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THE CHERN-SIMONS TERM AND THE DYNAMICS OF THE CP^{n-1} MODEL IN THREE DIMENSIONS

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Abstract

In this work we analyse the phase structure of the CP^{n-1} model in three dimensional space time coupled to fermions, with special attention to the role played by the Chern Simons term generated by the fermions. A rich phase structure arises from the large n expansion.

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1. Introduction.

The CP^{n-1} model in three dimensional space time has been considered long ago by Aref'eva and Azakov[1], who made an important study, proving that the model has two phases. In the lower phase the $SU(n)$ symmetry is broken to $SU(n-1)$, while in the upper phase, the theory is $SU(n)$ symmetric, with spontaneous mass generation for the fundamental bosonic fields, which however in this phase are confined by a long range force arising from the gauge field interaction.

We shall show that the CP^{n-1} model may be coupled to minimal or supersymmetric fermions; the Chern Simons term[2,3,4] plays an important role plays an important role in modifying the infrared structure of the model. This modification depends on whether the fermion is massive or not. In the minimal case the fermion mass is a parameter at our disposal, that can be fixed arbitrarily, and the cases of massive and massless fermions must be discussed separately. In the supersymmetric case on the other hand, the fermionic mass must be equal to the bosonic one, and there is less freedom to play with this parameter.

Our aim is to provide a nontrivial model displaying a rich phase structure, but which is possible to deal with in the framework of a reasonable aproximation scheme, preferably including nonperturbative effects, and which is important for the discussion of superconductivity at high temperature[5,6]; the CP^{n-1} model may be used with this purpose[7].

This paper is divided as follows. In section 2 we derive the $\frac{1}{n}$ expansion, deriving explicitely the gauge field propagator for each phase. In section 3 we briefly discuss the renormalization properties of the model. We discuss the results and draw conclusions in section 4.

2.The CP^{n-1} model with fermions, and the $\frac{1}{n}$ expansion.

The model we are going to study consists of the usual CP^{n-1} Lagrangian and a fermionic minimal interaction

$$\mathcal{L}_{min} = \overline{D^\mu z} D_\mu z + \overline{\psi} i \not{D} \psi - M \overline{\psi}\psi \qquad (2.1)$$

where $D_\mu = \partial_\mu + A_\mu$, and $z = (z_1, \cdots, z_n)$, is a $U(n)$ multiplet satisfying the constraint

$$\bar{z}z = \frac{n}{2f} \tag{2.1a}$$

where f is a coupling constant. The fermion is also an n component multiplet $\psi = (\psi_1, \cdots, \psi_n)$, with a mass M, which is a free parameter.

If $M = 0$, we can obtain a supersymmetric model, if we include a quartic interaction for the fermion

$$\mathcal{L}_{susy} = \mathcal{L}_{min}^{M=0} + \frac{1}{2} f \left(\overline{\psi}\psi\right)^2, \tag{2.2}$$

as long as we enlarge the set of constraints, to include

$$\bar{z}\psi = 0 = \overline{\psi}z \tag{2.2a}$$

Since the spinor structure in tree dimensional space time is the same as that of two dimensional space time, no further interaction arises, and the two dimensional "γ_5" interaction, migrates now to the current current interaction.

The $\frac{1}{n}$ expansion of the model may now be obtained by well known methods[1,8,9,10], and we shall not do it in full detail here. In fact, our main interest concerns the vector meson two point function, and this is the point where we shall make a more detailed discussion.

The $\frac{1}{n}$ expansion is obtained from the following auxiliary Lagrangian in the minimal case[9,10,8]

$$\mathcal{L}_{min} = \bar{z}\left(-D^\mu D_\mu - m^2\right) z + \overline{\psi}\left(i\, \not{D} - M\right)\psi + \frac{\alpha}{\sqrt{n}}\left(\bar{z}z - \frac{n}{2f}\right) \tag{2.3}$$

with $D_\mu = \partial_\mu - \frac{i}{\sqrt{n}} A_\mu$, while in the supersymmetric case, we need a further auxiliary field σ, in order to implement the quartic fermion self interaction in terms of an interaction quadratic in the fermion fields, as well as an auxiliary field c, to implement the constraint (2.2a); thus we have

$$\mathcal{L}_{susy} = \bar{z}\left(-D^\mu D_\mu - m^2\right) z + \overline{\psi} i \not{D}\psi + \frac{\alpha}{\sqrt{n}}\left(\bar{z}z - \frac{n}{2f}\right) + \frac{1}{2fn}\sigma^2 \frac{1}{\sqrt{n}}\sigma\overline{\psi}\psi + \frac{1}{\sqrt{n}}\overline{c}\bar{z}\psi + \frac{1}{\sqrt{n}}\overline{\psi}zc. \tag{2.4}$$

Notice again the absence of the γ_5 self interaction in three dimensions which migrated to the current current interaction.

In the large n limit we have to impose a saddle point condition on the expectation value of α, fixing the bosonic mass (which can also be interpreted as originating from the expectation value of this field); we have to make a similar procedure for $\hat{\sigma} = \sigma - \sqrt{n}M$, fixing the fermionic mass M, in the supersymmetric case. These parameters are fixed in terms of the ultraviolet cut-off Λ as well as the coupling constant f. This is the typical procedure in two dimensional space time. However, in three dimensions, we have room for a new phase[1]. Indeed, integrating over $n-1$ degrees of freedom z_i (and in the supersymmetric case ψ_i) with $i = 1, \cdots n-1$, after expanding also around the classical field z_n^{cl} (and about the classical fermion field ψ_n^{cl} in the supersymmetric case), we obtain, using a Pauli Villars regularization, the result

$$\frac{1}{2\pi}(\Lambda - m) + \frac{1}{f} + \bar{z}_n^{cl} z_n^{cl} = 0 \tag{2.5a}$$

$$\bar{z}_n^{cl}\alpha = z_n^{cl}\alpha = 0 \tag{2.5b}$$

for the bosonic piece of the action. On the other hand, for the expectation value of the σ field in the supersymmetric case, we have

$$\frac{1}{2\pi}(\Lambda - M) + \frac{1}{f} + \overline{\psi}_n^{cl}\psi_n^{cl} = 0 \tag{2.6a}$$

$$\overline{\psi}_n^{cl}\sigma = \psi_n^{cl}\sigma = 0 \tag{2.6b}$$

We identify two phases arising from (2.5) (and (2.6)). In the upper, unbroken phase, $f > f_c = \frac{2\pi}{\Lambda}$, we have

$$m = 2\pi\left(\frac{1}{f_c} - \frac{1}{f}\right), \tag{2.7}$$

which is the generated mass, for the bosonic field, and in this phase

$$\bar{z}_n^{cl} = z_n^{cl} = 0 \tag{2.8}$$

In this phase the $SU(n)$ symmetry is restaured (with analogous result for the fermion in the supersymmetric case, where $M = m$). This is the only existing phase in two dimensional space time.

For the other solution to equations (2.5-6), we have the broken phase, where

$$m = 0 \tag{2.9}$$

$$| z_n^{cl} |^2 \equiv \mu = \frac{1}{2}\left(\frac{1}{f} - \frac{1}{f_c}\right) . \tag{2.10}$$

This solution is valid when $f < f_c$. Notice that neither the minimal nor the supersymmetric couplings alter the above phase structure. Indeed, in the supersymmetric case, (2.9) is valid, and (2.10) is realized by the expectation value $\langle \overline{\psi}_n^{cl} \psi_n^{cl} \rangle$, in the place of $\overline{z}_n^{cl} z_n^{cl}$ The propagators of the fundamental particles are immediately computed. For all cases we have

$$\langle z^i(k) \overline{z}^j(-k) \rangle = \frac{i}{k^2 - m^2} \delta^{ij} \tag{2.11}$$

$$\langle \psi^i(p) \overline{\psi}^j(-p) \rangle = \frac{i}{\not p - M} \delta^{ij} \tag{2.12}$$

where in the upper phase m is given by (2.7), $i, j = 1, \cdots, n$, while in the lower phase $m = 0$, $i, j = 1, \cdots, n-1$; moreover, in the supersymmetric case we are constrained to the equality $M = m$, while M was a free parameter in the minimal case.

Let us discuss the $SU(n)$ symmetric phase, i.e., $f > f_c$, where the fundamental fields have a (transmuted) mass. The gauge field two point function is computed as follows

$$\Gamma_{\mu\nu} = \Gamma_{\mu\nu}^B + \Gamma_{\mu\nu}^F \tag{2.13a}$$

and

$$\Gamma_{\mu\nu}^B = -\frac{\delta^2}{\delta A_\mu \delta A_\nu} \mathrm{tr} \ln \left(-\partial^2 - m^2 + \frac{i}{\sqrt{n}} A_\mu \partial^\mu - \frac{1}{n} A^\mu A_\mu \right) |_{A_\mu = 0} \tag{2.13b}$$

$$\Gamma_{\mu\nu}^F = +\frac{\delta^2}{\delta A_\mu \delta A_\nu} \mathrm{tr} \ln (\not\partial - i \not A - M) |_{A_\mu = 0} \tag{2.13c}$$

For the bosonic contribution we obtain from the above, the inverse propagators in the large n limit. We have

$$\Gamma_{\mu\nu}^B(p) = 2 g_{\mu\nu} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{k^2 - m^2} - \int \frac{d^3 k}{(2\pi)^3} \frac{(2k+p)_\mu (2k+p)_\nu}{(k^2 - m^2)[(k+p)^2 - m^2]} . \tag{2.14}$$

Although these are linearly divergent integrals, the use of a Pauli-Villars regulator eliminates the divergence in a gauge invariant way, leaving the finite result

$$\Gamma_{\mu\nu}^B(p) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \left\{ \frac{i}{8\pi\sqrt{-p^2}} (-p^2 + 4m^2) \, \mathrm{atan} \sqrt{\frac{-p^2}{4m^2}} - \frac{im}{4\pi} \right\} . \tag{2.15}$$

The above function has an infra red (IR) behavior given by the expression

$$\Gamma_{\mu\nu}(p) \approx \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{i}{24\pi} \frac{p^2}{m} . \tag{2.16}$$

The contribution of the fermions is given by the expression

$$\Gamma_{\mu\nu}^F(p) = \mathrm{tr} \int \frac{d^3 k}{(2\pi)^3} \frac{\gamma_\mu (\not k - M) \gamma_\nu [\not k + \not p - M]}{(k^2 - M^2)[(k+p)^2 - M^2]} , \tag{2.17}$$

which corresponds to the one loop fermionic approximation.

There are two main contributions to the above, one arising from the product of an even number of gamma matrices, which we call $\Gamma_{\mu\nu}^{F1}$, and a Chern Simons type term, $\Gamma_{\mu\nu}^{CS}$. They are given by the following expresions, which can be computed using a Feynman parametrization of the one loop integrals, and elementtary integration[11]:

$$\Gamma_{\mu\nu}^{F1}(p) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{-i}{2\pi} \sqrt{-p^2} \left[\left(\frac{1}{4} + \frac{M^2}{p^2} \right) \mathrm{atan} \sqrt{\frac{-p^2}{4M^2}} + \frac{1}{2} \sqrt{\frac{M^2}{-p^2}} \right]$$
$$= \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) F(p^2) \tag{2.18}$$

for fermions of mass M, and

$$\Gamma_{\mu\nu}^{F1}(p) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{-i}{16} \sqrt{-p^2} , \tag{2.19}$$

for $M = 0$. On the other hand, for the Chern Simons term we obtain the following expression

$$\Gamma_{\mu\nu}^{CS}(p) = \frac{i}{4\pi} \left(\frac{M}{|M|} \sqrt{-p^2} - 2M \, \mathrm{atan} \sqrt{\frac{-p^2}{4M^2}} \right) \epsilon_{\mu\nu\rho} \frac{p^\rho}{\sqrt{-p^2}}$$
$$= B(p^2) \epsilon_{\mu\nu\rho} \frac{p^\rho}{\sqrt{-p^2}} \tag{2.20}$$

for massive fermions, and

$$\Gamma_{\mu\nu}^{CS}(p) = \frac{ie}{4\pi} \mathrm{sign}\, M \epsilon_{\mu\nu\rho} p^\rho \tag{2.21}$$

for massless fermions. Notice the ambiguity due to the sign of the mass. We shall suppose from now on that $M \geq 0$. We have for the total two point function the result

$$\Gamma_{\mu\nu}(p) = (F(p^2) + B(p^2)) \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) + A(p^2) \epsilon_{\mu\nu\rho} \frac{p^\rho}{\sqrt{-p^2}} \tag{2.22}$$

In order to obtain the propagator we invert the above expression 22, obtaining (after a suitable gauge fixing procedure[10]

$$D_{\mu\nu}(p) = \frac{F(p^2)+B(p^2)}{A(p^2)^2+\left(F(p^2)+B(p^2)\right)^2}$$
$$\left[\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)-\frac{A(p^2)}{F(p^2)+B(p^2)}\epsilon_{\mu\nu\rho}\frac{p^\rho}{\sqrt{-p^2}}\right] \quad (2.23)$$

The IR behavior of the functions can be easily obtained. For massless fermions in the minimal theory we have

$$A(p^2)\approx\frac{i}{4\pi}\sqrt{-p^2} \quad (2.24a)$$

$$B(p^2)\approx-\frac{i}{16}\sqrt{-p^2}\cdot \quad (2.24b)$$

$$F(p^2)\approx\frac{i}{24\pi}\frac{p^2}{m}\,. \quad (2.24c)$$

Therefore

$$D_{\mu\nu}\approx-\frac{16i\pi^2}{(\pi^2-16)\sqrt{-p^2}}\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}+\frac{4}{\pi}\epsilon_{\mu\nu\rho}\frac{p^\rho}{\sqrt{-p^2}}\right]. \quad (2.25)$$

For massive fermions, on the other hand, we have, for small momentum, the behavior

$$A(p^2)\approx\frac{i}{4\pi}\sqrt{-p^2}\varepsilon\,, \quad (2.26a)$$

$$B(p^2)\approx-\frac{i}{12\pi}\frac{p^2}{M}\,, \quad (2.26b)$$

and

$$F(p^2)\approx\frac{i}{24\pi}\frac{p^2}{m}\,. \quad (2.26c)$$

The constant ε is regularization dependent[2]. Naively it is given by $\varepsilon=\frac{\Lambda}{|\Lambda|}-\frac{M}{|M|}$, where Λ is the ultra violet cut-off, but in fact it can have another value depending on the regularization employed[2]. For the gauge field propagator we have now

$$D_{\mu\nu}=\frac{12\pi i}{p^2}\left(\frac{1}{2m}-\frac{1}{M}\right)^{-1}\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right) \quad (2.27)$$

for $\varepsilon=0$, while for non zero values of ε we have:

$$D_{\mu\nu}(p)=-\frac{i\pi\left(\frac{1}{m}-\frac{2}{M}\right)}{\frac{1}{24}p^2\left(\frac{1}{m}-\frac{2}{M}\right)^2-\frac{3}{2}\varepsilon^2}\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}+\frac{6\varepsilon}{\frac{1}{m}-\frac{2}{M}}\epsilon_{\mu\nu\rho}\frac{p^\rho}{p^2}\right] \quad (2.28)$$

Therefore, if ε is non zero (as is usually the case, if we which to mantain gauge invariance[4]) the pole in the propagator disappears, giving rise to a cut in the case of massless fermions, and to a massive propagator in the massive fermion case, as usual, when we have a Chern Simons term. The discussion of the supersymmetric case is identical with that of the massive fermions, with equal mass, i.e., $M=m$.

In the supersymmetric case, we compute also the σ field propagator, finding

$$\Gamma_\sigma(p)=\frac{i}{\pi}\left\{\frac{m}{2}+\frac{-p^2+4m^2}{8\sqrt{-p^2}}\left(2\mathrm{atan}\sqrt{\frac{4m^2}{-p^2}}-\pi\right)\right\} \quad (2.29)$$

while for the α two point function we have:

$$\Gamma_\alpha(p)=\frac{-i}{4\pi\sqrt{-p^2}}\mathrm{atan}\sqrt{\frac{-p^2}{4m^2}} \quad (2.30)$$

The auxiliary (fermionic) field c, which enforces the contraint (2.2b) has a two point function

$$\Gamma_{c\bar c}(p)=\frac{i}{8\pi}\left(m+\frac{i}{2}\not p\right)\left[2\mathrm{atan}\sqrt{\frac{4m^2}{-p^2}}-\pi\right] \quad (2.31)$$

Let us now turn to the broken ($SU(n)$) phase. Here, the fundamental fields form a $SU(n-1)$ massless multiplet, with propagators

$$\langle z_i(p)\bar z_j(p)\rangle=\frac{i}{p^2}\delta_{ij} \quad (2.32)$$

$$\langle\psi_i(p)\bar\psi_j(p)\rangle=\frac{i}{\not p-M}\delta_{ij} \quad (2.33)$$

where $M=0$ in the supersymmetric case. The propagator of the auxiliary α field is now given by the zero mass limit of the previous expression 30, corrected by the expectation value of the n^{th} z field, and is given by

$$D_\alpha(p)=\frac{16p^2}{\sqrt{-p^2}+16\mid\sigma\mid^2} \quad (2.34)$$

Let us now concentrate on the gauge field propagator. The important point is allways whether the fermions are massive or not. Using the same notation as before, we have, for the Chern Simons term

$$A(p^2) = \frac{i}{4\pi}\sqrt{-p^2}\varepsilon \tag{2.35}$$

The bosonic contribution is rather simple, being given by the bosonic massless loop, and the constant term $|\sigma|^2$ arising from the vacuum expectation value of $\overline{z}_n z_n$, and we have the result

$$F(p^2) = \frac{\sqrt{-p^2}}{16} + |\sigma|^2 . \tag{2.36}$$

whereas from the fermions we obtain one of the previous contributions, that is either (19,21) (for $M = 0$), or (18,20) (for massive fermions). Notice that the supersymmetric case corresponds to massless fermions.

Therefore, for massless fermions (we take $\varepsilon = 1$) we have

$$D_{\mu\nu}(p) = \frac{16i\pi^2 |\sigma|^2}{p^2 - 16|\sigma|^4\pi^2}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} + \frac{i}{4\pi}\epsilon_{\mu\nu\rho}p^\rho\right] , \tag{2.37}$$

which has no pole at $p^2 \approx 0$, but rather a well defined mass term. A different charge for the fermionic field as compared to the one of the fermion, would imply a more complicated structure for this propagator.

In the minimal massive case, we have, first for the case $\varepsilon = 0$

$$D_{\mu\nu}(p) = \frac{16i}{\sqrt{-p2} - 16|\sigma|^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \tag{2.38}$$

while for ε non zero, we obtain (we put for definiteness $\varepsilon = 2$):

$$D_{\mu\nu}(p) = \frac{16\pi^2 i}{\pi^2\left(\sqrt{-p^2} - 16|\sigma|^2\right)^2 + 64p^2}\times \left\{\left(\sqrt{-p^2} - 15|\sigma|^2\right)\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + \frac{8i}{\pi}\epsilon_{\mu\nu\rho}p^\rho\right\} . \tag{2.39}$$

Therefore we must have a complex spectrum, which cannot be analysed with naive arguments, from the above expression.

3. Renormalization.

A short note on renormalization is in order. First notice the behavior of the auxiliary c and σ propagators for large momentum

$$\Gamma_c(p) \approx -\frac{i}{8}\left(m + \frac{i}{2}\not{p}\right) + \frac{i}{4\pi}\frac{\not{p}}{\sqrt{-p^2}} \tag{3.1}$$

$$\Gamma_\sigma(p) \approx \frac{i}{\pi}\left[\frac{M}{2} - \frac{\pi}{8}\sqrt{-p^2}\right] \tag{3.2}$$

$$\Gamma_\alpha(p) \approx \frac{i}{8\sqrt{-p^2}} \tag{3.3}$$

These results shows that the fermionic as well as the bosonic interaction, after we take into account also the gauge field propagator behaviour for large momentum, are renormalizable. Its is clear also, that since the decay behavior of the propagators with the momentum is not stronger than that of the free propagators, we do not have any positivity problem.

In any case, renormalizability is preserved. Indeed, there are cancellations of ultraviolet divergencies due to the constraints (2.1a) and (2.2a), generalizing arguments used in [12]. In fact, for Green's functions having more than two external isospin carrying legs, the following argument holds. Follow the isospin of a given external line; it must end in another isospin carrying external line. Consider the diagram where the two lines came from a complicated blob (as in figure 3.1) and the comon internal line interacted more than once. Construct now, as in figure 3.2, a diagram where the same lines now come out of the same blob join into an auxiliary line (α if we are dealing with two z lines, ϕ if we are dealing with two fermions, and c if it is a boson fermion pair), and afterwards, that auxiliary line generates the previous pair again. For the divergent part of the above two diagrams there is a cancellation, since when the blob schrincks to a constant, the second diagram gives a minus sign, due to the fact that the auxiliary field propagator is minus the inverse of the two point function corresponding to the fields it gave origin to. This gives a cancellation of divergencies for each pair of external line, and a simple counting of the remaining divergencies shows that we are left only with the usual mass and wave function renormalization (the coupling constant renormalization is englobed by the mass renormalization, due to the mass transmutation phenomenon).

4.Conclusions.

We conclude this paper, in a way analogous to the two dimensional situation, where the long range force arising from the CP^{n-1} bosonic self interaction is screened by the fermionic fields. The nature of the screening is however more complicated not only due to the phase structure of the model, but also due to the regularization dependence of the Chern Simons coefficient. In any case, we learned that massless fermions play a very special role in the upper (unbroken) phase, since the gauge field propagator displays a square root cut rather that any pole. In this case, the construction may be inverted in the sense that a square root behavior of a gauge field permits the construction of a fermionic field, analogously to the two dimensional construction[13]. In the supersymmetric case, on the other hand, we allways end up with a massive gauge field, showing that the screening mechanism works in tree dimensions as well[14]. Therefore, the supersymmetric CP^{n-1} model in three dimensions, has in both phases, a liberation mechanism for the fundamental fields degrees of freedom. We have not analysed whether there is a mechanism analogous to the two dimensional one, where the chirality of the fermions decouple, in order that the gauge field has its pole shifted from zero.

In the purely bosonic theory, we have two phases, an $SU(n)$ symmetric, confining phase, and an $SU(n-1)$ symmetric (or $SU(n)$ broken), non confining massless phase. the two phase structure is preserved when coupling the model with fermions, either minimally, or supersymmetrically.

In the confining phase, if we couple minimally to massless fermions, the long range properties of the gauge fields are screened by the fermions, and the z-partons are liberated, in a mechanism similar to the two dimensional case. The gauge field, on the other hand, does not present any pole.

In the massive (and also in the supersymmetric) case, a mass term is generated for the gauge field, which now presents a massive pole. Again, but by another mechanism, the partons are liberated in the process.

In the broken phase, on the other hand, the z partons are, in any case, unconfined. however, the situation for the analytic properties of the gauge field propagator is inverted: in the massless (and supersymmetric) cases, the gauge field presents now a pole, that turns into a more complicated structure for massive fermions.

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References.

[1] I. Ya Aref'eva, S. I. Azakov Nucl. Phys. B162(1980)298.

[2] M. Lüscher Nucl. Phys. B326(1989)557.

[3] G. Semenoff Pys. Re. Lett. 61(1988)517.

[4] A.N. Redlich Phys. Rev. Lett. 52(1984)118.

[5] E. Witten et al Int. J. Mod. Phys.B.

[6] E. Manossaki, R. Salvador Phys. Rev. Lett. 62(1989)1310.

[7] I. Dzyaloshinski, A. M. Polyakov, P. B. Wiegman Phys. Lett. 127A(1988)112.

[8] A. D'Adda, P. Di Vecchia, M. Lüscher, Nucl. Phys. B146(1978), B152(1979).

[9] E. Abdalla, M. C. B. Abdalla, M. Gomes Phys. Rev D23(1981)1800, D27(1983)825.

[10] E. Abdalla, M. Forger, A. Lima-Santos Nucl. Phys. B.256(1985)145.

[11] S. Gradhstein, R. Ryzhik, Table of integrals.

[12] I. Ya. Areff'eva Theor. Math Phys. 36(1979)573.

[13] E. Marino, to appear.

[14] E. Abdalla, M.C.B. Abdalla, M. Gomes, Phys. Rev. D25(1982)452.
R. Köberle, V. Kurak, Phys. Rev. Lett.

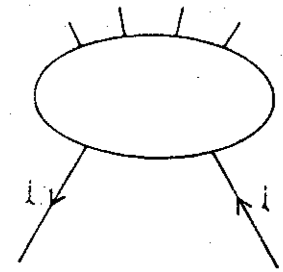

fig. 3.1

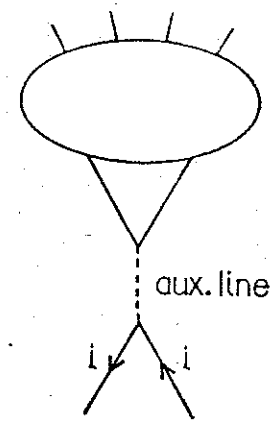

fig. 3.2