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IFUSP/P-877

ENERGY AND MASS-NUMBER DEPENDENCE OF THE DISSOCIATION TEMPERATURE IN HYDRODYNAMICAL MODELS

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Outubro/1990

Energy and Mass-Number Dependence of the Dissociation Temperature in Hydrodynamical Models

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Abstract

Transverse-momentum distributions of π and K have been analyzed to obtain the $\sqrt{s}$ dependence of the collective transverse flow and the dissociation temperature in pp- and $\bar{p}p$-induced multiparticle production reactions. A good fit of both the pion and kaon data has been obtained, in terms of a previously proposed simple parametrization of the collective transverse rapidity distribution. The main outcomes are the logarithmically increasing average transverse rapidity squared and a slowly decreasing dissociation temperature T_d as the incident energy $\sqrt{s}$ increases. This last behavior is in excellent agreement with early Landau's estimate. An extension of the same estimate to nucleus-nucleus collisions gives the correct low temperature component which has been observed in heavy-ion experiments.

1. The search for the quark-gluon plasma (QGP) in recent years has greatly revived the interests in studying relativistic hydrodynamical models for high-energy hadronic and nuclear collisions[1,2]. In such models, the system which arises in a high-energy collision starts from an *"initial"* thermalization and evolves collectively until the mean free path of the outgoing particles becomes large enough to escape from the system. The moment of dissociation is characterized by some temperature T_d and the collective velocity that the fluid has acquired by expansion. *Whether inequivocal signatures of such collective flow exists and how the dissociation temperature varies with the incident energy and nuclear size are questions which deserve a special attention in the high-energy hadron physics.*

In an earlier paper[3], one of the present authors has suggested the interpretation of the energy dependence of the pion transverse-momentum distribution as due to the increase of the transverse flow with the incident-energy growth. By parametrizing the *collective* velocity distribution as

$$u^0 \frac{d\rho}{d^3u} \equiv \frac{d\rho}{\mathrm{sh}\xi\mathrm{ch}\xi\, d\alpha d\xi d\phi} = A\exp(-C\alpha^2 - B\xi^2)\,, \tag{1}$$

where α and ξ are respectively the longitudinal and the transverse rapidities and ϕ is the azimuth, and by making a convolution with the thermal (Bose) distribution

$$\frac{D\, d\vec{p}_0}{\exp(\frac{p^\mu u_\mu}{T_d}) - 1}\,, \tag{2}$$

where $\vec{p}_0$ is the pion momentum in the proper frame of the fluid, it was shown that all the existing data at that time[4] could well be fitted with a fixed temperature T_d (which has been chosen to be $m_\pi/1.5$ there) and a logarithmically increasing parameter B^{-1}. Notice that, as far as $E(d\sigma/d\vec{p})|_{\theta=\pi/2}$ is concerned, A, D and C in the parametrization given above may be gathered together to give just one normalization constant G, which is in principle known. An extrapolation of this result to the $\bar{p}p$ collider energies[5)] has shown a quite good agreement with the first data.

2. More recently, the difference of the average transverse momenta $< p_T >$ of pions and kaons has been used to extract the mean transverse velocity $< v_T >$ and the dissociation temperature T_d in pp and $\bar{p}p$ collisions[6)]. The main conclusions of that work are: *i)* $T_d \simeq 124\,\mathrm{MeV}$ is more or less independent of the incident energy $\sqrt{s}$ and *ii)* the mean transverse velocity increases slowly with $\sqrt{s}$. Although the exact functional form is different, the conclusion *ii)* is qualitatively the same as the one which has been obtained in Ref. 3).

The essential improvement of that work over Ref. 3) would be that, in principle, the consideration of final particles of different masses might allow a better discrimination of the two competing factors, namely T_d and v_T, eliminating thus a good amount of ambiguity (this ambiguity has also been observed by other authors in a recent work[7)]). However, we think that an analysis just of $< p_T >$ as done there is too rough, when we do have p_T distributions of those particles. The main purpose of the present note is to discuss the $\sqrt{s}$ dependence of v_T and also of T_d, by analyzing the existent p_T*-distribution* data[4,8−11)] in terms of the model proposed in Ref. 3).

3. Thus, in what follows, we have parametrized π and K data at each incident energy $\sqrt{s}$, by using (1) and (2) given above. The particle-mass dependence enters evidently in (2), through p^μ. Besides $T_d(\sqrt{s})$ and $B(\sqrt{s})$, we have now two normalization parameters $G_\pi(\sqrt{s})$ and $G_K(\sqrt{s})$, corresponding to pions and kaons respectively. These parameters

have been determined by using χ^2 fit to the data. In doing so, we have arbitrarily fixed the maximum p_T value, which has been taken $p_{T max} = 1.5\,\mathrm{GeV}$ throughout the ISR energies because, at higher values of p_T, hard processes may become important and the separation of the soft, hydrodynamical, component becomes harder. At the collider's energies, however, we expect that $p_{T max}$ may be taken much larger (~ 3 GeV).

Another point which should be clarified is the following. In analyzing the data as function of $\sqrt{s}$, we are by no means accepting Landau's interpretation[1)], namely that in every collision all the incident energy is converted into hadronic matter, giving thus always a fireball of the same mass. It is well known that hadronic collisions are characterized by large inelasticity fluctuations and, actually, we have studied in a previous paper[12)] the well established correlation between $< p_T >$ and n_{ch}, interpreting the increase of $< p_T >$ as due to the augmentation of the transverse expansion with growing fireball mass. What we are doing in the present analysis is, in studying the inclusive distribution, to replace the average of the contributions coming from fireballs of different masses by a single term with average mass $< M >$, which increases with $\sqrt{s}$. The exact $\sqrt{s}$ dependence of $< M >$ is not well established[13,14)], but since every relevant parameter of this analysis is very weakly dependent on the energy, the final results remain valid if $< M > = \mathrm{const.}\sqrt{s}$ or close.

It should be emphasized that here we are taking T_d not a constant but an energy-dependent parameter, because in contrast to the critical hadronization temperature T_c, *the dissociation temperature is not an intrinsic property of the thermodynamic state but depends also on the size of the system.* This is a point which has often been neglected in the literature and we show (in **6.**) that a proper consideration of this question gives a unified interpretation, both of the above mentioned energy dependence of T_d in pp or $\bar{p}p$

collisions and the low-temperature component which appears in p_T distribution in heavy-ion collisions[15].

4. We give in Table I, the fitted values of all the parameters at several values of $\sqrt{s}$. The corresponding curves for π and K transverse-momentum distributions, together with the data, are shown in Figures 1 - 5. As seen, the π data are perfectly reproduced at every energy and in the whole t interval we have considered. Also the K data are quite well reproduced, if we recall that generally these data present large uncertainties. The concavity which is present in $E(d\sigma/d\vec{p})|_{\theta=\pi/2}$ of both π and K and at every energy can naturally be obtained as a consequence of the transverse collective-flow distribution.

The energy dependence of B^{-1} and T_d is shown in Figures 6 and 7, respectively. Although the data do not allow a determination of the precise functional form of $B^{-1}(\sqrt{s})$, it is seen that this parameter increases steadily, indicating growth of the transverse flow with $\sqrt{s}$. It may be fitted either by $\ln s$ or $\ln^2 s$ and, when compared with the result given in Ref. 6), it shows larger transverse velocities in the energy region we have considered.

What is *qualitatively new* in our results is the dissociation temperature T_d shown in Fig. 6. Namely, it decreases as the incident energy increases, in conformity with what we have mentioned at the end of **3.** That T_d should decrease with $\sqrt{s}$ is natural, because larger energy means larger expansion, so larger fluid volume too, and the density (and so does the temperature) has to get smaller in order that particles can escape from the fluid.

5. An estimate of T_d has been made by Belenkij and Landau[16] in 1955, and they did find its energy dependence, but since it is very weak and evidently they did not have at that time precise data to check it, they have simply neglected it in the subsequent discussions. We believe that now we have enough data to verify it.

Following Ref. 16), the transverse dimension of the fluid at the moment of dissociation may be evaluated as[17]

$$L \simeq \frac{z_d}{m_\pi}\left(\frac{\pi m_p a}{\Phi(z_d)}\right)^{1/4}\left(\frac{\sqrt{s}}{2m_p}\right)^{1/6}, \tag{3}$$

where $z_d = m_\pi/T_d$, a =proton radius and

$$\Phi(z_d) = z_d^4 \int_0^\infty \frac{x^2\sqrt{1+x^2}\,dx}{\exp(z_d\sqrt{1+x^2})-1}. \tag{4}$$

The main ingredients to obtain this result is, besides the asymptotic form of Khalatnikov's one-dimensional solution[18], the assumption of a conical motion and the conservation of both energy and entropy. In our opinion, these assumptions have sufficiently general validity so to be applied in the entire energy region we are considering, where probably the mixed phase is predominant during the whole expansion stage.

If we now take the mean free path $l = 1/n_d\sigma$, with $\sigma \simeq \sigma_{\pi\pi}$ and

$$n_d = \frac{3m_\pi^3}{2\pi^2 z_d^3}F(z_d), \tag{5}$$

where

$$F(z_d) = z_d^3 \int_0^\infty \frac{x^2\,dx}{\exp(z_d\sqrt{1+x^2})-1}, \tag{6}$$

then the dissociation temperature is fixed by the condition

$$\frac{l}{L} \simeq \frac{2\pi^2 z_d^2}{3m_\pi^2\sigma F(z_d)}\left[\frac{\Phi(z_d)}{\pi m_p a}\right]^{1/4}\left(\frac{2m_p}{\sqrt{s}}\right)^{1/6} \simeq 1. \tag{7}$$

So, the energy dependence of T_d reads

$$T_d \simeq \pi\sqrt{\frac{2}{3\sigma F(z_d)}}\left[\frac{\Phi(z_d)}{\pi m_p a}\right]^{1/8}\left(\frac{2m_p}{\sqrt{s}}\right)^{1/12}. \tag{8}$$

Notice that the right-hand side of (8) still contains a weak T_d dependence. As seen in Fig. 6, this formula, with a reasonable choice of a and σ, is in excellent agreement with the results of our analysis of pp and $\bar{p}p$ data from the ISR and the $\bar{p}p$-Collider.

6. The estimate of L, given in **5**, may equally be applied to nucleus-nucleus collisions with a small change, if pion dominance is assumed. For simplicity, we consider only the symmetrical case, in which two identical nuclei with mass number A collide each other and assume the collision to be central. In such a case, the nuclear radius is given by $R = A^{1/3}R_0$, where $R_0 \simeq 1.2fm \sim a$, so L in (3) is replaced by

$$L_{AA} \simeq A^{1/3}\frac{z_d}{m_\pi}\left(\frac{\pi m_p R_0}{\Phi(z_d)}\right)^{1/4}\left(\frac{\sqrt{s}}{2m_p}\right)^{1/6}, \qquad (3a)$$

where $\sqrt{s}$ is to be interpreted as the center-of-mass energy of a nucleon-nucleon system having the same relative velocity as the entire nucleus-nucleus system. The dissociation condition (7) becomes now

$$\frac{2\pi^2 z_d^2}{3m_\pi^2\sigma F(z_d)}\left[\frac{\Phi(z_d)}{\pi m_p R_0}\right]^{1/4}\left(\frac{2m_p}{\sqrt{s}}\right)^{1/6}\frac{1}{A^{1/3}} \simeq 1\,. \qquad (7a)$$

If we neglect the slowly varying functions $F(z_d)$ and $\Phi(z_d)$ in (7a), A dependence of T_d is approximately given by $T_{d,AA} \simeq A^{-1/6}T_{d,pp}$ at the same $\sqrt{s}$.

We can compare the result obtained above with the recently published 200A GeV $^{32}S+^{32}S$ data[15]. This laboratory energy corresponds to $\sqrt{s} = 19.4$ GeV (where $T_{d,pp} \simeq$ 125MeV). By using (7a), the dissociation temperature for $S + S$ system turns out to be $T_{d,SS} \simeq 78.2$MeV, which is in excellent agreement with the low-p_T component, seen in the data, and also in a hydrodynamical analysis of Sinyukov *et al.*[19] As for the large-p_T component, it naturally suffers widening due to the flow effect and also because, close to the fluid surface, particles can escape at higher temperature.

7. In the present note, we have analysed the existent pion and kaon p_T-distribution data in pp and $\bar{p}p$ collisions, in terms of a hydrodynamical model with Gaussian rapidity distribution. As expected, the average collective transverse rapidity increases slowly, but steadily with the incident energy. In terms of the average squared transverse rapidity, B^{-1} in (1), it increases as $\ln^2 s$. A new feature appears regarding the dissociation temperature T_d, which decreases slowly with $\sqrt{s}$, reflecting the increase in the size of the hadronic system in expansion. This system's size effect manifests itself also in nuclear collisions, where larger intrinsic volumes of the colliding systems cause a remarkable lowering of the final temperature. A unified description of these two effects, namely energy and mass-number dependences of T_d could be achieved, within a rough treatment of transverse expansion.

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TABLE I. χ^2-fitted values of the parameters B, T_d, G_π and G_K at several values of incident energy $\sqrt{s}$. (*) At 63 GeV, B has been fixed equal to 5.0.

$\sqrt{s}$ (GeV)	B	T_d (MeV)	G_π (mb/GeV^2)	G_K (mb/GeV^2)	χ^2/NDF
23	8.39±0.64	124±1	2301±140	972± 70	1.30
31	6.73±0.50	122±1	2094±120	1096± 57	0.50
45	6.57±0.43	118±1	2411±120	1331± 71	0.90
53	5.82±0.35	117±1	2329± 97	1325± 61	3.11
63	5.0 (*)	112±1	2312± 28	1409± 61	3.30
200	4.84±0.75	119±7	2503±167	1428± 200	0.40
540	2.28±0.12	98±4	2956±171	2509± 415	1.40
900	2.66±0.30	95±7	4197±454	3816±1164	0.90

Figure Captions

Fig. 1: Fits of low-p_T pion distribution data[8] at the ISR, as explained in the text.

Fig. 2: Fits of intermediate-p_T pion distribution data[4] at the ISR, as explained in the text.

Fig. 3: Fits of kaon p_T-distribution data[4,8] at the ISR, as explained in the text.

Fig. 4: Fits of pion p_T-distribution data[9−11] at the $\bar{p}p$ Collider, as explained in the text. The points at 200 and 900 GeV have been extracted from UA1 data, by subtracting p and K contributions from the charged distributions, with the use of p/π and K/π ratios given by UA2 Collaboration.

Fig. 5: Fits of kaon p_T-distribution data[9−11] at the $\bar{p}p$ Collider, as explained in the text.

Fig. 6: $\sqrt{s}$ dependence of the parameter B^{-1} (eq.(1)), as determined in the present analysis. The line corresponds to the values which reproduce the average velocities determined in Ref. 6).

Fig. 7: $\sqrt{s}$ dependence of the dissociation temperature T_d, as determined in the present analysis. The curve represents the theoretical T_d as determinbed by (8), with a =0.88 fm and σ =54 mb.

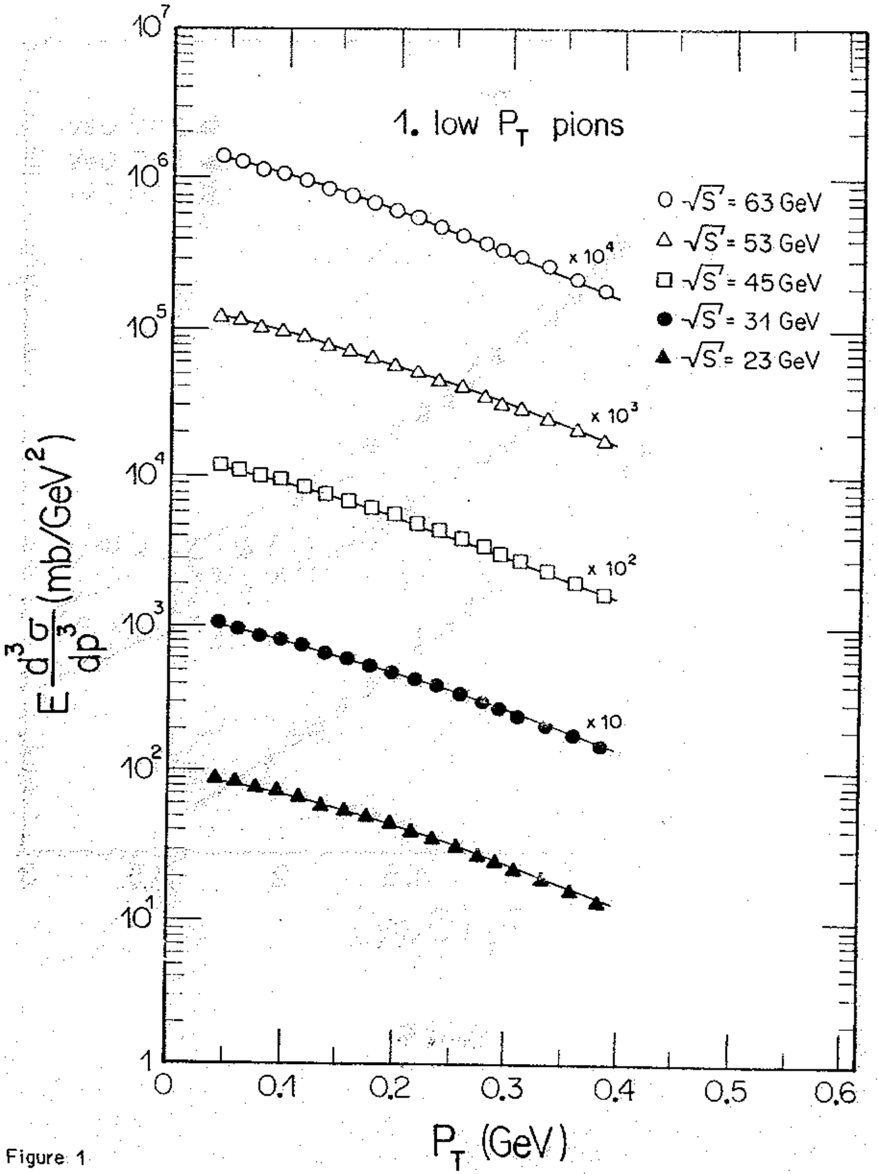


Figure 1

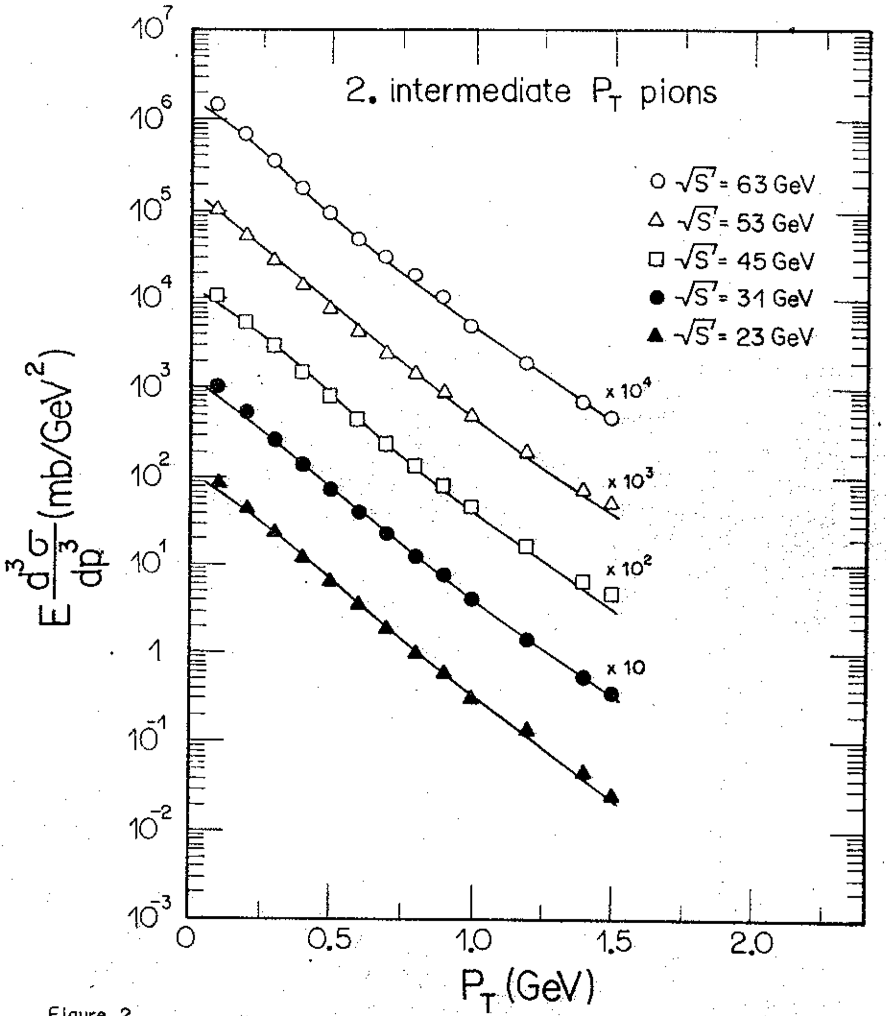


Figure 2

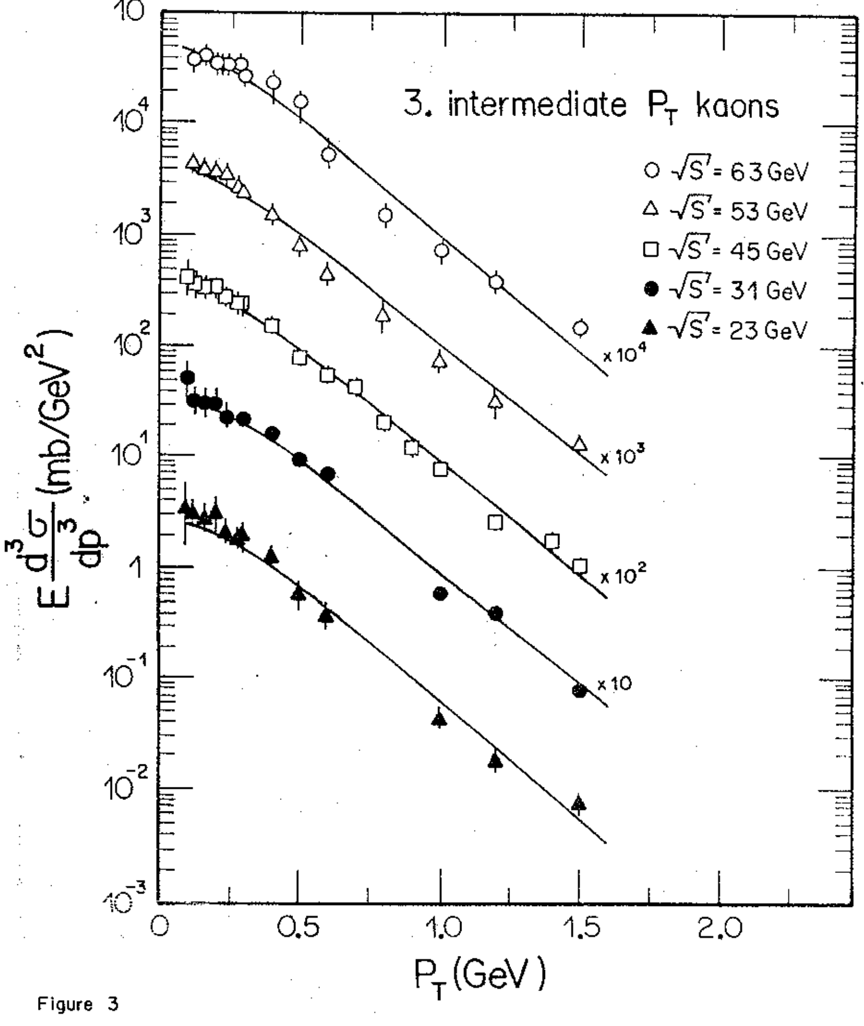


Figure 3

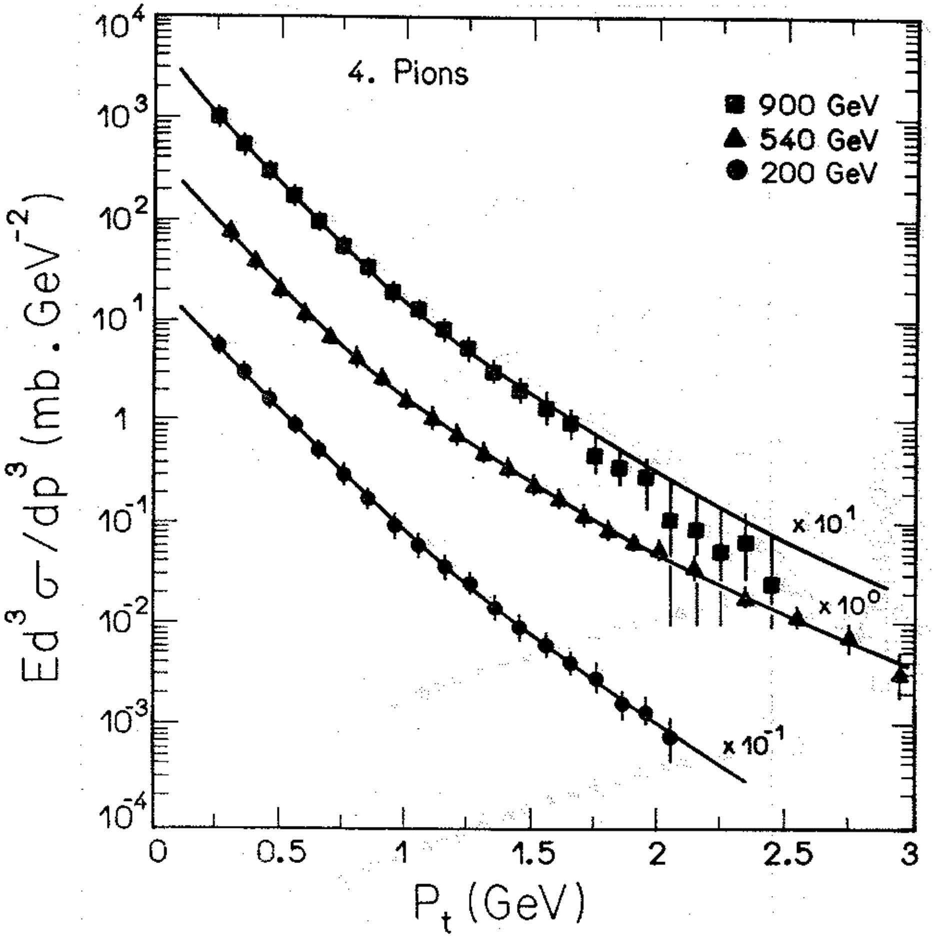


Figure 4

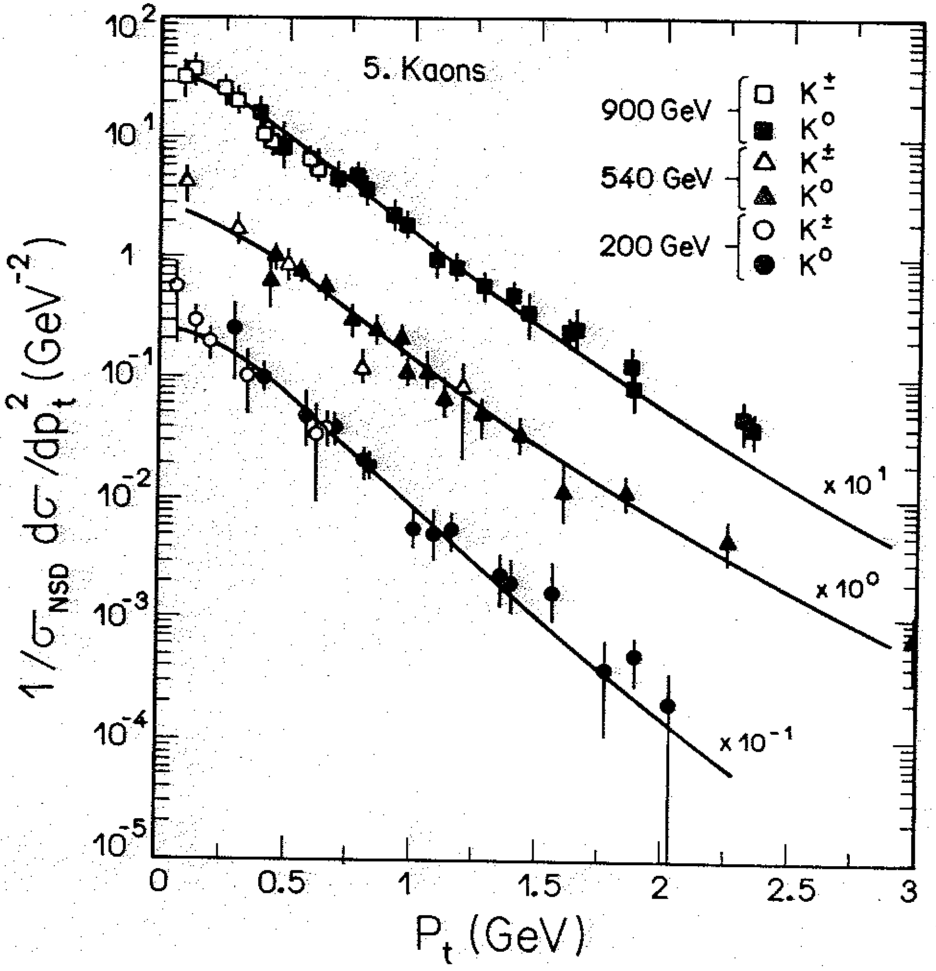


Figure 5

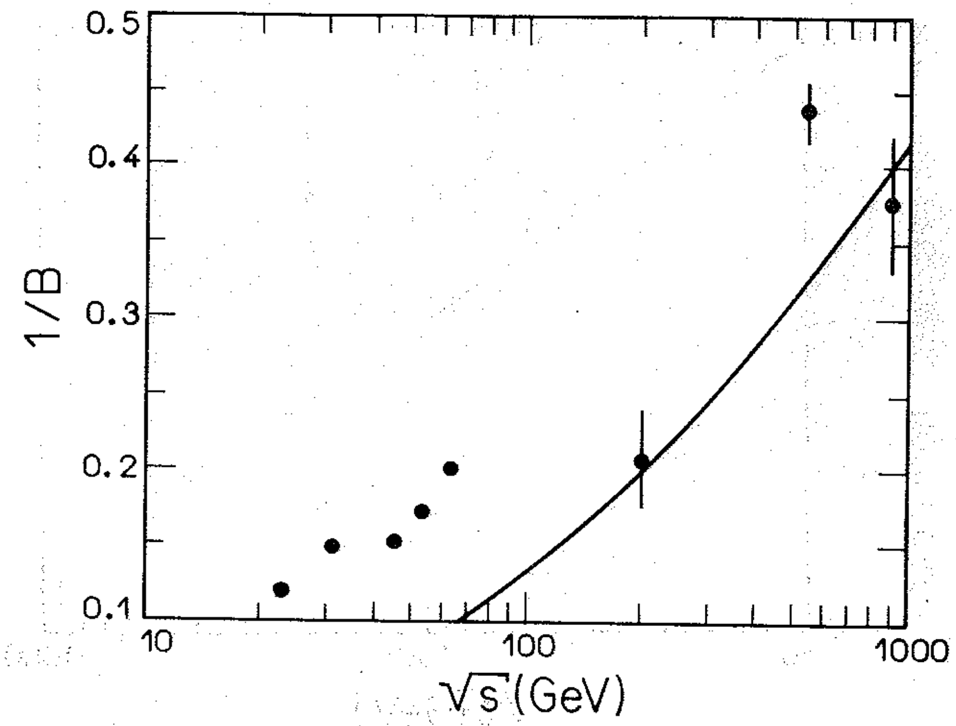
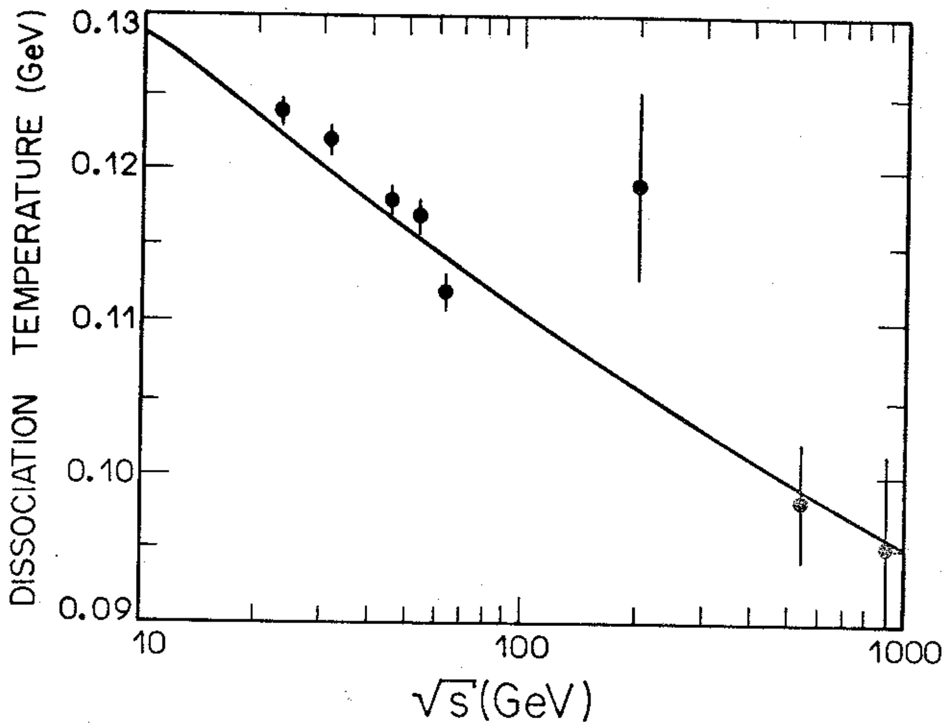


Figure 6



DISSOCIATION TEMPERATURE (GeV)
0.13
0.12
0.11
0.10
0.09
10
100
1000
√s (GeV)

Figure 7