bengtsson, Phys.Lett. 182B (1986) 321
R.Marnelius and U.Mårtensson, preprint Göteborg 89-55 (1989) and references therein.