UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE FÍSICA
CAIXA POSTAL 20516
01498 - SÃO PAULO - SP
BRASIL

PUBLICAÇÕES

IFUSP/P-882

UNITARY POLE APPROXIMATION FOR THE COULOMB-PLUS-YAMAGUCHI POTENTIAL AND APPLICATION TO A THREE-BODY BOUND STATE CALCULATION

K. Ueta
Instituto de Física, Universidade de São Paulo

G.W. Bund
Instituto de Física Teórica
Universidade Estadual Paulista
Rua Pamplona, 145, 01405 São Paulo, SP, Brasil

Dezembro/1990

UNITARY POLE APPROXIMATION FOR THE COULOMB-PLUS-YAMAGUCHI POTENTIAL AND APPLICATION TO A THREE-BODY BOUND STATE CALCULATION

K. Ueta
Instituto de Física da Universidade de São Paulo
C.P. 20516 - 01498 São Paulo, Brasil

G. W. Bund
Instituto de Física Teórica - Universidade Estadual Paulista
Rua Pamplona, 145 - 01405 São Paulo, Brasil

ABSTRACT

The Unitary Pole Approximation (UPA) is used to construct a separable representation for a potential U which consists of a Coulomb repulsion plus an attractive potential of the Yamaguchi type. The exact bound-state wave function is employed. U is chosen as the potential which binds the proton in the $1d_{5/2}$ single-particle orbit in ^{17}F. Using the separable representation derived for U, and assuming a separable Yamaguchi potential to describe the $1d_{5/2}$ neutron in ^{17}O, the energies and wave functions of the ground state (1^+) and the lowest 0^+ state of ^{18}F are calculated in the core-plus-two-nucleons model solving the Faddeev equations.

I. INTRODUCTION

In some three-body processes, the T-matrix associated to a pair of particles is dominated by the bound-state pole of the pair. In this case, the interaction U between these particles can be approximated by the separable potential

$$U_{sep} = U\,|\Psi^{BS}\rangle\,\langle\Psi^{BS}|U|\Psi^{BS}\rangle^{-1}\,\langle\Psi^{BS}|\,U\,, \tag{1}$$

$|\Psi^{BS}\rangle$ being the bound-state wave function. This approximation is known as Unitary Pole Approximation (UPA).

Our purpose is to construct, based on the UPA, a separable approximation for a potential U which consists of a short range attractive part, V, and of a Coulomb repulsion $V_C = (ZZ'e^2)/r$. As it is well known, the inclusion of the Coulomb interaction in the three-body problem presents many difficulties. Although it has been possible to extend the Faddeev formalism to include the Coulomb force[1], the numerical applications have been restricted to low values ($\lesssim 4$) of the product ZZ'[2,3]. The usual replacement of the Coulomb T-matrix by V_C (Born Approximation) becomes questionable as ZZ' increases.

In a recent calculation of the p-d break-up reaction[4], the long range tail of the Coulomb interaction is replaced by a short range potential and the EST method[5] is used to obtain a separable approximation for this cut-off Coulomb potential. The EST method is more general than the UPA. However, for the UPA it is not necessary to make any screening of the Coulomb tail.

II. CONSTRUCTION OF THE UPA FORM FACTOR

Having in mind applications to three-body systems consisting of two nucleons outside an inert and massive core, we consider U as being the single-particle potential of the proton.

For simplicity, it will be assumed that the short range part V is already separable and acts in a specific (ℓj) orbit of the shell model:

$$\langle\vec{p}|V|\vec{p}'\rangle = -\frac{\lambda_{\ell j}}{2m} g_{\ell j}(p)\, g_{\ell j}(p') \sum_{\mu} \langle \hat{p}|\mathcal{Y}_{\ell j\mu}\rangle \langle\mathcal{Y}_{\ell j\mu}|\hat{p}'\rangle, \qquad (2)$$

where $\vec{p}$ is the momentum of the proton (mass m) and

$$\langle\hat{p}|\mathcal{Y}_{\ell j\mu}\rangle = \sum_{m_\ell m_s} (\ell m_\ell\, \tfrac{1}{2}\, m_s|j\mu)\, Y^{m_\ell}_{\ell}(\hat{p})\, |\tfrac{1}{2}, m_s\rangle \cdot \qquad (3)$$

We assume also that the error made in considering the core as a point charge, which leads to an excess Coulomb repulsion in the region corresponding to the interior of the core, is compensated for by making the potential V more attractive.

It is convenient to choose a form factor of the Yamaguchi type:

$$g_{\ell j}(p) = \frac{p^{\ell}}{(p^2+\beta^2)^{\ell+1}} \cdot \qquad (4)$$

With this choice, the two-body problem corresponding to the potential $V+V_C$ can be solved exactly[6,7]. The energy $\epsilon_{\ell j}$ of the bound-state is determined by the equation

$$\lambda^{-1}_{\ell j} = \frac{\pi}{2} 4^{-\ell} \binom{2\ell+1}{\ell} \frac{\ell+1}{\ell+1-\frac{s}{\kappa}} (2\beta)^{-2\ell-1} (\beta+\kappa)^{-2} \cdot$$

$$\cdot\ {}_2F_1\left[1, -1-\frac{s}{\kappa};\ \ell+2-\frac{s}{\kappa};\ \left(\frac{\beta-\kappa}{\beta+\kappa}\right)^2\right], \qquad (5)$$

where $s = -[2mZZ'e^2]/2$ and $\kappa = (2m|\epsilon_{\ell j}|)^{1/2}$.
The corresponding wave function is given by

$$\Psi^{BS}_{\ell j\mu}(\vec{p}) = N_{\ell j} \frac{1}{p^2+\kappa^2} [g_{\ell j}(p) - v^C_{\ell j}(p)]\, \mathcal{Y}_{\ell j\mu}(\hat{p}), \qquad (6)$$

where

$$v^C_{\ell j}(p) = -\frac{1}{p}\left[\frac{4p\kappa^2}{(p^2+\kappa^2)(\beta^2-\kappa^2)}\right]^{\ell+1} \cdot$$

$$\sum_{n=\ell+1}^{\infty} \frac{\frac{s}{\kappa}}{n-\frac{s}{\kappa}} \left(\frac{\beta-\kappa}{\beta+\kappa}\right)^n C^{\ell+1}_{n-\ell-1}\left(\frac{p^2-\kappa^2}{p^2+\kappa^2}\right) \qquad (7)$$

and $N_{\ell j}$ is a normalization factor. The $C^m_n(z)$ are the Gegenbauer polynomials.

From expression (6), we see that the separable potential U_{sep} which generates the same bound state as the potential $V+V_C$ is given by

$$\langle\vec{p}|U_{sep}|\vec{p}'\rangle = -\frac{\Lambda_{\ell j}}{2m} g^C_{\ell j}(p)\, g^C_{\ell j}(p') \cdot$$

$$\cdot \sum_{\mu} \langle\hat{p}|\mathcal{Y}_{\ell j\mu}\rangle \langle\mathcal{Y}_{\ell j\mu}|\hat{p}'\rangle, \qquad (8)$$

with

$$g^C_{\ell j}(p) = g_{\ell j}(p) - v^C_{\ell j}(p), \qquad (9)$$

and $\Lambda_{\ell j}$ is determined by requiring that the bound state has the energy eigenvalue $\epsilon_{\ell j}$. Expression (8) is the UPA, Eq. (1), when the degeneracy introduced by the quantum number μ is included.

As an example, we consider U as the interaction which describes the $1d_{5/2}$ single-particle bound state of the proton in ^{17}F. The energy of the bound state ($\epsilon_{2\,5/2} = -0.596$ MeV) is reproduced if we take $\lambda_{2\,5/2} = 945\ fm^{-7}$ and $\beta = 1.464\ fm^{-1}$. in equations (2) and (4). The value of β is the same as the one appropriate for the neutron $1d_{5/2}$ single-particle state in ^{17}O (see

Sec. III). In Fig. 1, we show the radial function

$$u(r) = Ar^3 e^{-\kappa r} \int_0^1 x^{2+i\gamma} \left(1 + \frac{\beta-\kappa}{2\kappa} x\right)^{2-i\gamma} e^{-(\beta-\kappa)rx} dx, \qquad (10)$$

where A is a normalization constant and $\gamma = is/\kappa$ (Ref. 7, page 432). The corresponding mean-square radius is 3.74 fm. This value is very close to the value 3.69 fm given in Ref. 8. In Fig. 2, we plot the functions $g_{2\ 5/2}(p)$, $v^C_{2\ 5/2}(p)$ and $g^C_{2\ 5/2}(p)$ (Eqs. (4), (7) and (9)). It can be shown analytically that $v^C_{2\ 5/2}(p)$ behaves as p^2 when $p \to 0$ and as p^{-4} for $p \to \infty$. Finally, we mention that the coupling constant $\Lambda_{2\ 5/2}$ which appears in Eq. (8) results equal to 1103 fm^{-7}.

III. APPLICATION TO A THREE-BODY BOUND STATE CALCULATION

We consider the nucleus ^{18}F as a three-body system composed of an ^{16}O core plus a proton (particle 1) and a neutron (particle 2). We restrict ourselves to bound states dominated by the $(1d_{5/2}, 1d_{5/2})$ configuration and, in fact, consider only the ground state (1^+) and the lowest 0^+ state (excitation energy 1.042 MeV).

For the proton-^{16}O interaction, $U_1 = V_1 + V_C$, we use the UPA potential described in the previous section. For the neutron-core interaction, V_2, a potential of same form as the short-range part V_1 of the proton interaction (Eqs. (2)-(4)) is used. The parameters are chosen in such a way to reproduce the energy (-4.146 MeV) and the radius[8] (3.464 fm) of the $1d_{5/2}$ single-particle state in ^{17}O. Thus, the values $\lambda^{(2)}_{2\ 5/2} = 924$ fm^{-7} and $\beta^{(2)}_{2\ 5/2} = 1.464$ fm^{-1} are obtained. We here make the remark that since $\lambda^{(1)}_{2\ 5/2}$ (= 945 fm^{-7}) is larger than $\lambda^{(2)}_{2\ 5/2}$, V_1 is more attractive than V_2. This is expected since, as was pointed out in Sec. II, V_1 has an artificial attractive part to compensate for the

excess Coulomb repulsion.

For the neutron-proton interaction, V_{12}, we use the separable s-wave Yamaguchi potential:

$$\langle\vec{q}|V_{12}|\vec{q}\,'\rangle = \sum_{S=0,1} -\frac{\Lambda_S}{m} g_S(q)\, g_S(q') \sum_{M_S} |SM_S\rangle\langle SM_S| , \qquad (11)$$

$$g_S(q) = \frac{1}{q^2+\beta_S^2} \qquad (12)$$

In Eq. (11), $\vec{q}$ is the relative momentum $\frac{1}{2}(\vec{P}_1-\vec{P}_2)$, $\vec{P}_1$ being the momentum of particle 1, and $|SM_S\rangle$ is the spin wave function for the proton-neutron system. The values used for the parameters are $\Lambda_0 = 0.149$ fm^{-3}, $\beta_0 = 1.165$ fm^{-1}, $\Lambda_1 = 0.382$ fm^{-3} and $\beta_1 = 1.406$ fm^{-1}, which are determined from the values a_s=-23.71 fm, r_{os} = 2.70 fm, a_t = 5.42 fm and r_{ot} = 1.76 fm for the scattering length and the effective range of the neutron-proton scattering.

Performing the Faddeev decomposition of the total wave function, $\Psi = \Psi^{(1)}+\Psi^{(2)}+\Psi^{(3)}$, we obtain the following expressions for the components $\Psi^{(i)}$:

$$\Psi^{(1)}(\vec{P}_1, \vec{P}_2) = \frac{2m}{2mE-P_1^2-P_2^2} \sum_{\ell' j'} g^C_{2\ 5/2}(P_1) \frac{H^{(1)}_{2\ 5/2,\ell' j';J}(P_2)}{P_2} \cdot$$

$$\cdot\ \mathcal{Y}_{2\ 5/2,\ \ell' j';JM_J}(\hat{P}_1, \hat{P}_2), \qquad (13)$$

$$\Psi^{(2)}(\vec{P}_1, \vec{P}_2) = \frac{2m}{2mE-P_1^2-P_2^2} \sum_{\ell' j'} g^{(2)}_{2\ 5/2}(P_2) \frac{H^{(2)}_{2\ 5/2,\ell' j';J}(P_1)}{P_1} \cdot$$

$$\mathcal{Y}_{\ell' j',2\ 5/2;JM_J}(\hat{P}_1, \hat{P}_2), \qquad (14)$$

$$\Psi^{(3)}(\vec{P},\vec{p}) = \frac{2m}{2mE - \frac{1}{2}P^2 - 2p^2} \sum_{LS} \sqrt{4\pi}\; g_S(p)\, \frac{H_{LS;J}(P)}{P} \cdot$$

$$\cdot\; \mathcal{Y}_{LO(L)S;JM_J}(\hat{P},\hat{p}) \cdot \qquad (15)$$

In Eqs. (13)-(15), E is the energy of the three-body bound state, (J,M_J) denotes the total angular momentum, $\vec{P}$ is the center of mass momentum $\vec{P}_1+\vec{P}_2$ and the $\mathcal{Y}$'s are the usual total angular momentum eigenfunctions.

The spectator functions H satisfy the homogeneous integral equations given in Refs. 10 and 11. For the 1^+ state, we have $(\ell',j') = (2, 5/2), (2, 3/2), (4, 7/2)$, $L = 0,2$ and $S = 1$ in expansions (13)-(15) and, for the 0^+ state, $(\ell',j') = (2, 5/2)$, $L = 0$ and $S = 0$. The coupled integral equations are transformed into a system of algebraic equations by applying the N point Gauss quadrature method for the integrals. The vanishing of the associated determinant gives the energy eigenvalue. In this way, we get $E_{1^+} = -10.31$ MeV and $E_{0^+} = -7.93$ MeV. These numbers are close to the experimental values[12] $E^{exp}_{1^+} = -9.75$ MeV and $E^{exp}_{0^+} = -8.71$ MeV, despite of the fact that only the $1d_{5/2}$ interaction is considered.

In order to evaluate the contribution of the Coulomb force to the three-body bound state energy, we replace the valence proton by a neutron and calculate the energy of the 0^+ ground state of ^{18}O obtaining $E_{0^+}(^{18}O) = -11.38$ MeV. Therefore, in our model, the switching-on of the proton-core Coulomb interaction raises the energy by an amount $E_{0^+}(^{18}F) - E_{0^+}(^{18}O) = -7.93$ MeV + 11.38 MeV = 3.45 MeV. Experimentally, one has $E^{exp}_{0^+}(^{18}F) - E^{exp}_{0^+}(^{18}O) = -8.71$ MeV + 12.19 MeV = 3.48 MeV. This shows that the UPA is able to yield a correct value of the Coulomb energy.

In Figs. 3 and 4, we plot the spectator functions versus momentum. From the closeness of $H^{(1)}$ and $H^{(2)}$, we conclude that the asymmetry introduced in the total wave function (Eqs. (13)-(15)) by the Coulomb force lies mainly in the difference

$v^C_{2\ 5/2}$ between the form factors $g^C_{2\ 5/2}$ and $g^{(2)}_{2\ 5/2}$ ($g_{2\ 5/2}$ in Fig. 2) and is about twenty percent. We make here the remark that in the numerical calculations we found it convenient to multiply $g^C_{2\ 5/2}$ and $g^{(2)}_{2\ 5/2}$ by a factor 15.50 and, accordingly divide $\Lambda_{2\ 5/2}$ and $\lambda^{(2)}_{2\ 5/2}$ by $(15.50)^2$. Therefore, the actual values of $H^{(1)}$ and $H^{(2)}$ are 15.50 times those shown in Figs. 3 and 4.

It is our purpose to extend the present calculation to other levels of ^{18}F and further we expect to be able to apply the UPA to describe the (d,n) stripping reaction on ^{16}O.

REFERENCES

1. E.O. Alt, W. Sandhas and H. Ziegelmann, Phys. Rev. C17, 1981 (1978).

2. C.E.M. Aguiar, J.R. Brinati and M.H.P. Martins, Nucl. Phys. A460, 381 (1986).

3. G.H. Berthold, A. Stadler and H. Zankel, Phys. Rev. C41, 1365 (1990).

4. P. Doleschall, H. Kröger and R.J. Slobodrian, Phys. Rev. C37, 927 (1988).

5. D.J. Ernst, C.M. Shakin and R.M. Thaler, Phys. Rev. C8, 46 (1973).

6. H. van Haeringen, J. Math. Phys. 24, 1274 (1983).

7. H. van Haeringen, *Charged-Particle Interactions* (Coulomb Press, Leyden, 1985).

8. B.A. Brown, S.E. Massen and P.E. Hodgson, J. Phys. G5, 1655 (1979).

9. Y. Akiyama, Prog. Theor. Phys. 23, 903 (1960).

10. K. Ueta, H. Miyake and A. Mizukami, Phys. Rev. C27, 389 (1983).

11. H. Miyake, A. Mizukami and K. Ueta, Nuovo Cimento 84A, 225 (1984).

12. F. Ajzenberg-Selove, Nucl. Phys. A190, 1 (1972).

FIGURE CAPTIONS

Figure 1 - Exact radial wave function for the $1d_{5/2}$ state in the Coulomb-plus-Yamaguchi potential.

Figure 2 - Form factors $g^C_{2\ 5/2}$ of UPA, $g_{2\ 5/2}$ of the Yamaguchi potential and the difference $v^C_{2\ 5/2}$ between $g_{2\ 5/2}$ and $g^C_{2\ 5/2}$.

Figure 3 - Spectator functions for the ground state of ^{18}F.

Figure 4 - Spectator functions for the lowest 0^+ state of ^{18}F.

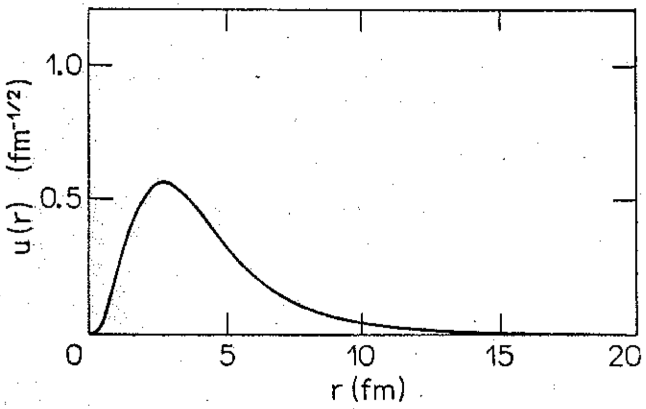

Fig. 1

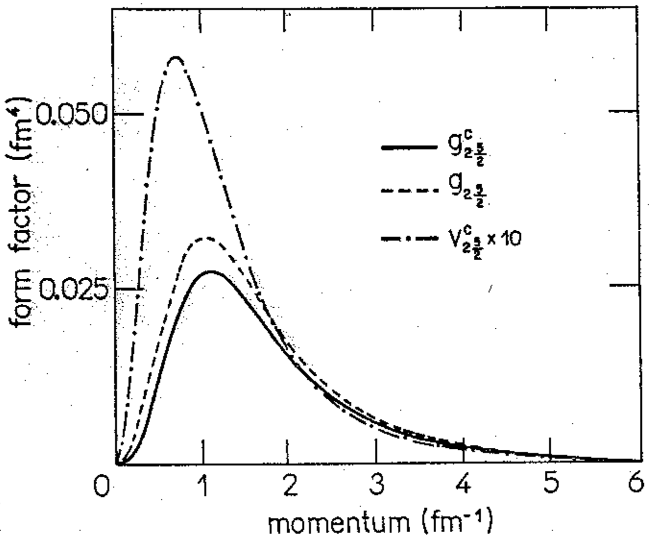

Fig. 2

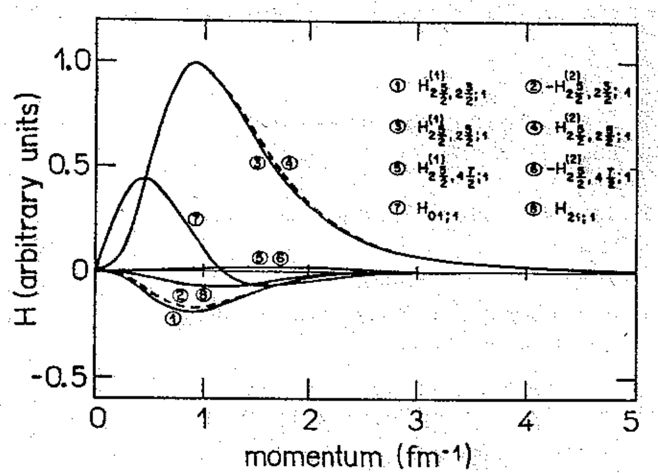

Fig. 3

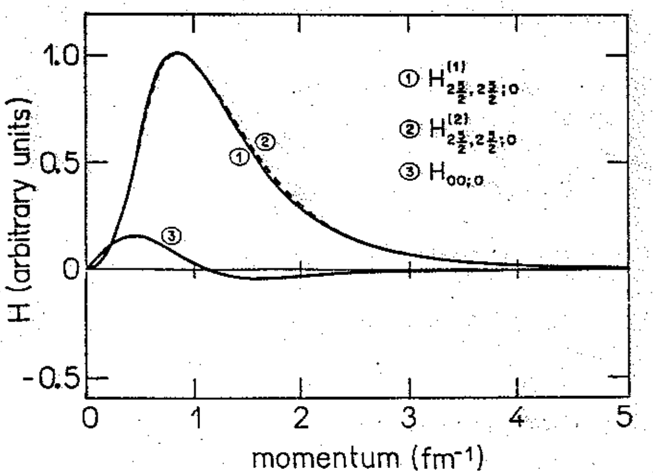

Fig. 4