UNIVERSIDADE DE SÃO PAULO

INSTITUTO DE FÍSICA
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PUBLICAÇÕES

IFUSP/P-883

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Dezembro/1990

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Abstract: A classical relativistic approach based in the Vlasov equation is applied to the study of infinite nuclear matter. Hadronic matter couples to massive scalar and vector fields. The small amplitude oscillations around a stationary state are studied. Orthogonality and completeness relations and the energy weighted sum rule are obtained for the longitudinal modes. The appearance of the zero sound mode and the distribution of the strength among the different modes are discussed. It is seen that the scalar field only couples to the low energy excitations at high densities and for wavelengths not too long. The vector field couples strongly to the low energy excitations. The long wavelength limit and high density limit are studied.

1. INTRODUCTION

A model relativistic field theory of nuclear matter, known as quantum hadrodynamics (QHD), was proposed by Walecka [1,2] some years ago. The aim of this model is to study high density matter, which might be attained in the core region of neutron stars and in high-energy heavy-ion collisions. Most of the previous work in QHD has been performed in either the mean-field theory (MFT) or at the one-loop level, which includes the shift in the baryon vacuum energy [2,3]. The motivation for these studies was that the MFT should become increasingly valid as the density increases.

When a two-loop calculation is performed for the ground-state [4], the one-loop results change drastically. However, as pointed out by Furnstahl et al [4], the failure of the loop expansion does not necessarily imply that the MFT results are inaccurate representations of the underlying quantum field theory, specially at high density.

It is the purpose of this work to study the QHD in a classical approach, using the relativistic Vlasov equation, which may be regarded as the classical limit of the MFT. The relativistic Vlasov equation based on QHD has been used lately to study heavy-ion collisions [5,7], and its predictions are similar to the more difficult calculations based on the time-dependent Dirac equations [5]. This means that the use of the relativistic Vlasov equation appears as an alternative way to study relativistic systems.

We are interested in studying The RPA collective modes corresponding to small amplitude oscillations around a stationary state in nuclear matter. In section 2 we formulate the Vlasov equation based on QHD. The dispersion relation, orthogonality and completeness relations that the RPA normal modes fulfill are given in section 3. The numerical results and conclusions are presented in section 4.

2. THE VLASOV EQUATION FORMALISM

Following Walecka we consider a system of baryons, with mass, m interacting with and through a neutral scalar field Φ with mass m_s, and a neutral vector field $V^\mu = (V_0, \mathbf{V})$ with mass m_v. If g_v and g_s are the vector and scalar coupling constants respectively, and denoting by $f(\mathbf{x}, \mathbf{p}, t)$ the one-body phase-space distribuction function, the energy of the system is

$$E = 2\int \frac{d^3x d^3p}{(2\pi)^3} f(\mathbf{x},\mathbf{p},t)\{[(\mathbf{p}-g_v\mathbf{V})^2 + (m-g_s\Phi)^2]^{1/2} + g_v V_0\}$$
$$+ \frac{1}{2}\int d^3x (\Pi_\Phi^2 + \nabla\Phi\cdot\nabla\Phi + m_s^2\Phi^2)$$
$$+ \frac{1}{2}\int d^3x [\Pi_{V_i}^2 - 2\Pi_{V_i}\partial_i V_0 + \nabla V_i\cdot\nabla V_i - \partial_j V_i \partial_i V_j + m_v^2(\mathbf{V}^2 - V_0^2)], \tag{2.1}$$

where $\Pi_{\Phi(V_i)}$ is the field canonically conjugated to $\Phi(V_i)$.

The time evolution of the distribution function is described by the Vlasov equation

$$\frac{\partial f}{\partial t} + \{f, h\} = 0 , \tag{2.2}$$

where $\{,\}$ means the usual Poisson bracket, and h, the classical effective one-body hamiltonian, is given by

$$h = \sqrt{(\mathbf{p} - g_v \mathbf{V})^2 + (m - g_s \Phi)^2} + g_v V_0 , \tag{2.3}$$

It has been argued in ref.[8] that eq.(2.2) expresses the conservation of the number of particles in phase space and is, therefore, covariant.

From Hamilton's equations we can derive the equations describing the time evolution of the fields Φ and V^μ :

$$\frac{\partial^2 \Phi}{\partial t^2} - \nabla^2 \Phi + m_s^2 \Phi = g_s \rho_s(\mathbf{x}, t) , \tag{2.4a}$$

$$\frac{\partial^2 V_0}{\partial t^2} - \nabla^2 V_0 + m_v^2 V_0 = g_v j_0(\mathbf{x}, t) + \frac{\partial}{\partial t}\left(\frac{\partial V_0}{\partial t} + \nabla \cdot \mathbf{V}\right) , \tag{2.4b}$$

$$\frac{\partial^2 V_i}{\partial t^2} - \nabla^2 V_i + m_v^2 V_i = g_v j_i(\mathbf{x}, t) + \frac{\partial}{\partial x_i}\left(\frac{\partial V_0}{\partial t} + \nabla \cdot \mathbf{V}\right) , \tag{2.4c}$$

where

$$\rho_s(\mathbf{x}, t) = 2\int \frac{d^3p}{(2\pi)^3} f(\mathbf{x}, \mathbf{p}, t) \frac{m - g_s \Phi}{\epsilon} , \tag{2.5}$$

is the scalar density,

$$j_0(\mathbf{x}, t) = 2\int \frac{d^3p}{(2\pi)^3} f(\mathbf{x}, \mathbf{p}, t) , \tag{2.6}$$

$$\mathbf{j}(\mathbf{x}, t) = 2\int \frac{d^3p}{(2\pi)^3} f(\mathbf{x}, \mathbf{p}, t) \frac{\mathbf{p} - g_v \mathbf{V}}{\epsilon} , \tag{2.7}$$

are the components of the four-current, and

$$\epsilon = \sqrt{(\mathbf{p} - g_v \mathbf{V})^2 + (m - g_s \Phi)^2} . \tag{2.8}$$

It can be easily seen that the four-current satisfies the continuity equation. From eq.(2.3) we can write

$$\mathbf{j}(\mathbf{x}, t) = 2\int \frac{d^3p}{(2\pi)^3} f(\mathbf{x}, \mathbf{p}, t) \frac{\partial h}{\partial \mathbf{p}}$$

and therefore it is straightforward to show that

$$\frac{\partial j_0}{\partial t} + \nabla \cdot \mathbf{j} = 2\int \frac{d^3p}{(2\pi)^3} \left(\frac{\partial f}{\partial t} + \{f, h\}\right) . \tag{2.9}$$

Using eq.(2.2) it follows that

$$\partial_\mu j^\mu = 0 . \tag{2.10}$$

This continuity equation also gives (from eqs.(2.4b and c)) the following relation between the components of the vector field :

$$\frac{\partial V_0}{\partial t} + \nabla \cdot \mathbf{V} = 0 . \tag{2.11}$$

At zero temperature and for particles obeying Fermi-Dirac statistics, the value of the distribution function is either 1 or 0, since the single particle state is either occupied by one particle or empty. The state which minimizes the energy of the system is characterized by the Fermi momentum P_F, and is described by the distribution function

$$f_0(\mathbf{x}, \mathbf{p}) = \xi\, \Theta[P_F^2 - p^2] , \tag{2.12a}$$

by a constant scalar field given by the self-consistent equation

$$m_s^2 \Phi_0 = g_s \rho_s^0(M) , \tag{2.12b}$$

with M denoting the effective baryon mass :

$$M = m - g_s \Phi_0 , \tag{2.12c}$$

and by a constant vector field V_0^μ, where

$$V_0^0 = g_v j_0^0 , \tag{2.12d}$$

$$V_0^i = 0 . \tag{2.12e}$$

In equation (2.12a) ξ stands for the isospin degeneracy : $\xi = 1$ for neutron matter and $\xi = 2$ for nuclear matter.

Collective modes in the present approach correspond to small oscillations around the equilibrium state. The linearized equations of motion describe small deviations from the equilibrium state. Therefore, collective modes will be given as solutions of the linearized equations of motion. To construct these equations let

$$f = f_0 + \delta f , \tag{2.13a}$$

$$\Phi = \Phi_0 + \delta\Phi , \tag{2.13b}$$

$$V_0 = V_0^0 + \delta V_0 , \tag{2.13c}$$

$$V_i = \delta V_i . \tag{2.13d}$$

As in ref.[8] we introduce a generating function $S(\mathbf{x}, \mathbf{p}, t)$ such that

$$\delta f = \{S, f_0\} = -\xi\{S, p^2\}\delta(P_F^2 - p^2) . \tag{2.14}$$

In terms of this generating function, the linearized Vlasov equation for δf is equivalent to the following time evolution equation

$$\frac{\partial S}{\partial t} + \{S, h_0\} = \delta h = -g_s \delta\Phi \frac{M}{\epsilon_0} + g_v \delta V_0 - g_v \frac{\mathbf{p}}{\epsilon_0} \cdot \delta \mathbf{V} , \tag{2.15}$$

which has to be satisfied only for $p = P_F$. In eq.(2.15)

$$h_0 = \sqrt{p^2 + M^2} + g_v V_0^0 = \epsilon_0 + g_v V_0^0 . \tag{2.16}$$

The linearized equations of the fields are

$$\frac{\partial^2 \delta\Phi}{\partial t^2} - \nabla^2 \delta\Phi + m_s^2 \delta\Phi = g_s \delta\rho_s , \tag{2.17a}$$

$$\frac{\partial^2 \delta V_0}{\partial t^2} - \nabla^2 \delta V_0 + m_v^2 \delta V_0 = g_v \delta j_0 , \tag{2.17b}$$

$$\frac{\partial^2 \delta V_i}{\partial t^2} - \nabla^2 \delta V_i + m_v^2 \delta V_i = g_v \delta j_i , \tag{2.17c}$$

with

$$\delta\rho_s = \frac{2}{(2\pi)^3} \int d^3p\, \delta f \frac{M}{\epsilon_0} - g_s \delta\Phi \frac{d}{dM}(\rho_s^0) , \tag{2.18a}$$

$$\delta j_0 = \frac{2}{(2\pi)^3} \int d^3p\, \delta f , \tag{2.18b}$$

$$\delta j_i = \frac{2}{(2\pi)^3} \int d^3p\, \delta f \frac{p_i}{\epsilon_0} - \frac{2 g_v}{(2\pi)^3} \int d^3p\, f_0 \left(\frac{\delta V_i}{\epsilon_0} - \frac{p_i \mathbf{p} \cdot \delta \mathbf{V}}{\epsilon_0^3} \right) . \tag{2.18c}$$

Of particular interest on account of their physical relevance are the longitudinal modes, with momentum k and frequency ω, described by the ansatz

$$\begin{pmatrix} S(\mathbf{x},\mathbf{p},t) \\ \delta\Phi \\ \delta V_0 \\ \delta V_i \end{pmatrix} = \begin{pmatrix} S_\omega(\cos\theta) \\ \delta\Phi_\omega \\ \delta V_\omega^0 \\ \delta V_\omega^i \end{pmatrix} e^{i(\omega t - \mathbf{k}\cdot\mathbf{x})} , \tag{2.19}$$

where θ is the angle between p and k. For these modes, we get $\delta V_\omega^x = \delta V_\omega^y = 0$, and calling $\delta V_\omega^z = \delta V_\omega$, the equations of motion become

$$i(\omega - \omega_0 x) S_\omega(x) = -g_s \frac{M}{\epsilon_F} \delta\Phi_\omega + g_v \delta V_\omega^0 - g_v \frac{P_F}{\epsilon_F} x \delta V_\omega , \tag{2.20a}$$

$$\left(\omega^2 - k^2 - m_s^2 - g_s^2 \frac{d\rho_s^0}{dM}\right) \delta\Phi_\omega = -\frac{2i\xi}{(2\pi)^2} g_s \omega_0 P_F M \int_{-1}^{1} dx x S_\omega(x) , \tag{2.20b}$$

$$\left(\omega^2 - k^2 - m_v^2\right) \delta V_\omega^0 = -\frac{2i\xi}{(2\pi)^2} g_v \omega_0 P_F \epsilon_F \int_{-1}^{1} dx x S_\omega(x) , \tag{2.20c}$$

$$\left(\omega^2 - k^2 - m_v^2 - \Omega^2\right) \delta V_\omega^0 = -\frac{2i\xi}{(2\pi)^2} g_v \omega_0 P_F^2 \int_{-1}^{1} dx x^2 S_\omega(x) , \tag{2.20d}$$

where $\omega_0 = kP_F/\epsilon_F$, $\epsilon_F = \sqrt{P_F^2 + M^2}$ and

$$\Omega^2 = \frac{2g_v^2}{(2\pi)^3} \int d^3p\, f_o \left(\frac{1}{\epsilon_0} - \frac{p^2 \cos^2(\theta)}{\epsilon_0^3} \right) = \frac{4}{3} \frac{\xi}{(2\pi)^2} g_v^2 \frac{P_F^3}{\epsilon_F} . \tag{2.21}$$

It is important to notice that the continuity equations (in form of eqs.(2.11)) is contained in eqs.(2.20). If we integrate x times eq.(2.20a) from -1 to 1 we get

$$\omega \int_{-1}^{1} dx\, x S_\omega(x) - \omega_0 \int_{-1}^{1} dx\, x^2 S_\omega(x) = i \frac{2}{3} g_v \frac{P_F}{\epsilon_F} \delta V_\omega .$$

Using this relation in eqs.(2.20c) and (2.20d), we obtain

$$\omega \delta V_\omega^0 = k \delta V_\omega , \tag{2.22}$$

which is equivalent to the continuity equation.

The solutions of eqs.(2.20) form a complete set of eingenmodes [9] which may be used to construct a general solution for an arbitrary longitudinal perturbation

3. GENERAL SOLUTION OF THE EINGENMODES EQUATIONS

Defining the dimensionless quantities

$$Q_{1\omega} = \frac{\delta\Phi_\omega}{m} , \; Q_{2\omega} = \frac{\delta V_\omega^0}{m} , \; Q_{3\omega} = \frac{\delta V_\omega}{m} , \; S_\omega(x) = \sqrt{\frac{\xi P_F \epsilon_F}{2\pi^2}} \frac{S_\omega(x)}{m}$$

$$G_1 = \frac{g_s}{\omega_0} \sqrt{\frac{\xi P_F}{2\pi^2 \epsilon_F}} M , \tag{3.1a}$$

$$G_2 = \frac{g_v}{\omega_0} \sqrt{\frac{\xi P_F \epsilon_F}{2\pi^2}} , \tag{3.1b}$$

$$G_3 = \frac{g_v}{\omega_0} \sqrt{\frac{\xi P_F}{2\pi^2 \epsilon_F}} P_F = \frac{P_F}{\epsilon_F} G_2 , \tag{3.1c}$$

$$\omega_1 = \frac{1}{\omega_0} \left(k^2 + m_s^2 + \frac{d\rho_s^0}{dM} \right) , \tag{3.1d}$$

$$\omega_2 = \frac{1}{\omega_0} (k^2 + m_v^2) , \tag{3.1e}$$

$$\omega_3 = \frac{1}{\omega_0} (k^2 + m_v^2 + \Omega^2) = \omega_2^2 + \frac{2}{3} G_3^2 , \tag{3.1f}$$

and $\overline{\omega} = \omega/\omega_0$, we can rewrite eqs.(2.20) as

$$(\overline{\omega} - x) S_\omega(x) = i G_1 Q_{1\omega} - i G_2 Q_{2\omega} + i G_3 x Q_{3\omega} , \tag{3.2a}$$

$$(\overline{\omega}^2 - \omega_1^2) Q_{1\omega} = -i G_1 \int_{-1}^{1} dx x S_\omega(x) , \tag{3.2b}$$

$$(\overline{\omega}^2 - \omega_2^2)Q_{2\omega} = -iG_2 \int_{-1}^{1} dx\, x\, S_\omega(x) , \tag{3.2c}$$

$$(\overline{\omega}^2 - \omega_3^2)Q_{3\omega} = -iG_3 \int_{-1}^{1} dx\, x^2 S_\omega(x) . \tag{3.2d}$$

Equation (2.22) is now equivalent to the relation

$$\overline{\omega} Q_{2\omega} = \frac{\epsilon_F}{P_F} Q_{3\omega} . \tag{3.3}$$

These equations may be derived from the Lagrangian :

$$L = \sum_{j=1}^{3} (-1)^{j+1} (P_j^* \dot{Q}_j + P_j \dot{Q}_j^*) - i \int_{-1}^{1} S^*(x) \dot{S}(x) x dx - \int_{-1}^{1} |S(x)|^2 x^2 dx$$

$$+ \sum_{j=1}^{3} (-1)^j (|P_j|^2 + \omega_j^2 |Q_j|^2) - iG_1 Q_1 \int_{-1}^{1} S^*(x) x dx + iG_1 Q_1^* \int_{-1}^{1} S(x) x dx + iG_2 Q_2 \int_{-1}^{1} S^*(x) x dx$$

$$- iG_2 Q_2^* \int_{-1}^{1} S(x) x dx - iG_3 Q_3 \int_{-1}^{1} S^*(x) x^2 dx + iG_3 Q_3^* \int_{-1}^{1} S(x) x^2 dx , \tag{3.4}$$

where all time derivatives are considered with respect to $\tau = \omega_0 t$. In eq.(3.4) P_j is the momentum canonically conjugate to Q_j.

It is convenient to work in a Lagrangian formalism because the Euler- Lagrange equations are a natural way to introduce the normal modes, and because orthogonality relations and sum rules are easily obtained from the Lagrangian. This Lagrangian is, however, only formal. It was written in this way in order to derive eqs.(3.2). Therefore, the presence of the time derivative of the time component of the vector field ($\dot{Q}_2$), and its canonically conjugate momentum (P_2), that does not appear in a conventional Lagrangian formalism, does not imply that Q_2 and P_2 are independent dynamic variables.

Using the ansatz

$$\Psi_\omega(\tau) = \begin{pmatrix} Q_{1\omega}(\tau) \\ P_{1\omega}(\tau) \\ Q_{2\omega}(\tau) \\ P_{2\omega}(\tau) \\ Q_{3\omega}(\tau) \\ P_{3\omega}(\tau) \\ S_\omega(x,\tau) \end{pmatrix} = \begin{pmatrix} Q_{1\omega} \\ P_{1\omega} \\ Q_{2\omega} \\ P_{2\omega} \\ Q_{3\omega} \\ P_{3\omega} \\ S_\omega(x) \end{pmatrix} e^{i\overline{\omega}\tau} , \tag{3.5}$$

in the Euler-Lagrange equations we get for the normal modes

$$i\overline{\omega} Q_{j\omega} = P_{j\omega} , \ (j = 1,2,3), \tag{3.6a}$$

$$i\overline{\omega} P_{1\omega} + \omega_1^2 Q_{1\omega} = iG_1 \int_{-1}^{1} S_\omega(x) x dx , \tag{3.6b}$$

$$i\overline{\omega} P_{2\omega} + \omega_2^2 Q_{2\omega} = iG_2 \int_{-1}^{1} S_\omega(x) x dx , \tag{3.6c}$$

$$i\overline{\omega} P_{3\omega} + \omega_3^2 Q_{3\omega} = iG_3 \int_{-1}^{1} S_\omega(x) x^2 dx , \tag{3.6d}$$

$$(\overline{\omega} - x) S_\omega(x) = iG_1 Q_{1\omega} - iG_2 Q_{2\omega} + iG_3 x Q_{3\omega} . \tag{3.6e}$$

There are two types of solutions of eqs.(3.6) [10]. An even finite number (2N) of discrete modes $\overline{\omega} = \pm\overline{\omega}_{xn}$ if $|\overline{\omega}| > 1$, and a continuum of solutions if $|\overline{\omega}| < 1$. The dispersion relation for the discrete modes is

$$\overline{\omega}_{xn}^2 - \omega_2^2 - G_2 \int_{-1}^{1} \frac{x}{\overline{\omega}_{xn} - x} (f(\overline{\omega}_{xn}) + x\alpha(\overline{\omega}_{xn})) dx = 0 , \tag{3.7}$$

where

$$f(\overline{\omega}) = \frac{G_1^2}{G_2} \frac{\overline{\omega}^2 - \omega_2^2}{\overline{\omega}^2 - \omega_1^2} - G_2 , \tag{3.8a}$$

$$\alpha(\overline{\omega}) = G_3 \left(\frac{P_F}{\epsilon_F} \right)^2 \overline{\omega} . \tag{3.8b}$$

The solutions of eq.(3.7) are equivalent to the longitudinal undamped collective modes studied by Chin in a quantal one-loop approach [3]. The discrete modes are described by

$$\Psi_{\pm n} = \begin{pmatrix} Q_{1\pm n} \\ P_{1\pm n} \\ Q_{2\pm n} \\ P_{2\pm n} \\ Q_{3\pm n} \\ P_{3\pm n} \\ S_{\pm n}(x) \end{pmatrix} = \begin{pmatrix} -i\frac{G_1}{G_2}\frac{\overline{\omega}_{xn}^2 - \omega_2^2}{\overline{\omega}_{xn}^2 - \omega_1^2} \\ \pm\overline{\omega}_{xn}\frac{G_1}{G_2}\frac{\overline{\omega}_{xn}^2 - \omega_2^2}{\overline{\omega}_{xn}^2 - \omega_1^2} \\ -i \\ \pm\overline{\omega}_{xn} \\ \pm i\overline{\omega}_{xn}\frac{P_F}{\epsilon_F} \\ \overline{\omega}_{xn}^2\frac{P_F}{\epsilon_F} \\ \frac{1}{\pm\overline{\omega}_{xn} - x}(f(\overline{\omega}_{xn}) + x\alpha(\overline{\omega}_{xn})) \end{pmatrix} . \tag{3.9}$$

In the continuum the normal modes are given by

$$\Psi_\omega = \begin{pmatrix} Q_{1\omega} \\ P_{1\omega} \\ Q_{2\omega} \\ P_{2\omega} \\ Q_{3\omega} \\ P_{3\omega} \\ S_\omega(x) \end{pmatrix} = \begin{pmatrix} -i\frac{G_1}{G_2}\frac{\overline{\omega}^2 - \omega_2^2}{\overline{\omega}^2 - \omega_1^2} a(\overline{\omega}) \\ \overline{\omega}\frac{G_1}{G_2}\frac{\overline{\omega}^2 - \omega_2^2}{\overline{\omega}^2 - \omega_1^2} a(\overline{\omega}) \\ -ia(\overline{\omega}) \\ \overline{\omega} a(\overline{\omega}) \\ -i\overline{\omega}\frac{P_F}{\epsilon_F} a(\overline{\omega}) \\ \overline{\omega}^2\frac{P_F}{\epsilon_F} a(\overline{\omega}) \\ \delta(\overline{\omega} - x) + (f(\overline{\omega}) + x\alpha(\overline{\omega}))\frac{a(\overline{\omega})}{\overline{\omega} - x} \end{pmatrix} , \tag{3.10}$$

with $a(\overline{\omega})$ satisfying the equation

$$a(\overline{\omega}) = \frac{G_2 \overline{\omega}}{\overline{\omega}^2 - \omega_2^2 - G_2 \int_{-1}^{1} \frac{x}{\overline{\omega} - x} (f(\overline{\omega}) + x\alpha(\overline{\omega})) dx} . \tag{3.11}$$

In eq.(3.11) and in what follows, integrals involving the factor $\frac{1}{\overline{\omega} - x}$ have to be interpreted as principal value integrals.

From eqs.(3.6), (3.9) and (3.10), we can see that the normal modes satisfy the following orthogonality relations :

$$i \sum_{j=1}^{3} (-1)^{j+1} (P_{j\pm n}^* Q_{j\pm n} - Q_{j\pm n}^* P_{j\pm n}) + \int_{-1}^{1} x S_{\pm n}^*(x) S_{\pm n}(x) dx = \pm\eta_n , \tag{3.12a}$$

$$i\sum_{j=1}^{3}(-1)^{j+1}(P^*_{j\omega'}Q_{j\omega} - Q^*_{j\omega'}P_{j\omega}) + \int_{-1}^{1} xS^*_{\omega'}(x)S_\omega(x)dx = \overline{\omega}\delta(\overline{\omega} - \overline{\omega}') , \qquad (3.12b)$$

$$i\sum_{j=1}^{3}(-1)^{j+1}(P^*_{j\pm n}Q_{j\omega} - Q^*_{j\pm n}P_{j\omega}) + \int_{-1}^{1} xS^*_{\pm n}(x)S_\omega(x)dx = 0 , \qquad (3.12c)$$

where

$$\eta_n = 2\overline{\omega}_{xn}\left[\left(\frac{G_1}{G_2}\frac{\overline{\omega}^2_{xn} - \omega_2^2}{\overline{\omega}^2_{xn} - \omega_1^2}\right)^2 - 1 + \overline{\omega}^2_{xn}\left(\frac{P_F}{\epsilon_F}\right)^2\right] + \int_{-1}^{1}\frac{x}{(\overline{\omega}_{xn} - x)^2}(f(\overline{\omega}_{xn}) + x\alpha(\overline{\omega}_{xn}))^2 dx . \qquad (3.13)$$

This set of solutions is complete [9,10]. For any arbitrary initial state

$$\Psi_0 = \Psi(\tau = 0) = \begin{pmatrix} Q_{01} \\ P_{01} \\ 0 \\ i\frac{\epsilon_F}{P_F}Q_{03} \\ Q_{03} \\ P_{03} \\ H(x) \end{pmatrix} , \qquad (3.14)$$

there is a function $c(\overline{\omega})(|\overline{\omega}| < 1)$ and N numbers c_{+n}, c_{-n} such that

$$\Psi_0 = \int_{-1}^{1} c(\overline{\omega})\Psi_\omega d\overline{\omega} + \sum_{n=1}^{N}(c_{+n}\Psi_{+n} + c_{-n}\Psi_{-n}) . \qquad (3.15)$$

As it has been pointed out before, Q_2 and P_2 are not independent dynamic variables. To define the initial state of the system we only need to fix the initial values of Q_1, P_1, Q_3, P_3 and $H(x)$. In fact, from eq.(3.3) we obtain

$$P_{20} = i\frac{\epsilon_F}{P_F}Q_{30}$$

and we must have $Q_{20} = 0$.

Following van Kampen [9] we introduce the auxiliary functions :

$$F_\pm(y) = \frac{G_2}{2\pi i(y^2 - \omega_2^2)}\int_{-1}^{1} x\frac{f(y) + x\alpha(y)}{x - y \mp i\delta}dx , \qquad (3.16a)$$

$$G_\pm(y) = \frac{G_2}{2\pi i(y^2 - \omega_2^2)}\left(\int_{-1}^{1} xH(x)\frac{f(y) + x\alpha(y)}{x - y \mp i\delta}dx + \sum_{j=1}^{3}(A_j + yB_j)\right) , \qquad (3.16b)$$

and

$$K_\pm(y) = \frac{G_\pm(y)}{1 + 2\pi i F_\pm(y)} , \qquad (3.16c)$$

where A_j and B_j are constants to be determined by the initial values P_{0j} and Q_{0j} respectively. Using the standard formula

$$\frac{1}{x - y \mp i\delta} = \frac{\mathcal{P}}{x - y} \pm i\pi\delta(x - y) ,$$

we can show that

$$c(\overline{\omega}) = \frac{1}{a(\overline{\omega})(f(\overline{\omega}) + \overline{\omega}\alpha(\overline{\omega}))}(K_+(\overline{\omega}) - K_-(\overline{\omega})) . \qquad (3.17)$$

Using eqs.(3.16) and (3.17) we can write

$$c(\overline{\omega}) = \frac{\tilde{c}(\overline{\omega})}{1 + \pi^2(f(\overline{\omega}) + \overline{\omega}\alpha(\overline{\omega}))^2 a^2(\overline{\omega})} , \qquad (3.18)$$

with

$$\tilde{c}(\overline{\omega}) = \frac{1}{\overline{\omega}}\left(i\sum_{j=1}^{3}(-1)^{j+1}(P^*_{j\omega}Q_{0j} - Q^*_{j\omega}P_{0j}) + \int_{-1}^{1} xS^*_\omega(x)H(x)dx\right) . \qquad (3.19)$$

As shown in Ref.[10], it is not permissible to use the orthogonality relations to derive $c(\overline{\omega})$ on account of the singularities in $S_\omega(x)$. However, the expressions for $c_\pm$ are obtained directly from these relations. We get

$$c_{\pm n} = \pm\frac{1}{\eta_n}\left(i\sum_{j=1}^{3}(-1)^{j+1}(P^*_{j\pm n}Q_{0j} - Q^*_{j\pm n}P_{0j}) + \int_{-1}^{1} xS^*_{\pm n}(x)H(x)dx\right) . \qquad (3.20)$$

The solutions of the initial value problem is therefore

$$\Psi(\tau) = \int_{-1}^{1} c(\overline{\omega})\Psi_\omega e^{\overline{\omega}\tau}d\overline{\omega} + \sum_{n=1}^{N}(c_{+n}\Psi_{+n}e^{\overline{\omega}_{xn}\tau} + c_{-n}\Psi_{-n}e^{-\overline{\omega}_{xn}\tau}) . \qquad (3.21)$$

The amplitudes $\tilde{c}(\overline{\omega})$, c_{+n} and c_{-n} defined by eqs. (3.19) and (3.20) satisfy [10] the following energy weighted sum rule (EWSR) , which is known to be preserved by the mean field approximation

$$\int_{-1}^{1}\frac{\overline{\omega}^2|\tilde{c}(\overline{\omega})|^2}{1 + \pi^2(f(\overline{\omega}) + \overline{\omega}\alpha(\overline{\omega}))^2 a^2(\overline{\omega})}d\overline{\omega} + \sum_{n=1}^{N}\omega_n\eta_n(|c_{+n}|^2 + |c_{-n}|^2) =$$

$$\int_{-1}^{1} x^2|H(x)|^2 dx + \sum_{j=1}^{3}(-1)^{j+1}(|P_{0j}|^2 + \omega_j^2|Q_{0j}|^2) + iG_1Q_{01}\int_{-1}^{1} H^*(x)x dx +$$

$$-iG_1Q^*_{01}\int_{-1}^{1} H(x)x dx - iG_2Q_{02}\int_{-1}^{1} H^*(x)x dx + iG_2Q^*_{02}\int_{-1}^{1} H(x)x dx +$$

$$+iG_3Q_{03}\int_{-1}^{1} H^*(x)x^2 dx - iG_3Q^*_{03}\int_{-1}^{1} H(x)x^2 dx = m_1 , \qquad (3.22)$$

The strength function is defined in the interval (0,1[and is given by

$$s(\overline{\omega}) = \frac{2\overline{\omega}^2|\tilde{c}(\overline{\omega})|^2}{1 + \pi^2(f(\overline{\omega}) + \overline{\omega}\alpha(\overline{\omega}))^2 a^2(\overline{\omega})} , \qquad (3.23)$$

and the fraction exhausted by the discrete modes is

$$F(\overline{\omega}_x) = \frac{1}{m_1}\sum_{n=1}^{N}\overline{\omega}_{xn}\eta_n(|c_{+n}|^2 + |c_{-n}|^2) . \qquad (3.24)$$

4. NUMERICAL RESULTS AND CONCLUSIONS

The parameters of QHD used in all numerical calculation are given in ref.[2], i.e., $g_s^2 = 91.64$, $g_v^2 = 136.2$, $m_s = 550 MeV$ and $m_v = 738 MeV$. The dispersion relation eq.(3.7) has always an even number of solutions $\pm\overline{\omega}_{sn}$. At saturation density of symmetric nuclear matter ($P_F = 1.42 fm^{-1}$, $\xi = 2$ and $M/m = 0.556$) if $k/m < 0.4$ eq.(3.7) has only four solutions ($N = 2$). Two related with the scalar mode, $\pm\overline{\omega}_{z1}$, and another two related with the vector mode, $\pm\overline{\omega}_{z2}$.

Equation (3.7) was written in units of ω_0. It can be rewritten as ($\omega_z = \omega_0\overline{\omega}_z$)

$$\left(\omega_z^2 - k^2 - m_s^2 - g_s^2\frac{d\rho_s^0}{dM}\right)(\omega_z^2 - k^2 - m_v^2) + \left[2 + \frac{\epsilon_F\omega_z}{kP_F}ln\left(\frac{\omega_z - kP_F/\epsilon_F}{\omega_z + kP_F/\epsilon_F}\right)\right]$$
$$\times\left[\xi\frac{g_s^2}{2\pi^2}\frac{P_F M^2}{\epsilon_F}(\omega_z^2 - k^2 - m_v^2) + \xi\frac{g_v^2}{2\pi^2}P_F\epsilon_F\left(\omega_z^2 - k^2 - m_s^2 - g_s^2\frac{d\rho_s^0}{dM}\right)\left(\frac{\omega_z^2}{k^2} - 1\right)\right] = 0 . \quad (4.1)$$

In the limit $k \to 0$ we get

$$(\omega_z^2 - \mathcal{M}_s^2)(\omega_z^2 - \mathcal{M}_v^2) = 0 , \quad (4.2)$$

with solutions

$$\omega_{z1} = \pm\mathcal{M}_s = \pm\sqrt{m_s^2 + g_s^2\frac{d\rho_s^0}{dM}} , \quad (4.3a)$$

$$\omega_{z2} = \pm\mathcal{M}_v = \pm\sqrt{m_v^2 + \Omega^2} . \quad (4.3b)$$

These are exactly the solutions obtained by Chin [3] in the same limit, and show that the mesons behave as if they had an effective mass of $m_{s(v)}^{eff} = \mathcal{M}_{s(v)}$. In Fig.1 we plot ω_{zn}^2 as a function of k^2 for both modes. From this figure we see that, for these values of k, the relation between ω_z^2 and k^2 is almost linear, but, unlike the results obtained in ref.[3] (in the low k limit), the slopes of these curves are different from 1. We have

$$\omega_{z1}^2 \simeq 1.08k^2 + \mathcal{M}_s^2 , \quad (4.4a)$$

$$\omega_{z2}^2 \simeq 0.87k^2 + \mathcal{M}_v^2 . \quad (4.4b)$$

If we consider values of k/m greater than 1.0, the relation is no longer linear.

For $k/m > 0.4$ we still have the solutions $\pm\omega_{z1}$ and $\pm\omega_{z2}$ related with the scalar and vector modes, but two more solutions,$\pm\omega_{z3}$, appear now ($N = 3$). This solution is the one which Chin has identifyed as zero-sound, and Fig.2 shows the behaviour of this solution as a function of k. As we will see later analysing the EWSR, this solution is only important for $0.5 \leq k/m \leq 2.0$, at nuclear saturation density.

The appearance of this solution depends on the value of k. In refs. [10,11], where a study of Landau damping in infinity nuclear matter is done, the zero-sound mode will only appear for certain values of the interaction strength. Our present problem is more complex because together with the generating function,$S(\mathbf{x}, \mathbf{p}, t)$, we also have the meson fields. However,it can be seen from eq.(4.1) that changing k corresponds to changing the interaction strength in refs.[10,11]. In fact, rewriting eq.(4.1) as

$$1 + \left[2 + \frac{\epsilon_F\omega_z}{kP_F}ln\left(\frac{\omega_z - kP_F/\epsilon_F}{\omega_z + kP_F/\epsilon_F}\right)\right]\left[\xi\frac{g_s^2}{2\pi^2}\frac{P_F M^2}{\epsilon_F}\frac{1}{\omega_z^2 - k^2 - \mathcal{M}_s^2}\right.$$
$$\left. + \xi\frac{g_v^2}{2\pi^2}P_F\epsilon_F\frac{1}{\omega_z^2 - k^2 - m_v^2}\left(\frac{\omega_z^2}{k^2} - 1\right)\right] = 0 , \quad (4.5)$$

we conclude that the zero-sound only appears for values of k for which the second term in eq.(4.5) is negative, i.e., $k/m > 0.4$ for $P_F = 1.42 fm^{-1}$. This is in agreement with the conclusion of Matsui [12], that at nuclear saturation density for symmetric nuclear matter, in the limit of small frequencies and large wavelengths, no zero-sound mode appears. Taking the same limit (long wavelength and low frequency) in eq.(4.5) we obtain precisely the expression derived by Matsui from a microscopic calculation of the meson propagator (eq.(B.16) in ref.[12]). Our equation allows, however, the study of the appearance of zero-sound mode for any value of k. We observe that as P_F increases the zero mode will turn up at larger and larger wavelengths. From $P_F = 1.55 fm^{-1}$ on, this mode exists for any value of k, in agreement with results of ref.[12].

The stability of these collective modes can be deduced directly from the form of the dispersion relation [13]. The condition

$$\omega_1^2\omega_2^2 > 2(G_1^2\omega_2^2 - G_2^2\omega_1^2) ,$$

must be verified. For stability we understand that the collective excitation spectra admits no solution with $\omega^2 < 0$ for real k^2. If ω were imaginary, then there would always be exponentially growing collective modes, or density fluctuations, which are physically unacceptable.

To analyse the collective modes in the continuum we will consider the simplest example of initial condition which favours this mode [8],namely

$$\Psi_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} . \quad (4.6)$$

For this condition the strength function becomes

$$s(\overline{\omega}) = \frac{2a^2(\overline{\omega})(\overline{\omega}^2 - \omega_2^2)^2}{G_2^2(1 + \pi^2(f(\overline{\omega}) + \overline{\omega}\alpha(\overline{\omega}))^2a^2(\overline{\omega}))} , \quad (4.7a)$$

the fraction exhausted by the discrete modes is

$$F(\overline{\omega}_z) = \frac{1}{m_1}\sum_{n=1}^{N}\frac{2\overline{\omega}_{zn}(\overline{\omega}_{zn}^2 - \omega_2^2)^2}{G_2^2\eta_n} , \quad (4.7b)$$

with $\overline{\omega}_m$ given by eq.(3.7), and from eq.(3.22) we get $m_1 = 2/3$.

In Fig.3, the percentages of the EWSR exhausted by the continuum and discrete modes are plotted as a function of k, at nuclear saturation density. From this figure we see that the scalar mode remains unexcited by this initial condition. However, the vector field is strongly coupled to the continuum. We also see that, despite the fact that the zero-sound mode (ω_{z3}) exists from $k/m = 0.4$ onward, it only exhausts a significant part of m_1 for $0.5 < k/m < 2.5$.

In Fig.4 we plot the strength function for the initial condition eq.(4.6) , for $k/m = 0.1$ and $P_F = 1.42 fm^{-1}$. The curves obtained for other values of k and P_F, and other initial conditions are similar.

An initial condition which favours the scalar modes is given by

$$\Psi_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} . \tag{4.8}$$

For this condition we have

$$s(\overline{\omega}) = \frac{G_1^2}{G_2^2} \frac{2\overline{\omega}^2(\overline{\omega}^2 - \omega_3^2)^2 a^2(\overline{\omega})}{(\overline{\omega}^2 - \omega_1^2)^2(1 + \pi^2(f(\overline{\omega}) + \overline{\omega}\alpha(\overline{\omega}))^2 a^2(\overline{\omega}))} , \tag{4.9a}$$

$$F(\overline{\omega}_z) = \frac{1}{m_1} \sum_{n=1}^{N} \frac{2G_1^2}{G_2^2} \left(\frac{\overline{\omega}_m^2 - \omega_2^2}{\overline{\omega}_m^2 - \omega_1^2} \right)^2 \frac{\overline{\omega}_m^3}{\eta_n} , \tag{4.9b}$$

and $m_1 = \omega_1^2$.

The numerical results obtained for this condition are displayed in Table I. For low k/m values only the scalar mode is excited and a very collective state is obtained at this frequency.

From both initial conditions, eqs.(4.6) and (4.8), we observe that, at this density ($P_F = 1.42 fm^{-1}$), the system behaves basically as if the scalar mode and the continuum were decoupled. However, this is no longer the case as P_F increases. In Fig.5 the percentages of the EWSR exhausted by each mode are plotted as a function of k for $P_F = 4.0 fm^{-1}$ and for the initial conditon eq.(4.6). It is surprising that at high density and for k/m values in the range $1.0 \leq k/m \leq 5.0$ the scalar mode becomes the most important one for this condition. It shows that at high density the scalar mode is also strongly coupled to the continuum. This behaviour is completely different from the one observed in the long wavelength limit. In this case the zero-sound mode exhausts only 0.07% of the EWSR for all values of k.

To conclude, in the present work we have constructed a relativistic Vlasov equation based on QHD. Applying the Vlasov equation to study the mesonic longitudinal normal modes in nuclear

matter, we have obtained results similar (in the low k limit) to the more difficult calculations based on the one-loop expansion.

We have obtained a set of stationary linear modes of excitation satisfying orthogonality and completeness relations, of the form characteristic to the RPA. An energy weighted sum rule was also calculated.

Our results show that the scalar and the continuum modes are essentially decoupled at normal densities, but both of them are coupled to the vector mode. In the high density limit all of them are coupled. We also conclude that in the long wavelength limit the continuum and vector field are strongly coupled while in the short wavelength limit they are almost decoupled. Therefore, vector meson oscillations can be interpreted as collective modes in the long wavelength limit and meson propagation in the short wavelength limit.

This work was partially supported by Junta Nacional de Investigação Científica e Tecnológica (JNICT), Portugal (project # 87148/C:FIS) and by FAPESP-Brazil. One of the authors, M.N., would like to thank the hospitality of CFTUC during her stay in Portugal.

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FIGURE CAPTIONS

Fig.1- The dispersion relation for the scalar mode (full line) and for the vector mode (dashed line) at nuclear saturation density. The frequency ω_s and wave vector k are in units of m.

Fig.2- The dispersion relation for zero-sound, at nuclear saturation density, as a function of k. The frequency ω_s and wave vector k are in units of m.

Fig.3- Percentages of the EWSR exhausted by the continuum and (full line)discrete modes as a function of k, for the initial condition eq.(4.6), and for $P_F = 1.42fm^{-1}$. The dashed, dotted and long-dashed lines represent the scalar, vector and zero-sound modes respectively.

Fig.4- Strenght function, as a function of ω (in units of ω_0) for the initial condition eq.(4.6), for $P_F = 1.42fm^{-1}$, and for $k/m = 0.1$.

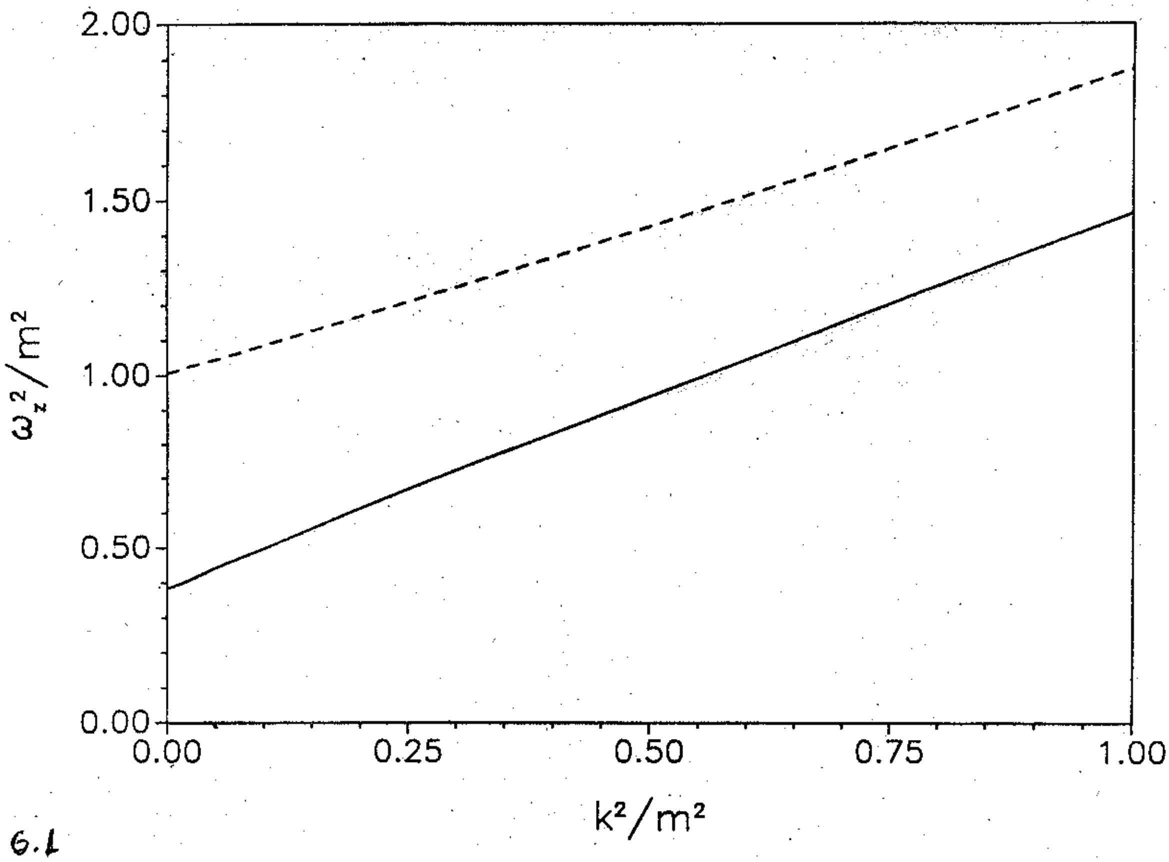
Fig.5- Same as fig.3 for $P_F = 4.0fm^{-1}$.

TABLES CAPTIONS

Table I- For the values of K indicated, percentages of the EWSR are given for the initial condition eq.(4.8) at nuclear saturation density.

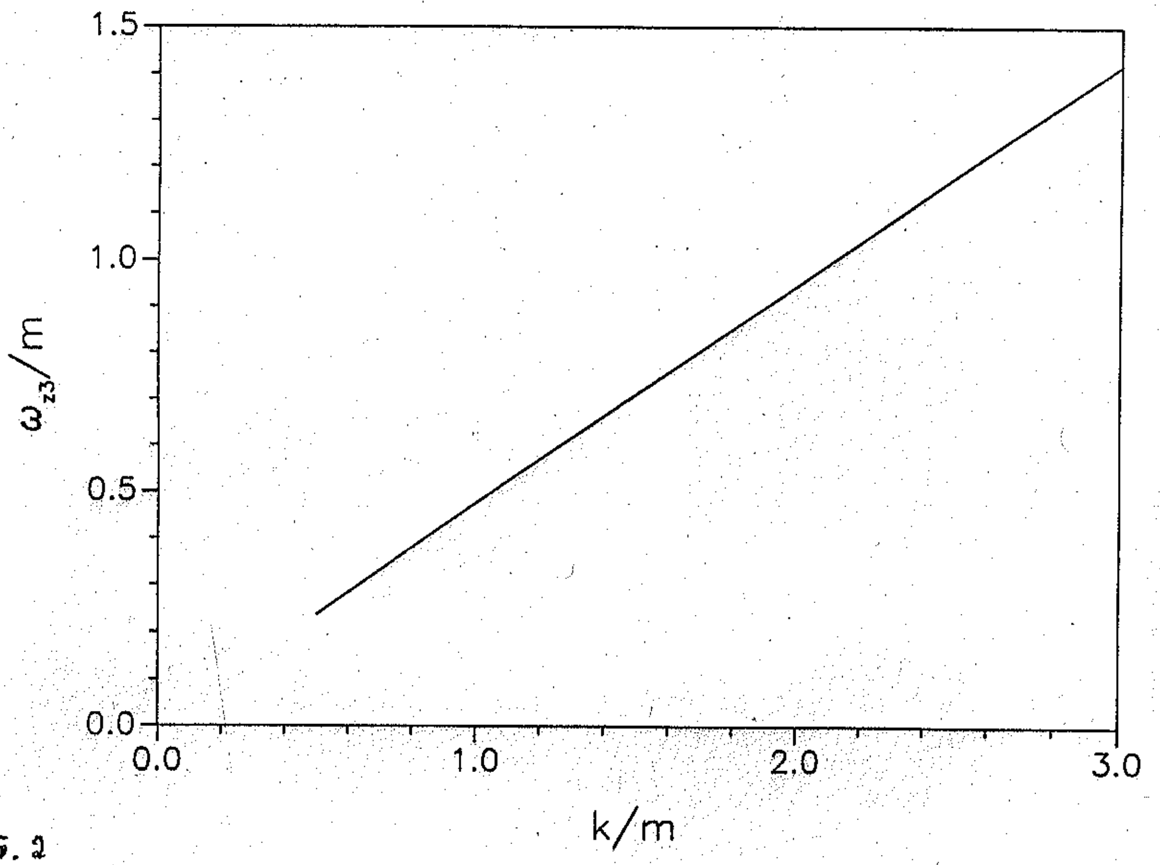
TABLE I

K/m	$\frac{100}{m_1}\int_0^1 s(\omega)d\omega$	$\%\omega_{z1}$	$\%\omega_{z2}$	$\%\omega_{z3}$
0.01	0	100	0	/
0.1	0	99.48	0.52	/
1.0	0.86	77.93	20.53	0.68
10.	0.04	55.33	44.63	0



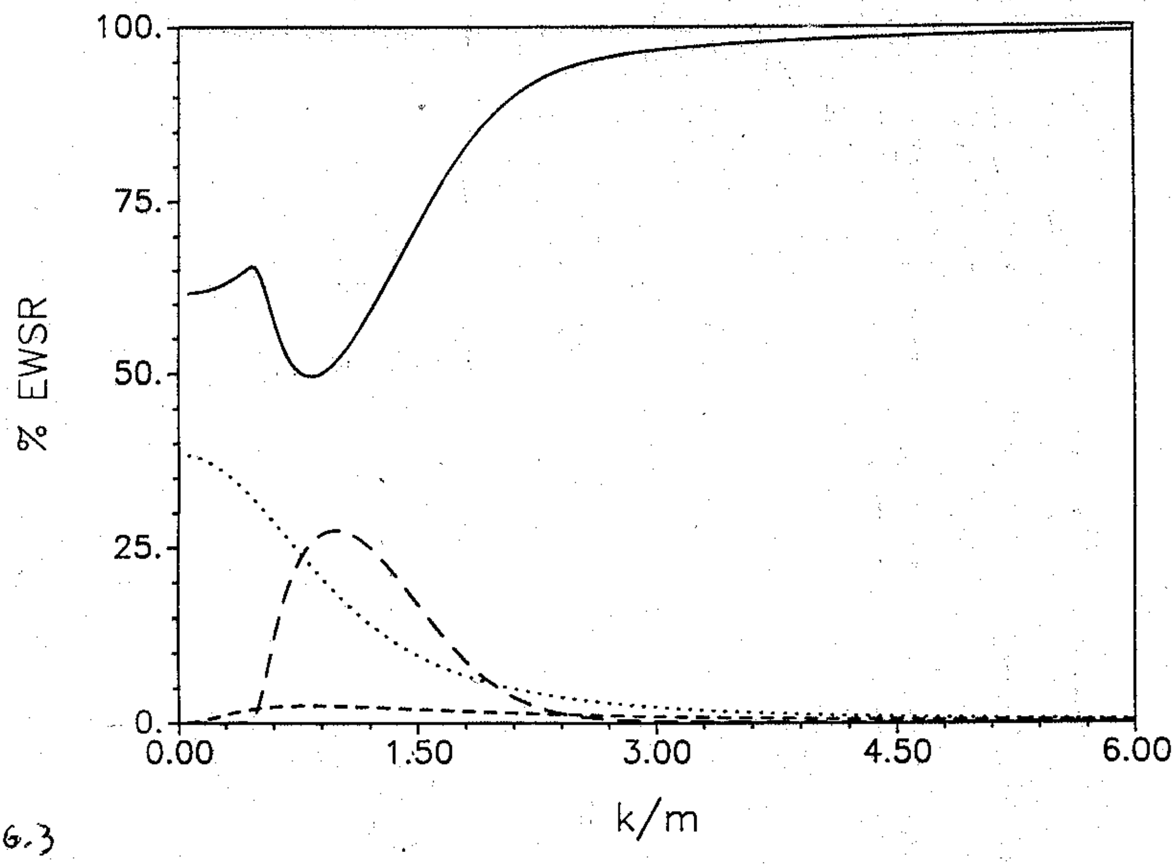
2.00
1.50
1.00
0.50
0.00
ω_z^2/m^2
0.00
0.25
0.50
0.75
1.00
k^2/m^2

FIG. 1



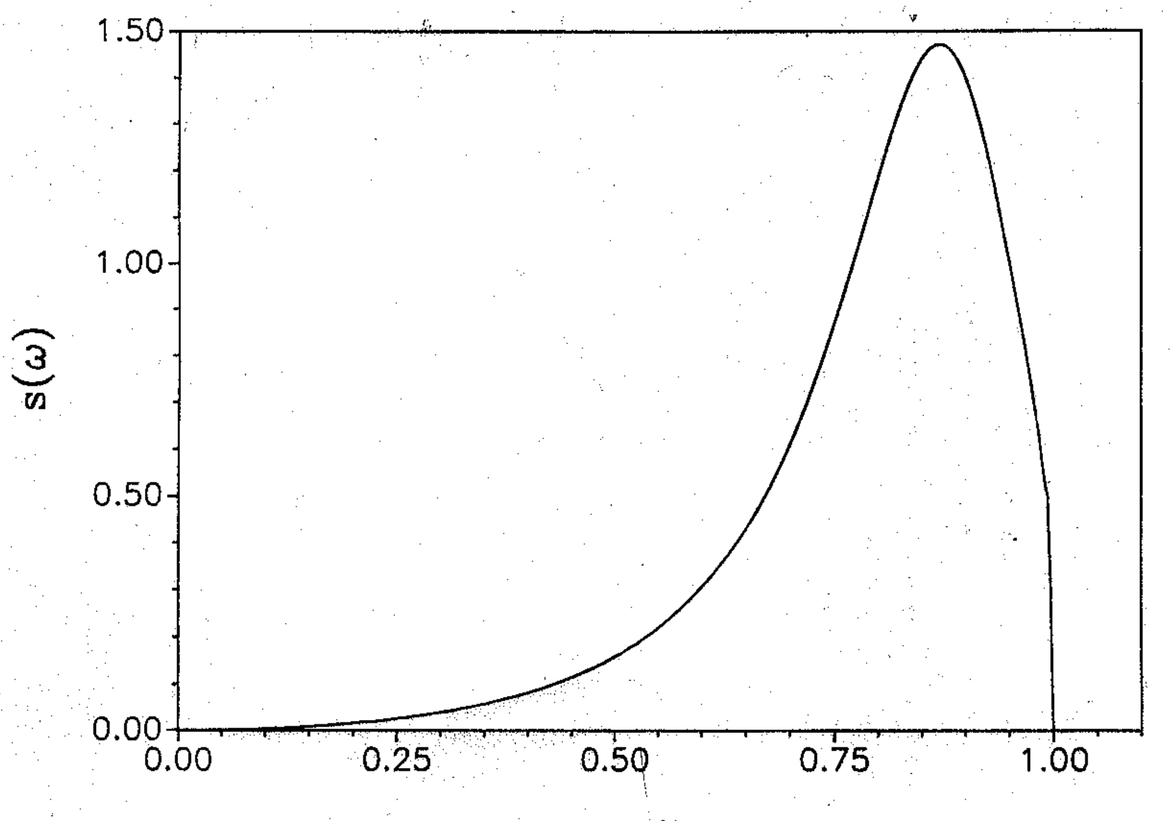
1.5
1.0
0.5
0.0
ω_{z3}/m
0.0
1.0
2.0
3.0
k/m

FIG. 2



100.
75.
50.
25.
0.
% EWSR
0.00
1.50
3.00
4.50
6.00
k/m

FIG.3



1.50
1.00
0.50
0.00
s(ω)
0.00
0.25
0.50
0.75
1.00
ω

FIG.4

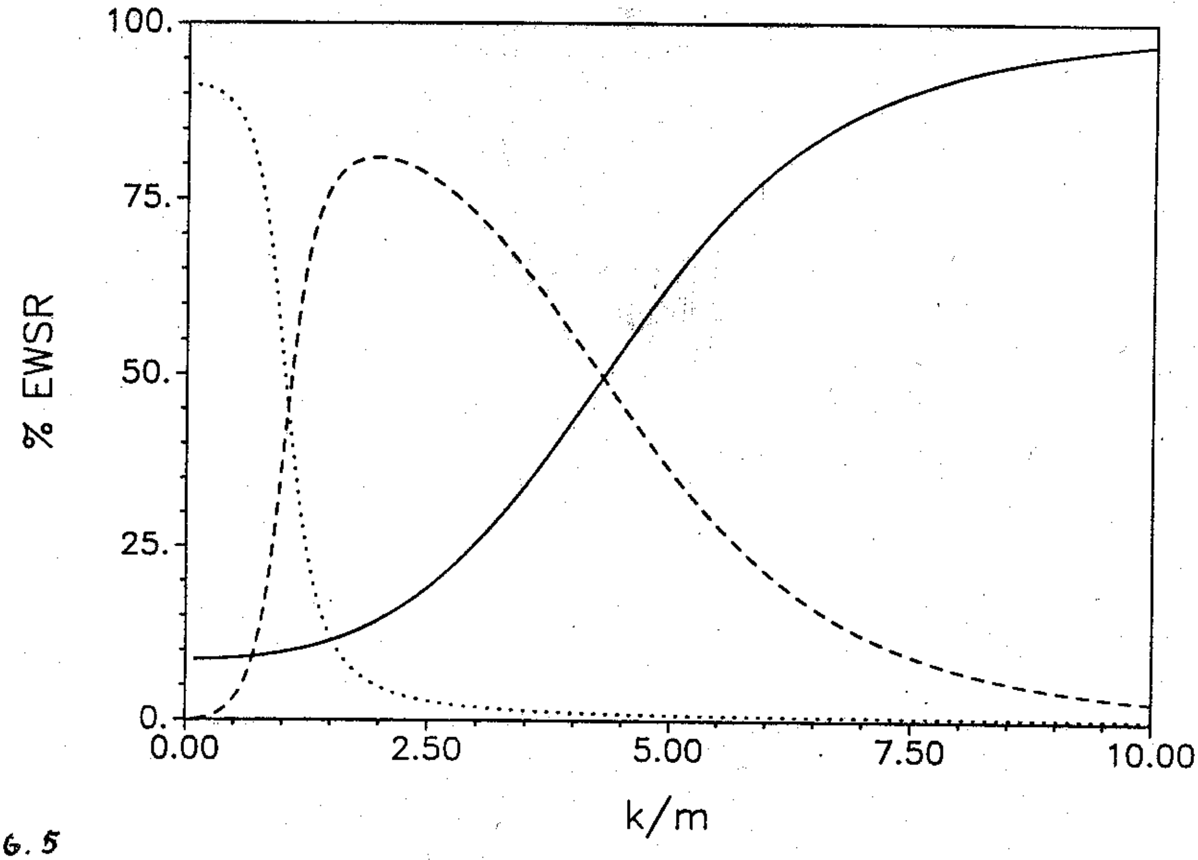


FIG. 5