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NEW METHOD FOR OBTAINING THE E0 CONTRIBUTION VIA γ_1-γ_3 ANGULAR CORRELATION: APPLICATION TO ^{100}Ru AND ^{110}Cd

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NEW METHOD FOR OBTAINING THE E0 CONTRIBUTION VIA γ_1–γ_3 ANGULAR CORRELATION: APPLICATION TO ^{100}Ru AND ^{110}Cd

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The use of the γ_1–γ_3 indirect angular correlation for the determination of the E0 component of the second transition is reviewed. A complete analysis based upon the least squares method is presented, and sensibility limits to the E0/E2 intensity ratio in even–even nuclei are calculated for several combinations of spin sequences and E2/M1 multipole mixtures of the first and second transitions. The statistical bias introduced by the least squares method is mapped and correction values are determined. The method allows one to stipulate the minimum counting statistics beyond which the E0/E2 intensity ratio can be determined with a standard deviation less than 100%. The E0/E2 intensity ratio of the 822 keV, $2' \to 2$ transition in ^{100}Ru was determined using this method, and it is 0.051(34). Older data from other authors on the 818 keV, $2' \to 2$ transition in ^{110}Cd were reanalized with this method, and the value 0.19(6) is suggested for the E0/E2 intensity ratio.

1. INTRODUCTION

The possibility of using indirect gamma angular correlations for the determination of the E0/E2 intensity ratio was previously examined, based upon the values of the angular correlation coefficients A_{22} and/or A_{44} [1]. The work of Gardulski and Wiedenbeck [2] has shown that the sensibility of this kind of experiment to this property is usually low, causing the standard deviation to be greater than or equal to 100%, and in some cases only an upper limit is possible to quote. This caused the abandon of that kind of measurement, and compilations like that of Lange et al. [3] do not even quote its results.

Our purpose here is to show that there is a statistical treatment that, allied to Monte Carlo simulations, permits the study of the counting statistics required for a selected sensibility to the E0/E2 intensity ratio, prior to the execution of the experiment.

2. ANGULAR CORRELATION FUNCTIONS

The angular correlation function usually fitted to the experimental data can be written

$$W_{exp}(\theta) = \alpha_0\left[1 + A_{22}\,\Omega_{22}\,P_2(\cos\theta) + A_{44}\,\Omega_{44}\,P_4(\cos\theta)\right] \quad , \tag{1}$$

where α_0 is the number of coincidences which would be observed in the isotropic case, A_{kk} are the anisotropy coefficients, Ω_{kk} are the solid angle and finite source size corrections, $P_k(\cos\theta)$ are the Legendre polynomials of order k, and θ is the angle between the detectors. Figure 1 shows a schematic diagram of a direct gamma cascade, where J, π are the angular momentum and parity quantum numbers of the nuclear states respectively, and L, L' are the two lowest possible quantum numbers of the angular momentum carried by the emitted radiation.

In the case of gamma angular correlations, the A_{kk} coefficients are given by

$$A_{kk} = B_k(\gamma_1)\,A_k(\gamma_2) \quad , \tag{2}$$

where

$$B_k(\gamma_1) = \frac{F_k(L_1L_1J_iJ) - 2\delta_1F_k(L_1L_1'J_iJ) + \delta_1^2F_k(L_1'L_1'J_iJ)}{1 + \delta_1^2} \quad , \tag{3}$$

and

$$A_k(\gamma_2) = \frac{F_k(L_2L_2J_fJ) + 2\delta_2F_k(L_2L_2'J_fJ) + \delta_2^2F_k(L_2'L_2'J_fJ)}{1 + \delta_2^2} \quad . \tag{4}$$

$B_k(\gamma_1)$ is the orientation parameter or statistical tensor of the initial state, and $A_k(\gamma_2)$ is the directional distribution coefficient for the second transition. The F_k symbols are the Frauenfelder coefficients [4], and δ_i is the multipole mixture of transition i.

In the particular case where the final nuclear spin is $J_f = 0$, the second transition is of pure multipolarity and $A_k(\gamma_2)$ reduces to

$$A_k(\gamma_2) = F_k(L_2L_2J_fJ) = F_k(JJ0J) \quad . \tag{5}$$

When the measured coincidence involves the first and third gamma rays of a triple cascade, as in figure 2, the A_{kk} coefficients acquire an extra factor, $U_k(J_1J_2)$, the deorientation coefficient, representing the sum over all possibilities for transition 2 to occur (gamma, conversion electron, etc., of all relevant multipolarities), not observed. The A_{kk} are written

$$A_{kk} = A_{kk}^{1-3} = B_k(\gamma_1)\,U_k(J_1J_2)\,A_k(\gamma_3) \quad , \tag{6}$$

where the third transition is supposed to be of pure multipolarity.

If the intermediate spins and parities are equal, i.e. $J_1 = J_2 = J$ $(> 1/2)$ and

$\pi_1 = \pi_2 = \pi$, the intermediate transition will have three relevant multipolarities, namely E0, M1, and E2. In this case, the $U_k(JJ)$ have the following form [1,5]

$$U_k(JJ) = (2J+1) \frac{\begin{Bmatrix} JJk \\ JJ0 \end{Bmatrix} Q^2 - \begin{Bmatrix} JJk \\ JJ1 \end{Bmatrix} \Delta^2 + \begin{Bmatrix} JJk \\ JJ2 \end{Bmatrix}}{Q^2 + \Delta^2 + 1} , \qquad (7)$$

normalized to $U_0(JJ) = 1$, where

$$Q^2 = Q^2(E0/E2) = \frac{\alpha}{1+\alpha} \left[\frac{<E0>_e}{<E2>_\gamma} \right]^2 = \frac{\alpha}{1+\alpha} q^2(E0/E2) = \frac{T(E0)}{T(E2)} , \qquad (8)$$

and

$$\Delta^2 = \frac{1+\beta}{1+\alpha} \left[\frac{<M1>_\gamma}{<E2>_\gamma} \right]^2 = \frac{1+\beta}{1+\alpha} \frac{1}{\delta_2^2} = \frac{T(M1)}{T(E2)} , \qquad (9)$$

$T(\lambda L)$ being the total transition probability for the multipole λL of the second transition, the curly bracketted symbols are 6j coefficients, β and α are the M1 and E2 conversion coefficients respectively, the $<\lambda L>_\gamma$ and $<\lambda L>_e$ are the multipolar nuclear matrix elements of the γ and conversion electron transitions respectively, and $q(E0/E2)$ is the multipole mixture as defined by Lange et al. [3].

3. METHOD OF ANALYSIS

The method is based upon the minimization of

$$S^2 = \sum_{i=1}^{N} \frac{1}{\sigma_i^2} \left[W_i - W_{exp}(\theta_i ; \alpha_0 , A_{22}^{1\text{-}3} , A_{44}^{1\text{-}3}) \right]^2 \qquad (10)$$

with respect to α_0, $A_{22}^{1\text{-}3}$ and $A_{44}^{1\text{-}3}$. W_i and σ_i are the experimental values and their uncertainties, respetively. Nevertheless, the $A_{kk}^{1\text{-}3}$ are not independent, but eqs. (6) involving δ_1, δ_2^2 and Q^2 do represent a constraint between them, and the fit of both as independent parameters is not recommended. For the determination of Q^2 we also need the multipole mixtures of the first and second transitions, namely δ_1 (E2/M1 in general) and δ_2(E2/M1). The problem can be made simpler if these two mixtures are previously determined in alternative direct correlations, generally present in the decay scheme. If this is done, we have only to minimize S^2 with respect to α_0 and Q^2. The extremum equations, $\partial S^2/\partial\alpha_0 = 0$ and $\partial S^2/\partial Q^2 = 0$, can be written in matrix form,

$$\tilde{U}B - \alpha_0 \tilde{U}AU = 0 \qquad (11a)$$

and

$$\frac{\partial}{\partial Q^2}(\tilde{U}B) - \frac{1}{2} \alpha_0 \frac{\partial}{\partial Q^2}(\tilde{U}AU) = 0 \qquad (11b)$$

respectively, where the tilde represents transposition, and the column vectors **U**, **B** and the symmetric matrix **A** are given by

$$U = \begin{bmatrix} 1 \\ U_2 \\ U_4 \end{bmatrix} , \quad B = \begin{bmatrix} W_i/\sigma_i^2 \\ W_i p_{2i}/\sigma_i^2 \\ W_i p_{4i}/\sigma_i^2 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 1/\sigma_i^2 & p_{2i}/\sigma_i^2 & p_{4i}/\sigma_i^2 \\ & p_{2i}^2/\sigma_i^2 & p_{2i}p_{4i}/\sigma_i^2 \\ & & p_{4i}^2/\sigma_i^2 \end{bmatrix} , \qquad (12)$$

where there are implicit sums over the $i = 1,2,...,N$ experimental points, and the p_{ki} are given by

$$p_{ki} = B_k(\gamma_1) A_k(\gamma_3) \Omega_{kk} P_k(\cos\theta_i) . \qquad (13)$$

From the fact that **A** is symmetric, it follows

$$\frac{\partial}{\partial Q^2}(\tilde{U}AU) = 2\frac{\partial \tilde{U}}{\partial Q^2}AU \quad , \tag{14}$$

and we rewrite eq. (11b)

$$\frac{\partial \tilde{U}}{\partial Q^2}(B - \alpha_0 AU) = 0 \quad . \tag{15}$$

It is easily verified that

$$\frac{\partial U}{\partial Q^2} = \frac{1}{Q^2+\Delta^2+1}(1 - U) \quad , \tag{16}$$

with

$$1 \equiv \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad ,$$

and eq. (15) becomes

$$\frac{1}{Q^2+\Delta^2+1}(\tilde{1} - \tilde{U})(B - \alpha_0 AU) = 0 \quad . \tag{17}$$

Since the denominator of eq. (17) is generally finite, it can be left behind, and the remainder of the equation might be seen as a linear combination of eq. (11a) with a simpler one, and we can adopt a new set of equations,

$$\tilde{U}(B - \alpha_0 AU) = 0 \tag{18a}$$

and

$$\tilde{1}(B - \alpha_0 AU) = 0 \quad . \tag{18b}$$

The quantity α_0 can be isolated in eq. (18b) and its insertion in (18a) ends with a remarkable, if more or less previsible result: (18a) turns out to be a first degree polynomial in Q^2, which means that the exact solution of the least squares problem for its determination might be achieved without the use of iterative methods, but is linear and then analytic. The standard deviations of α_0 and Q^2 are obtained in the usual way, calculating their covariance matrix.

Due to the quantity of arithmetical operations, the computer code must be all built in DOUBLE PRECISION, when written in FORTRAN 77 or equivalent languages.

4. STUDY OF THE SENSIBILITY LIMITS

With the analytic solution of the least squares problem, we did a study of the effect of the counting statistics (α_0, σ_α), and of δ_1, δ_2 and their uncertainties, for several spin sequences. This was performed via a simulation of the Q^2 distribution. Choosing $\overline{Q^2}$, $\overline{\alpha}_0$, $\overline{\delta}_1$, σ_{δ_1}, $\overline{\delta}_2$ and σ_{δ_2}, generating δ_1 and δ_2 with a gaussian random number generator (g.g.), and calculating the A_{kk}^{1-3} coefficients and the $\overline{W}_i$. Finally, each W_i is generated with a g.g., around $\overline{W}_i$, with standard deviation $\sqrt{\overline{W}_i}$. This data set is then submitted to the analysis of the previous section, returning Q^2 and α_0 values. Repeating this procedure many times, we are able to study the frequency distribution (f.d.) of Q^2, calculating its mean value and standard deviation. The f.d. of Q^2 is slightly asymmetric in general, as shown in figure 3.

The simulations were done for α_0 ranging from 10^3 to 5.10^4, Q^2 from 0 to 1, δ_1 from -10 to 10, δ_2 from 10^{-3} to 10, and for the following spin sequences: J_i ranging from 0 to 4, when J, J_f are 2 and 0 respectively, and J_i from 2 to 6 when J, J_f are 4 and 2. Each simulation involved seven $W(\theta)$ and σ_W pairs, with θ ranging from 90° to 270° in steps of 30°.

The study of the Q^2 distributions for the combinations of the parameters mentioned above, allowed us to write down an expression for the detection limit, defined as the standard deviation of Q^2 or, in other words, the Q^2 value which would be measured with a 100% standard deviation (~ 16% probability of being vanishing small). This limit is given by

$$Q^2_{1\,im} = \frac{1}{\sqrt{\alpha_0}} \left\{ \frac{1}{\delta_2^2} \left[(a \cdot J_i + b) \log|\delta_1| + (c \cdot J_i + d) \right] + e \right\} \qquad (19)$$

where the term involving $\log|\delta_1|$ is not present when the first transition is of pure multipolarity or of M3/E2 character. a, b, c, d and e are coefficients given in table 1, as functions of the intermediate and final spins of the cascade, J_i and J_f respectively. The $1/\sqrt{\alpha_0}$ factor in eq. (19) might be substituted by the mean relative standard deviation of one's data set, σ_W/W.

When determining the Q^2 value from one's data, one main point is the statistical bias introduced in the results by the least squares method due to the standard deviations of the intervening quantities (mainly the counting statistics). We have mapped this bias for the various cases simulated, and some of the results are shown in table 2. Values quoted in table 2 have to be subtracted from the Q^2 produced by the least squares method, in order to show the correct value. This bias was studied in all simulations performed, showing low sensibility to the Q^2 value.

It was also observed that the sensibility of this method rapidly decreases as the absolute value of one or both the multipole mixtures δ_1, δ_2 goes to values less than 1, as can be seen from eq. (19).

5. APPLICATIONS: 2' → 2 TRANSITIONS IN ^{100}Ru AND ^{110}Cd

A. ^{100}Ru

A study of ^{100}Rh β^+ and EC decays to ^{100}Ru was concluded recently by G. Kenchian [6], and a number of angular correlation data became available. Two indirect gamma cascades were chosen, with the 822 keV 2' → 2 transition as the unobserved one, and the angular correlation data are given in table 3, together with other information about the cascades.

The multipole conversion coefficients for the 822 keV transition were taken from the work of Hager and Seltzer [7], resulting $\alpha(E2) = 1.42 \times 10^{-3}$ and $\beta(M1) = 1.49 \times 10^{-3}$. The E2/M1 multipole mixture of the same transition was taken from Kenchian's work, δ (822 keV) = 3.70(41), and it is consistent with older data [3].

Application of eq. (19) to these data produces $Q^2_{1\,im} = 0.041$ for the detection limit of the E0/E2 intensity ratio. The resulting Q^2 values are 0.077(32) and 0.037(46) from the 1107–539 and 1153–539 cascades respectively, corrected for the statistical bias. The weighted mean of the previous values is the accepted E0/E2 intensity ratio of the 822 keV transition in ^{100}Ru, and it is 0.051(34), which has only about 7% probability of being vanishing small.

B. ^{110}Cd

Gardulski and Wiedenbeck [2] had used the method or ref. 1 in order to determine the E0 content of the 2' → 2 transition in several nuclei. All results shown in their work are affected by a 100% standard deviation. Since they furnish enough experimental detail in their work, we decided to reanalize the greatest value quoted there, that of ^{110}Cd.

Table 4 shows the cascades they had used, their anisotropy coefficients and resulting Q^2 values. With the anisotropy coefficients and their standard deviations we were able to recover the counting statistics, simulate the measured data under the conditions described by the authors, and reanalyze them.

Application of eq. (19) to the counting statistics of both cascades resulted $Q^2_{1\,im} = 0.13$ and $Q^2_{1\,im} = 0.073$ for the 3220 and 4220 cascades, respectively. Analysis of simulated data produced $Q^2 = 0.183(69)$ and $Q^2 = 0.24(17)$ for these cascades, and the weighted mean of 0.19(6) is a recommended value for this intensity ratio.

6. CONCLUSIONS

The analytic solution of the extremum equations of the least squares method applied to the determination of the E0/E2 intensity ratio enabled us to study the sensibility of the indirect gamma angular correlation to $Q^2(E0/E2)$ as well as to map the statistical bias present in the result. We were also able to express this sensibility in a closed form (eq. (19)), representing an estimate of the standard deviation that results from a chosen counting statistics, for fixed values of δ_1, δ_2 and spin sequence of the cascade. This expression can be trusted within a factor of two, as can be seen from the applications done in section 5.

By applying this method, we have determined the E0/E2 intensity ratio for the 822 keV transition in ^{100}Ru and also produced a recommended value for the 818 keV transition in ^{110}Cd, based upon measurements of Gardulski and Wiedenbeck. We recommend the reanalysis of their data within the framework of the method described here.

The loss of sensibility as δ_2 approaches zero is related to the reduction of the E2 intensity and its presence in the denominator of the Q^2 ratio. In these cases, the sensibility might be restored by defining an E0/M1 intensity ratio and applying the same analysis done in this work.

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FIGURE CAPTIONS

FIG. 1. Schematic representation of a direct γ_1–γ_2 cascade.

FIG. 2. Schematic representation of an indirect γ_1–γ_3 cascade.

FIG. 3. Absolute frequency vs. Q^2 obtained by simulating the 1553–539 cascade of ^{100}Ru, with $\overline{Q^2} = 0.051$, $\alpha_0 = 7200$ and 20000 trials.

TABLE 1. Coefficients of eq. (19).

JJ_f	a	b	c	d	e
20	− 2.23	0	2.30	0	1.10
42	0	− 1.14	0	26.9	4.57

TABLE 2. Q^2 statistical bias for various combinations of the other parameters. For each δ_1, the first value is related to $\delta_2 = \pm 1$ and the second to $\delta_2 = \pm 10$.

spin sequence / δ_1	0220	1220	2220	3220	4220	2442	3442	4442	5442	6442
10^{-3}	.039[a]	−.059	−.056	−.031	−.040	−.092[a]	−.020	−.057	−.011	−.0035[a]
	.0010	−.0010	0.	.0046	.0033	.0082	.010	.008	.010	.0069
10^{-1}		−.058	−.047	–			.064	−.0023[a]	–	
		−.0014	−.0019	–			.020	.022	–	
1		−.017	0.[a]	−.044			.058	–	–	
		−.0037	−.014[a]	−.0058			.022	–	.067	
10		.057	.068	−.0040[a]			−.079	−.039	–	
		.0056	.0032	−.0054[a]			−.0072	−.0026	.036	
-10^{-1}		−.037	−.056	−.050			–	−.043[a]	–	
		−.0036	−.001	−.0023			–	.010[a]	.061	
−1		−.0005[a]	−.035	−.049			–	.053	.065	
		−.010[a]	−.0019	−.0046			–	.024	.039	
−10		.060	.056	−.048[a]			−.068	.036	−.063	
		.0028	.012[a]	.0008[a]			−.0055	.0032[a]	−.0024	

[a]This value needs more accurate verification.

TABLE 3. $W(\theta)$ values for cascades in ^{100}Ru.

θ (°)	Cascade[a] 2(1107)2(822)2(539)0	2(1553)2(822)2(539)0
90	6174(125)	7360(115)
120	6122(123)	7326(116)
150	6173(124)	7275(118)
180	5965(120)	6983(103)
210	6131(125)	7233(116)
240	6454(125)	7339(116)
270	6269(123)	7479(114)

[a]The notation contains the spin sequence and numbers between parentheses are gamma transition energies, in keV.

TABLE 4. Directional correlation results of Gardulski and Wiedenbeck [2] for ^{110}Cd.

	Cascade[a]	
	3(687)2(818)2(658)0	4(744)2(818)2(658)0
A_{22}	−0.051(21)	0.017(9)
A_{44}	−0.033(28)	0.006(13)
Q^2	$0.044^{+0.179}_{-0.044}$	$0.176^{+0.219}_{-0.176}$

[a]Same notation as in Table 3.

Figure 1

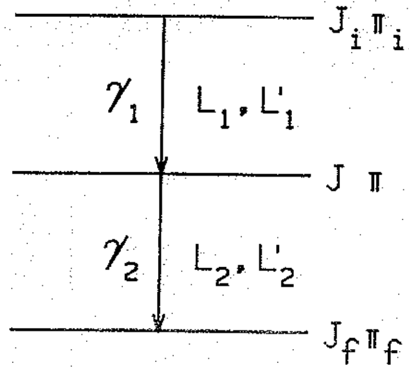


Figure 2

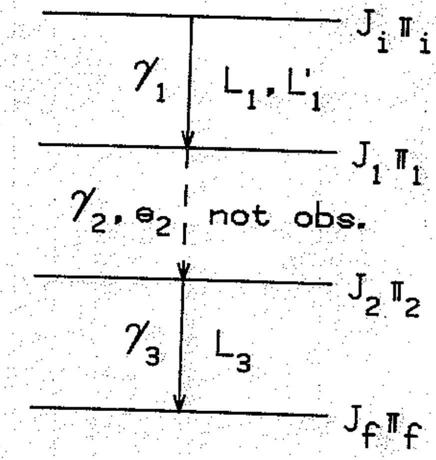
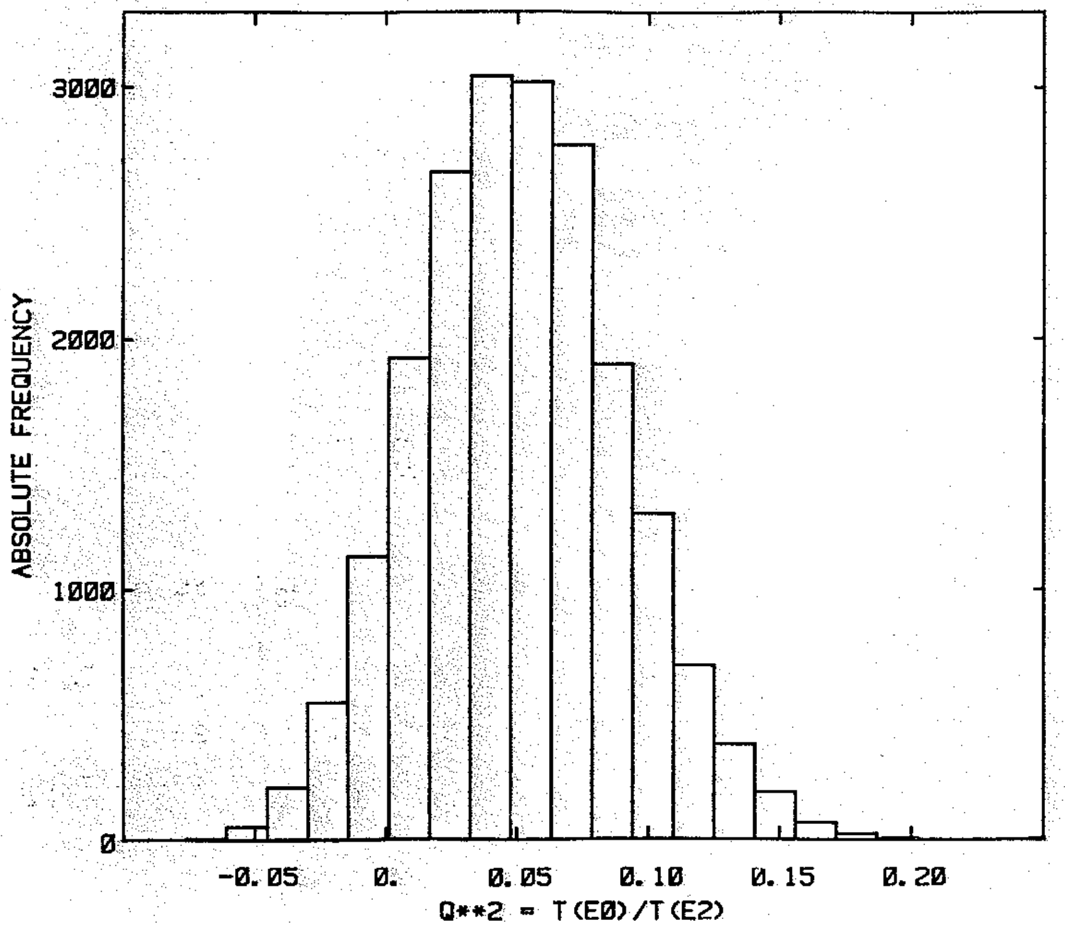


Figure 3



3000
2000
1000
0
ABSOLUTE FREQUENCY
-0.05
0.
0.05
0.10
0.15
0.20
Q**2 = T(E0)/T(E2)