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Abstract

We discuss an alternative method for computing finite-temperature effects in field theories, within the framework of the imaginary-time formalism. Our approach allows for a systematic calculation of the high temperature expansion in terms of Riemann Zeta functions. The imaginary-time result is analytically continued to the complex plane. We are able to obtain the real-time limit of the real *and* the imaginary parts of the Green functions.

1. Introduction

There have been several calculations of Green functions in thermal field theories. In particular the high-temperature limit of the two-point function is well known [1 – 4], and more recently higher-point functions have also been studied in the context of QCD and quantum gravity [5]. In general, these calculations employ the imaginary-time formalism and are restricted to the leading high-temperature behaviour. As far as we know there is no result for the next-to-leading behaviour in the context of the imaginary-time formalism. In the real-time formalism there is a calculation by Weldon [2] for the real part of the two-point function in QCD, which takes account of the complete high-temperature expansion. The purpose of this paper is to compute the real and the imaginary parts of the complete high-temperature expansion of the two-point function, employing an analytic continuation of the imaginary-time formalism. Our main result is given by the eq. (4.20).

The standard way to compute the finite-temperature Green functions in the imaginary-time formalism, is to employ the relation (see [1] and references therein)

$$T\sum_{n=-\infty}^{\infty} I\,(k_0=2\pi nT) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} dk_0\,\frac{1}{2}\left[I(k_0)+I(-k_0)\right] + \frac{1}{2\pi i}\int_{-i\infty+\delta}^{i\infty+\delta} dk_0\left[I(k_0)+I(-k_0)\right]\frac{1}{\exp\left(\frac{k_0}{T}\right)-1}, \tag{1.1}$$

where $I(k_0)$ is given by an integral over the space components $\mathbf{k}$. Eq. (1.1) has an interesting physical interpretation. The first term on the right is the vacuum (T-independent) Green function and the second term contains the temperature dependent Bose-Einstein distribution (we restrict ourselves to boson fields). The result for the vacuum piece is usually well known for many field theories. The contour in the k_0-plane of the second term is then closed in the right half plane, and one is left with an integral over $\mathbf{k}$ with the Bose-Einstein distribution. After the angular integration is performed, we are not able to proceed without restriction to the leading high-temperature limit. It was shown by Pisarski [6] that the leading high-temperature contribution comes from the region of high $|\,\mathbf{k}\,|$, in which case one can use an approximation for the integrand. However, in general the no-leading contributions come from the complete range of the $|\,\mathbf{k}\,|$-integration and there is no simple way to proceed with the calculation.

In this paper we will employ a different approach in order to compute the left hand side of eq. (1.1). This consists in first computing the integral over $|\mathbf{k}|$, and then to perform the sum over n. As we will see, this procedure allows one to obtain the complete high-temperature expansion in a systematic way. Besides that, we are able to obtain the most general analytic continuation and a real-time limit which gives both the real *and* the imaginary parts of the Green functions. For simplicity we will consider here only the two-point function, though the same technique can be employed to higher point functions.

One of the key elements of this calculation is a process of analytic continuation which makes it possible to use the relation

$$\sum_{n=1}^{\infty} n^{-\alpha} = \zeta(\alpha), \tag{1.2}$$

for *any* $\alpha \neq 1$. This is in the spirit of ζ-function regularization. The contributions with $\alpha =$ 1 correspond to the ultraviolet divergence. Using dimensional regularization, the terms with $\alpha = 1$ are transformed into contributions proportional to $\zeta(1-2\epsilon) \simeq -1/2\epsilon + \gamma + \mathcal{O}(\epsilon)$. These contributions give not only the pure pole and finite induced terms that are present at $T = 0$, but also some additional finite terms which are characteristic of the $T \neq 0$ configurations.

In the next two sections we will be considering the scalar $\lambda\phi^3$ theory. Using this model we will be able to illustrate the main points of our method in the simplest possible way. Some important steps will be clarified in the next section considering the even simpler 2-dimensional scalar model. Afterwards, in the section 3 we will treat the 6-dimensional $\lambda\phi^3$ theory, which has some similarities with QCD such as a dimensionless coupling constant and the asymptotic freedom. The techniques employed to the scalar theory will be required in the section 4 where we consider the pure Yang-Mills theory.

2. The 2-dimensional scalar theory

In order to illustrate our method without introducing unnecessary complications, we consider here the scalar $\lambda\phi^3$ theory. In this section we will make the computations even simpler by considering a 2-dimensional model. In two dimensions it is possible to compute the two-point function without using a high-temperature approximation. This will enable us to obtain a better understanding of the general analytic properties.

Before we restrict to a 2-dimensional space time, let us write the basic equations for a general D-dimensional space time. The two-point function is given by (see fig. 1 for notation)

$$\tilde{\Sigma}^D = \lambda^2(\mu^2)^{\frac{6-D}{2}} \frac{T}{2} \sum_{n=-\infty}^{\infty} \int \frac{d^{D-1}\mathbf{k}}{(2\pi)^{D-1}} \frac{1}{\mathbf{k}^2 + k_0^2 + m^2} \frac{1}{(\mathbf{k}+\mathbf{p})^2 + (k_0+p_0)^2 + m^2}, \tag{2.1}$$

where $k_0 = 2\pi nT$ and $p_0 = 2\pi lT$ (l is some fixed integer). Our metric convention in the Minkowski space is such that $g^{00} = +1$, $g^{11} = g^{22} = g^{33} = \ldots = g^{D-1\;D-1} = -1$. In eq. (2.1) both k_0 and p_0 were transformed from pure real to pure imaginary numbers, which is equivalent to consider a Euclidean metric.

The standard way to compute a finite temperature Green function like eq. (2.1) is well known. The sum over n is transformed into an integral in the complex plane around poles in the imaginary axis. In what follows we will proceed differently. We first perform the $(D-1)$-dimensional integral and only then sum over n. As we will see in the next section this procedure will make it possible to obtain not only the leading high temperature contribution, but also all the other powers of T in a systematic way. Besides that, our result will give both, the real and the imaginary parts of the two-point function.

Using the Feynman parametrization and integrating over $|\mathbf{k}|$, we obtain from eq. (2.1)

$$\tilde{\Sigma}^D = \lambda^2(\mu^2)^{\frac{6-D}{2}} \Gamma\left(\frac{5-D}{2}\right) \frac{1}{(4\pi)^{\frac{D-1}{2}}} \frac{T}{2} \sum_{n=-\infty}^{\infty} I(k_0, D), \tag{2.2}$$

where

$$I(k_0, D) = \int_0^1 (k_0^2 + bx - \mathbf{p}^2 x^2 + m^2)^{\frac{D-5}{2}}\, dx, \tag{2.3}$$

and $b = p_0(2k_0 + p_0) + \mathbf{p}^2$. Our task now is to compute the parametric integral in eq. (2.3) and then to perform the sum over n in eq. (2.2).

With $D = 2$, integration over x in eq. (2.3) yields [7]

$$I(k_0,2) = \frac{2}{(p_0^2+\mathbf{p}^2)\left[(2k_0+p_0)^2+\mathbf{p}^2\right]+m^2\mathbf{p}^2}\left[\frac{p_0(2k_0+p_0)+\mathbf{p}^2}{\sqrt{k_0^2+m^2}} - \frac{p_0(2k_0+p_0)-\mathbf{p}^2}{\sqrt{(k_0+p_0)^2+m^2}}\right]. \tag{2.4}$$

The mass m is only needed in order to regularize the contributions with $k_0 = 0$ and $k_0 = -p_0$. Therefore, for $n \neq 0, -l$ one may set $m = 0$ and the square roots become a moduli. The modulus $\mid k_0 + p_0 \mid$ from the second term in eq. (2.4) makes a difficult task to compute the sum over n. One would have to consider, for some positive p_0, two regions: $\mid k_0 \mid < p_0$ and $\mid k_0 \mid > p_0$. It is possible to avoid this difficulty by performing a shift $k_0 \to -k_0 - p_0$ in the second term of eq. (2.4). Nevertheless, one cannot be sure that the result with or without the shift, gives the same analytic continuation to a general complex p_0. Fortunately the 2-dimensional model is simple enough to provide an explicit comparison of the two procedures. One can compute the sum with or without making the shift and then to compare the two results. Let us first compute the sum of eq. (2.4) without performing the shift. Since the original expression has the symmetry $p_0 \to -p_0$ one may choose $p_0 > 0$. For $k_0 \neq 0, -p_0$, we may set $m = 0$ and the eq. (2.4) splits in three pieces:

$$I^+(k_0,2) = \frac{2}{p_0+ip}\left(\frac{1}{k_0}\frac{1}{2k_0+p_0-ip} - \frac{1}{k_0+p_0}\frac{1}{2k_0+p_0+ip}\right), \tag{2.5a}$$

for $k_0 > 0$,

$$I^-_>(k_0,2) = -\frac{2}{p_0-ip}\left(\frac{1}{k_0}\frac{1}{2k_0+p_0+ip} - \frac{1}{k_0+p_0}\frac{1}{2k_0+p_0-ip}\right), \tag{2.5b}$$

for $-p_0 < k_0 < 0$ and

$$I^-_<(k_0,2) = -2\left(\frac{1}{p_0-ip}\frac{1}{k_0}\frac{1}{2k_0+p_0+ip} + \frac{1}{p_0+ip}\frac{1}{k_0+p_0}\frac{1}{2k_0+p_0+ip}\right), \tag{2.5c}$$

for $k_0 < -p_0 < 0$.

The sum over n may now be written as

$$\begin{aligned}\sum_{n=-\infty}^{\infty} I(k_0,2)\big|_{p_0>0} = {} & \frac{4}{p_0^2+\mathbf{p}^2}\frac{1}{m} - \frac{4}{(p_0^2+\mathbf{p}^2)^2}\frac{p_0^2-\mathbf{p}^2}{p_0} \\ & + \sum_{n=1}^{\infty} I^+(k_0,2) + \sum_{n=-l-1}^{-\infty} I^-_>(k_0,2) \\ & + \sum_{n=-1}^{-l+1} I^-_<,\end{aligned} \tag{2.6}$$

where $l = p_0/2\pi T$ and the first two terms are the sum of the $n = 0$ and $n = -l$ contributions. Making the change of variable $n \to -n - l$ in the second sum of eq. (2.6) we obtain

$$\begin{aligned}\sum_{n=-\infty}^{\infty} I(k_0,2)\big|_{p_0>0} = {} & \frac{4}{p_0^2+\mathbf{p}^2}\frac{1}{m} - \frac{4}{(p_0^2+\mathbf{p}^2)^2}\frac{p_0^2-\mathbf{p}^2}{p_0} \\ & + \sum_{n=1}^{\infty}\left(I^+(k_0,2) + (p \to -p)\right) + \sum_{n=1}^{l-1} I^-_<(-k_0,2).\end{aligned} \tag{2.7}$$

Inserting eq. (2.5a) and eq. (2.5c) into eq. (2.7), and using the formulas [7]

$$\sum_{n=1}^{\infty} \frac{x}{n}\frac{1}{n+x} = \Psi(x) + \frac{1}{x} + \gamma, \tag{2.8a}$$

$$\sum_{n=1}^{\infty} \frac{x-y}{n+x}\frac{1}{n+y} = \Psi(x) - \Psi(y) + \frac{1}{x} - \frac{1}{y} \tag{2.8b}$$

and

$$\sum_{n=1}^{l-1} \frac{1}{n} = \Psi(l) + \gamma, \tag{2.8c}$$

where $\Psi(z) = d[\ln\Gamma(z)]/dz$, one get

$$\begin{aligned}\sum_{n=-\infty}^{\infty} I(k_0,2)\big|_{p_0>0} = {} & \frac{4}{p_0^2+\mathbf{p}^2}\frac{1}{m} + \frac{1}{\pi T}\frac{1}{p_0^2+\mathbf{p}^2}\Big[4\gamma + 2\Psi\Big(\frac{p_0-ip}{4\pi T}\Big) + 2\Psi\Big(\frac{p_0+ip}{4\pi T}\Big) \\ & + \frac{4\pi T}{p_0-ip} + \frac{4\pi T}{p_0+ip}\Big].\end{aligned} \tag{2.9}$$

The result for $p_0 < 0$ is easily obtained making $p_0 \to -p_0$ in the eq. (2.9). The correct analytic continuation to a function which is analytic off the imaginary p_0 axis is obtained simply by writing (2.9) in the form

$$\sum_{n=-\infty}^{\infty} I(k_0,2) = \frac{1}{p_0^2+\mathbf{p}^2}\left\{\frac{4}{m}+\frac{1}{\pi T}\left[\left(4\gamma+2\Psi(\frac{p_0-ip}{4\pi T})\ +2\Psi(\frac{p_0+ip}{4\pi T})\right)\ \theta(\mathrm{Re}\ p_0)\right.\right.$$
$$+\left(4\gamma+2\Psi(\frac{-p_0-ip}{4\pi T})+2\Psi(\frac{-p_0+ip}{4\pi T})\right)\theta(-\mathrm{Re}\ p_0)$$
$$\left.\left.+\left(\frac{4\pi T}{p_0-ip}+\frac{4\pi T}{p_0+ip}\right)\varepsilon(\mathrm{Re}\ p_0)\right]\right\}, \tag{2.10}$$

where $\varepsilon(\mathrm{Re}\ p_0)=\theta(\mathrm{Re}\ p_0)-\theta(-\mathrm{Re} p_0)$.

Let us now make the shift $k_0 \to -k_0 - p_0$ in the second term of eq. (2.4). The resulting expression may be written as

$$I^s(k_0,2)=\frac{2\varepsilon(k_0)}{p_0^2+\mathbf{p}^2}\left[(p_0-ip)\frac{1}{k_0}\frac{1}{k_0+\frac{p_0-ip}{2}}+\frac{1}{k_0+\frac{p_0-ip}{2}}-\frac{1}{k_0+\frac{p_0+ip}{2}}\right], \tag{2.11}$$

for $k_0 \neq 0$, and

$$I^s(0,2)=\frac{4}{p_0^2+\mathbf{p}^2}\frac{1}{m}, \tag{2.12}$$

where the superscript s denotes the shifted result. Eq. (2.11) is not as it stands in a form which allows analytic continuation directly. If p_0 is allowed to become complex in (2.11), there are poles in the complex plane which are not allowed. Therefore we first make use of the identity

$$\varepsilon(k_0)=2\theta(\mathrm{Re}\ p_0)\theta(k_0)-2\theta(-\mathrm{Re}\ p_0)\theta(-k_0)-\varepsilon(\mathrm{Re}\ p_0). \tag{2.13}$$

Using this identity and (2.8a) and (2.8b), we obtain

$$\sum_{n=-\infty}^{\infty} I^s(k_0,2)=\frac{1}{p_0^2+\mathbf{p}^2}\left\{\frac{1}{\pi T}\left[2\gamma+\Psi(\frac{p_0+ip}{4\pi T})+\Psi(\frac{p_0-ip}{4\pi T})+\frac{4\pi T}{p_0+ip}+\frac{4\pi T}{p_0-ip}\right]\right.$$
$$\left.\times\left(2\theta(\mathrm{Re}\ p_0)-\varepsilon(\mathrm{Re}\ p_0)\right)+\frac{2}{m}\right\}+(p_0\to -p_0). \tag{2.14}$$

Using the relation

$$\Psi(-z)-\Psi(z)=\pi\cot(\pi z)+\frac{1}{z}, \tag{2.15}$$

the contribution proportional to $\varepsilon(\mathrm{Re}\ p_0)$ can be written as

$$\varepsilon(\mathrm{Re}\ p_0)\frac{1}{\pi T}\frac{1}{p_0^2+\mathbf{p}^2}\left[\pi\frac{\sin(\frac{p_0}{2T})}{\sin(\frac{p_0-ip}{4T})\sin(\frac{p_0+ip}{4T})}-\frac{4\pi T}{p_0-ip}-\frac{4\pi T}{p_0+ip}\right]. \tag{2.16}$$

The first term in (2.16) vanishes at $p_0 = 2\pi l T$, and the unique analytic continuation satisfying the conditions of Carlson's theorem [8] is also zero. Therefore only the last two terms in (2.16) remain, and inserting them into (2.14) one gets an expression identical to (2.10).

One should also mention that eq. (2.10) has the expected general analytic properties in the complex p_0 plane off the imaginary axis, required in Carlson's theorem, see for example (3.1.8) of ref. [9] (since we are computing in the Euclidean space, the real and imaginary axis should be interchanged when comparing with ref. [9]). Indeed, using the properties of the Psi function one can verify that the eq. (2.10) has neither poles nor zeroes for Re $p_0 \neq 0$. For Im $p_0 = 0$ the result is real, positive and tends to zero when p_0 tends to infinity.

3. The 6-dimensional scalar theory

Let us now consider the 6-dimensional model. Integration over x in eq. (2.3) yields (in what follows we set $m = 0$) [7]

$$I(k_0,6)=-\frac{1}{4\mathbf{p}^2}\left\{|\ k_0+p_0\ |\left(p_0(2k_0+p_0)-\mathbf{p}^2\right)-|\ k_0\ |\left(p_0(2k_0+p_0)+\mathbf{p}^2\right)\right.$$
$$\left.+\frac{p_0^2+\mathbf{p}^2}{2i\ |\ \mathbf{p}\ |}\left[(2k_0+p_0)^2+\mathbf{p}^2\right]\ln\frac{2\ |\ k_0+p_0\ ||\ \mathbf{p}\ |+(2k_0p_0+p_0^2-\mathbf{p}^2)i}{2\ |\ k_0\ ||\ \mathbf{p}\ |+(2k_0p_0+p_0^2+\mathbf{p}^2)i}\right\}. \tag{3.1}$$

The simplest and perhaps the only way to proceed from eq. (3.1) is to use shifts in k_0. Performing the shift $k_0 \to -k_0 - p_0$ in the first term and in the numerator of the logarithm (the factor multiplying the logarithm is invariant under $k_0 \to -k_0 - p_0$) we obtain

$$I^s(k_0,6)=\frac{\varepsilon(k_0)}{4\mathbf{p}^2}\left\{2k_0\left(p_0^2+\mathbf{p}^2+2p_0k_0\right)-\frac{p_0^2+\mathbf{p}^2}{2i\ |\ \mathbf{p}\ |}\left[(2k_0+p_0)^2+\mathbf{p}^2\right]\right.$$
$$\left.\times\left[\ln\frac{i\ |\ \mathbf{p}\ |+p_0}{i\ |\ \mathbf{p}\ |-p_0}+\ln\frac{2k_0+p_0-i\ |\ \mathbf{p}\ |}{2k_0+p_0+i\ |\ \mathbf{p}\ |}\right]\right\}. \tag{3.2}$$

As in the 2-dimensional model we now use the identity (2.13). Again the first two terms in (2.13) give no singularity. This is because the logarithm in (3.2) is finite when Re p_0 and k_0 have the same sign. Let us consider the contribution proportional to $\varepsilon(\text{Re }p_0)$. In the 2-dimensional case we were able to perform the sum exactly, and we had verified explicitly that the singular contribution was zero for $p_0 = 2\pi l T$. In the present case one cannot perform the sum exactly. Even so, we may explore the fact that in the contribution proportional to $\varepsilon(\text{Re }p_0)$ shifts can be easily done. When we do the shift $k_0 \to -k_0 - p_0$, the quantity $2k_0 + p_0$ changes the sign. If we average over the two forms, before and after the shift, the last logarithm in eq. (3.2) vanishes. Note that since $k_0 = 2\pi n T$, the use of shifts is only meaningful if $p_0 = 2\pi l T$. Therefore, in the same way as in the 2-dimensional model, the singular contribution (which now is the logarithm) vanishes because we choose to analytically continue *after* using $p_0 = 2\pi l T$. Performing the sum over n of the resulting expression, we obtain

$$\sum_{n=-\infty}^{\infty} I(k_0,6) = \frac{p_0^2+\mathbf{p}^2}{4\mathbf{p}^2}\left\{\varepsilon(\text{Re }p_0)\left(2p_0+\frac{p_0^2+\mathbf{p}^2}{2i\mid\mathbf{p}\mid}\ln\frac{p_0+i\mid\mathbf{p}\mid}{p_0-i\mid\mathbf{p}\mid}\right)\right.$$
$$-\frac{4\pi T}{6}\left(1-\frac{p_0}{i\mid\mathbf{p}\mid}\ln\frac{i\mid\mathbf{p}\mid+p_0}{i\mid\mathbf{p}\mid-p_0}\right)$$
$$\left.-\left[2\theta(\text{Re }p_0)\sum_{n=1}^{\infty}\frac{(2k_0+p_0)^2+\mathbf{p}^2}{2i\mid\mathbf{p}\mid}\ln\frac{2k_0+p_0-i\mid\mathbf{p}\mid}{2k_0+p_0+i\mid\mathbf{p}\mid}+(p_0\to -p_0)\right]\right\}, \qquad (3.3)$$

where we have made use of the eq. (1.2) and $\zeta(-2)=0$, $\zeta(-1)=-1/12$, $\zeta(0)=1/2$ and $\sum_{n=-\infty}^{\infty} k_0 = 0$.

In order to be able to perform the remaining sum over n in the square bracket of eq. (3.3), one has to expand the logarithm in powers of k_0^{-1}. Using the definitions

$$W_k \equiv \frac{(-1)^k}{k}\frac{1}{2i\mid\mathbf{p}\mid}\left[\left(\frac{p_0+i\mid\mathbf{p}\mid}{2}\right)^k-\left(\frac{p_0-i\mid\mathbf{p}\mid}{2}\right)^k\right], \qquad (3.4a)$$

$$V_k \equiv 4W_{k+2}+4p_0W_{k+1}+\left(p_0^2+\mathbf{p}^2\right)W_k \qquad (3.4b)$$

and the eq. (1.2), we obtain the following result from eq. (3.3)

$$\sum_{n=-\infty}^{\infty} I(k_0,6) = \frac{p_0^2+\mathbf{p}^2}{4\mathbf{p}^2}\left\{\varepsilon(\text{Re }p_0)\left[\frac{p_0^2+\mathbf{p}^2}{2i\mid\mathbf{p}\mid}\ln\frac{p_0+i\mid\mathbf{p}\mid}{p_0-i\mid\mathbf{p}\mid}-2\sum_{j=1}^{\infty}\frac{V_{2j}}{(2\pi T)^{2j}}\zeta(2j)\right]\right.$$
$$\left.-\frac{4\pi T}{3}\left(1-\frac{p_0}{2i\mid\mathbf{p}\mid}\ln\frac{i\mid\mathbf{p}\mid+p_0}{i\mid\mathbf{p}\mid-p_0}\right)-\frac{1}{\pi T}\sum_{j=0}^{\infty}\frac{V_{2j+1}}{(2\pi T)^{2j}}\zeta(2j+1)\right\}, \qquad (3.5)$$

where we have made use of the following property

$$V_k(-p_0) = \begin{cases} -V_k(p_0) & \text{if } k = 2j; \\ V_k(p_0) & \text{if } k = 2j+1. \end{cases} \qquad (3.6)$$

The first term in the last sum of the eq. (3.5) contains a divergent contribution proportional to $\zeta(1) = \infty$. This will give a temperature-independent contribution to the two-point function (the eq. (2.2) has a T factor). In order to deal with this contribution one has to use some sort of regularization. Leaving this piece apart for a moment and inserting only the finite terms of eq. (3.5) into eq. (2.2) we obtain

$$(\tilde{\Sigma})^T = \frac{\lambda^2}{128\pi^2}\frac{p_0^2+\mathbf{p}^2}{\mid\mathbf{p}\mid^2}\left\{\varepsilon(\text{Re }p_0)\left[\frac{p_0^2+\mathbf{p}^2}{2i\mid\mathbf{p}\mid}\ln\frac{p_0+i\mid\mathbf{p}\mid}{p_0-i\mid\mathbf{p}\mid}T-\frac{1}{\pi}\sum_{j=1}^{\infty}\frac{V_{2j}}{(2\pi T)^{2j-1}}\zeta(2j)\right]\right.$$
$$\left.\frac{4\pi}{3}\left(1-\frac{p_0}{2i\mid\mathbf{p}\mid}\ln\frac{i\mid\mathbf{p}\mid+p_0}{i\mid\mathbf{p}\mid-p_0}\right)T^2+\frac{1}{\pi}\sum_{j=1}^{\infty}\frac{V_{2j+1}}{(2\pi T)^{2j}}\zeta(2j+1)\right\}. \qquad (3.7)$$

Let us now compute the divergent contributions. In order to do that we will use dimensional regularization. As we have seen, without using any regularization the only divergent contribution came from $\sum 1/k_0 \propto \zeta(1) = \infty$. Therefore, we only need to regularize the contributions proportional to $1/k_0$. Let us turn to eq. (2.3) and set $D = 6+2\epsilon$. The idea is to use an expansion in powers of $\mid k_0 \mid$ and collect the terms proportional to $\mid k_0 \mid^{-1+2\epsilon}$. After the sum over n is performed (using eq.(1.2)) we will obtain the regularized divergent contribution proportional to $\zeta(1+2\epsilon) = -1/2\epsilon + \gamma + \mathcal{O}(\epsilon)$.

With $m = 0$, eq. (2.3) may be rewritten as

$$I(k_0, 6+2\epsilon) = f(k_0,\epsilon)\tilde{I}(k_0,\epsilon), \qquad (3.8)$$

where

$$f(k_0,\epsilon)=\left(1+\frac{p_0^2}{\mathbf{p}^2}\right)^{1+\epsilon}\frac{1}{|\,\mathbf{p}\,|}\left[\frac{\mathbf{p}^2}{4}+\left(k_0+\frac{p_0}{2}\right)^2\right]^{1+\epsilon} \tag{3.9}$$

and

$$\tilde{I}\,(k_0,\epsilon)=\int_{\frac{-b}{\sqrt{b^2+4k_0^2\mathbf{p}^2}}}^{\frac{2\mathbf{p}^2-b}{\sqrt{b^2+4k_0^2\mathbf{p}^2}}}\left(1-x^2\right)^{\frac{1}{2}+\epsilon}dx. \tag{3.10}$$

The integral $\tilde{I}(k_0,\epsilon)$ may be expressed as

$$\tilde{I}(k_0,\epsilon)=\int_{\frac{-b}{\sqrt{b^2+4k_0^2\mathbf{p}^2}}}^{0}\ldots+\int_0^{\frac{2\mathbf{p}^2-b}{\sqrt{b^2+4k_0^2\mathbf{p}^2}}}\ldots=2\int_0^{\frac{b}{\sqrt{b^2+4k_0\mathbf{p}^2}}}\ldots, \tag{3.11}$$

where we have made $x\to -x$ in the first integral and $k_0\to -k_0-p_0$ in the second one. Using the substitution $x=\sin\theta$, one get

$$\tilde{I}(k_0,\epsilon)=2\int_0^{\arctan\frac{b}{2\,|\,k_0\,||\,\mathbf{p}\,|}}\left(\cos^2\theta\right)^{1+\epsilon}d\theta. \tag{3.12}$$

The power series for $f(k_0,\epsilon)$ up to $k_0^{-1+2\epsilon}$ is

$$f(k_0,\epsilon)=\left(1+\frac{p_0^2}{\mathbf{p}^2}\right)^{1+\epsilon}\frac{1}{|\,\mathbf{p}\,|}k_0^{2\epsilon}\left(k_0^2+(1+\epsilon)\,p_0k_0+\frac{(1+3\epsilon)\,p_0^2+(1+\epsilon)\,\mathbf{p}^2}{4}\right.$$
$$\left.+\frac{\epsilon}{4}p_0\left(\mathbf{p}^2+\frac{p_0^2}{3}\right)k_0^{-1}\right), \tag{3.13}$$

where we have neglected higher powers of ϵ.

In order to obtain the terms proportional to $|\,k_0\,|^{-1+2\epsilon}$ we have to expand $\tilde{I}(k_0,\epsilon)$ up to k_0^{-3}. Using

$$\arctan\frac{b}{2\,|\,k_0\,||\,\mathbf{p}\,|}=\arctan\left(\frac{k_0}{|\,k_0\,|}\frac{p_0}{|\,\mathbf{p}\,|}+\eta\right), \tag{3.14}$$

where

$$\eta=\frac{p_0^2-\mathbf{p}^2}{2\,|\,k_0\,||\,\mathbf{p}\,|}, \tag{3.15}$$

the power series for $\tilde{I}(k_0,\epsilon)$ may be written as

$$\tilde{I}(k_0,\epsilon)=\tilde{I}(k_0,\epsilon)\Big|_{\eta=0}+\tilde{I}'(k_0,\epsilon)\Big|_{\eta=0}\eta+\frac{\tilde{I}''(k_0,\epsilon)}{2!}\Big|_{\eta=0}\eta^2$$
$$+\frac{\tilde{I}'''(k_0,\epsilon)}{3!}\Big|_{\eta=0}\eta^3+\ldots, \tag{3.16}$$

where

$$\tilde{I}'(k_0,\epsilon)\Big|_{\eta=0}=2\left(1+\frac{p_0^2}{\mathbf{p}^2}\right)^{-2-\epsilon}, \tag{3.17a}$$

$$\tilde{I}''(k_0,\epsilon)\Big|_{\eta=0}=-4\,(2+\epsilon)\left(1+\frac{p_0^2}{\mathbf{p}^2}\right)^{-3-\epsilon}\frac{k_0}{|\,k_0\,|}\frac{p_0}{|\,\mathbf{p}\,|}, \tag{3.17b}$$

$$\tilde{I}'''(k_0,\epsilon)\Big|_{\eta=0}=4\left(1+\frac{p_0^2}{\mathbf{p}^2}\right)^{-3-\epsilon}\left[(12+10\epsilon)\left(1+\frac{p_0^2}{\mathbf{p}^2}\right)^{-1}-2-\epsilon\right]. \tag{3.17c}$$

In order to obtain an expression for $\tilde{I}(k_0,\epsilon)\Big|_{\eta=0}$ we first note that the term proportional to $k_0^{-1+2\epsilon}$ in eq. (3.13) is already of order ϵ. Therefore we may set $\epsilon=0$ in $\tilde{I}(k_0,\epsilon)\Big|_{\eta=0}$. The result is

$$\tilde{I}(k_0,0)\Big|_{\eta=0}=\frac{k_0}{|\,k_0\,|}\left[\left(1+\frac{p_0^2}{\mathbf{p}^2}\right)^{-1}\frac{p_0}{|\,\mathbf{p}\,|}+\frac{1}{2i}\ln\frac{i\,|\,\mathbf{p}\,|-p_0}{i\,|\,\mathbf{p}\,|+p_0}\right]. \tag{3.18}$$

Inserting eq. (3.13) and eq. (3.16) into eq. (3.8) we obtain

$$I(k_0,6+2\epsilon)=\frac{|\,k_0\,|^{-1+2\epsilon}}{12}\left\{p_0^2+\mathbf{p}^2+\epsilon\left[2\mathbf{p}^2+\left(p_0^2+\mathbf{p}^2\right)\left(3+\frac{p_0^2}{\mathbf{p}^2}\right)\right.\right.$$
$$\left.\left.\times\left(\left(1+\frac{p_0^2}{\mathbf{p}^2}\right)^{-1}+\frac{p_0}{2i\,|\,\mathbf{p}\,|}\ln\frac{i\,|\,\mathbf{p}\,|-p_0}{i\,|\,\mathbf{p}\,|+p_0}\right)\right]\right\}+\ldots, \tag{3.19}$$

where the dots represent powers of $|\,k_0\,|$ different from $-1+2\epsilon$. The sum over n may now be performed using

$$\sum_{n=-\infty}^{\infty}|\,k_0\,|^{-1+2\epsilon}=2(2\pi T)^{-1+2\epsilon}\zeta(1-2\epsilon)$$
$$=2(2\pi T)^{-1+2\epsilon}\left(-\frac{1}{2\epsilon}+\gamma+\mathcal{O}(\epsilon)\right). \tag{3.20}$$

Combining eq. (2.2), (3.19) and (3.20) we finally obtain, for the regularized divergent contribution and the induced finite terms, the following expression:

$$(\tilde{\Sigma})^{\epsilon} = \frac{\lambda^2}{384\pi^3}\left\{\left(\frac{1}{2\epsilon}+\frac{1}{2}\ln\frac{4\pi T^2}{\mu^2}-1-\frac{\gamma}{2}\right)(p_0^2+\mathbf{p}^2) + \mathbf{p}^2+\frac{p_0^2+\mathbf{p}^2}{2}\left(3+\frac{p_0^2}{\mathbf{p}^2}\right)\left(\left(1+\frac{p_0^2}{\mathbf{p}^2}\right)^{-1}+\frac{p_0}{2i\mid\mathbf{p}\mid}\ln\frac{i\mid\mathbf{p}\mid-p_0}{i\mid\mathbf{p}\mid+p_0}\right)\right\}. \tag{3.21}$$

Equation (3.21) has to be compared with the dimensionally regularized vacuum two-point function which is given by [10]

$$\tilde{\Sigma}\Big|_{vac} = \frac{\lambda^2}{384\pi^3}\left\{\left(\frac{1}{2\epsilon}+\frac{1}{2}\ln\frac{p_0^2+\mathbf{p}^2}{\mu^2}+\frac{1}{2}\ln\frac{1}{4\pi}-\frac{8}{6}+\frac{\gamma}{2}\right)(p_0^2+\mathbf{p}^2)\right\}. \tag{3.22}$$

If we define the finite part of $\tilde{\Sigma}\Big|_{vac}$ as

$$(\tilde{\Sigma})^f\Big|_{vac} = \frac{\lambda^2}{384\pi^3}\frac{p_0^2+\mathbf{p}^2}{2}\ln\frac{p_0^2+\mathbf{p}^2}{\mu^2}, \tag{3.23}$$

and use the same scheme of subtraction in (3.21), we are left with the following finite contribution at $T\neq 0$

$$(\tilde{\Sigma})^f = \frac{\lambda^2}{384\pi^3}\left\{\left(\ln\frac{4\pi T}{\mu}+\frac{1}{3}-\gamma\right)(p_0^2+\mathbf{p}^2)+\mathbf{p}^2 +\frac{p_0^2+\mathbf{p}^2}{2}\left(3+\frac{p_0^2}{\mathbf{p}^2}\right)\left(\left(1+\frac{p_0^2}{\mathbf{p}^2}\right)^{-1}+\frac{p_0}{2i\mid\mathbf{p}\mid}\ln\frac{i\mid\mathbf{p}\mid-p_0}{i\mid\mathbf{p}\mid+p_0}\right)\right\}, \tag{3.24}$$

The final result for the two-point function in the 6-dimensional $\lambda\phi^3$ theory is given by:

$$\tilde{\Sigma} = (\tilde{\Sigma})^T + (\tilde{\Sigma})^f, \tag{3.25}$$

where $(\tilde{\Sigma})^T$ and $(\tilde{\Sigma})^r$ are given by eq. (3.7) and eq. (3.24) respectively.

4. The Yang-Mills theory

The diagrams which contributes to the two-point gluon function are given in the fig. 2. In the Feynman gauge the $(4+2\epsilon)$-dimensional contribution of these diagrams reads

$$\tilde{\Sigma}^{ab}_{\mu\nu} = N\delta^{ab}g^2\mu^{-2\epsilon}\frac{T}{2}\sum_{n=-\infty}^{\infty}\left\{\int\frac{d^{3+2\epsilon}\mathbf{k}}{(2\pi)^{3+2\epsilon}}\left(\frac{A_{\mu\nu}}{k^2(k+p)^2}-4(\epsilon+1)\frac{g_{\mu\nu}}{k^2}\right)\right\}, \tag{4.1}$$

where N is the $SU(N)$ parameter and $a,b=1,...,N^2-1$. The second term in eq. (4.1) comes from the tadpole diagram (fig. 2b) and part of the gluon-loop diagram (fig. 2a). The tensor $A_{\mu\nu}$ is given by

$$A_{\mu\nu} = 4p^2 g_{\mu\nu}+8(1+\epsilon)k_\mu k_\nu+2(\epsilon-1)p_\mu p_\nu + (4\epsilon+5)p_\mu k_\nu+(4\epsilon+3)k_\mu p_\nu, \tag{4.2}$$

At finite-temperature the tadpole contributions like the second term in eq. (4.1) give a finite nonzero contribution. Indeed, integration over $\mathbf{k}$ yields [10]

$$\int\frac{d^{3+2\epsilon}\mathbf{k}}{(2\pi)^{3+2\epsilon}}\frac{1}{-\mathbf{k}^2-k_0^2} = -\frac{\Gamma\left(-\frac{1}{2}-\epsilon\right)}{(4\pi)^{\frac{3}{2}+\epsilon}}\mid k_0\mid^{1+2\epsilon}. \tag{4.3}$$

Performing the sum over n with help of eq. (1.2) and using $\zeta(-1-2\epsilon) = -1/12+\mathcal{O}(\epsilon)$ we get

$$\tilde{\Sigma}^{ab}_{\mu\nu} = N\delta^{ab}g^2\left[\frac{\mu^{-2\epsilon}T}{2}\sum_{n=-\infty}^{\infty}\int\frac{d^{3+2\epsilon}\mathbf{k}}{(2\pi)^{3+2\epsilon}}\frac{A_{\mu\nu}}{k^2(k+p)^2}+\frac{T^2}{6}g_{\mu\nu}\right]. \tag{4.4}$$

The longitudinal and the transverse components of $\tilde{\Sigma}^{ab}_{\mu\nu}$ are both independent and nonzero at finite-temperature [2]. These two components may be expressed in terms of $\tilde{\Sigma}^{ab}_{00}$ and $\tilde{\Sigma}^{ab\mu}_{\mu} = g^{\mu\nu}\tilde{\Sigma}^{ab}_{\mu\nu}$. Using the Feynman parametrization, the eq. (4.4) yields

$$\tilde{\Sigma}^{ab}_{00} = N\delta^{ab}g^2\left\{\frac{T^2}{6}-\frac{\Gamma\left(\frac{1}{2}-\epsilon\right)}{(4\pi)^{\frac{3}{2}+\epsilon}}\frac{\mu^{-2\epsilon}T}{2}\sum_{n=-\infty}^{\infty}\left[4\mathbf{p}^2+(2\epsilon+2)(2k_0+p_0)^2\right] \times I(k_0,4+2\epsilon)\right\}. \tag{4.5}$$

and

$$\tilde{\Sigma}^{ab\mu}_{\mu} = N\delta^{ab}g^2\left[\frac{T^2}{3}-\frac{\Gamma\left(\frac{1}{2}-\epsilon\right)}{(4\pi)^{\frac{3}{2}+\epsilon}}\frac{\mu^{-2\epsilon}T}{2}(10+6\epsilon)(p_0^2+\mathbf{p}^2)\sum_{n=-\infty}^{\infty}I(k_0,4+2\epsilon)\right], \tag{4.6}$$

where $I(k_0, 4+2\epsilon)$ is given by eq. (2.3).

The eq. (4.6) was obtained using

$$A_\mu{}^\mu = (4+4\epsilon)\left[(k+p)^2 + k^2\right] - (10+6\epsilon)\left(p_0^2 + \mathbf{p}^2\right), \tag{4.7}$$

and then making $k \to k+p$ in the first term, in order to obtain a tadpole integral like the one given by eq. (4.3). We have made use of the same procedure in order to transform part of the gluon-loop diagram (fig. 2a) into a tadpole integral. Note that, on dimensional grounds, the T^2 leading contribution to $\tilde\Sigma_\mu^{ab\mu}$ is completely given by the first term in eq. (4.6).

Let us proceed as in the scalar case and first compute the finite contributions with $\epsilon = 0$. Afterwards, we will compute the temperature-independent contributions proportional to $T\sum k_0^{-1+2\epsilon}$. Setting $\epsilon = 0$ in $I(k_0, 4+2\epsilon)$ and performing the integration over x in eq. (2.3) one get [7]

$$I(k_0,4) = \frac{1}{2i\mid\mathbf{p}\mid}\left[\ln\frac{2\mid k_0\mid\mid\mathbf{p}\mid + i\left(2k_0p_0 + p_0^2 + \mathbf{p}^2\right)}{2\mid k_0\mid\mid\mathbf{p}\mid - i\left(2k_0p_0 + p_0^2 + \mathbf{p}^2\right)} - \ln\frac{2\mid k_0+p_0\mid\mid\mathbf{p}\mid + i\left(2k_0p_0 + p_0^2 - \mathbf{p}^2\right)}{2\mid k_0+p_0\mid\mid\mathbf{p}\mid - i\left(2k_0p_0 + p_0^2 - \mathbf{p}^2\right)}\right]. \tag{4.8}$$

Performing the shift $k_0 \to -k_0 - p_0$ in the second term we obtain

$$I^s(k_0,4) = \frac{\varepsilon(k_0)}{i\mid\mathbf{p}\mid}\left(\ln\frac{i\mid\mathbf{p}\mid - p_0}{i\mid\mathbf{p}\mid + p_0} + \ln\frac{2k_0 + p_0 + i\mid\mathbf{p}\mid}{2k_0 + p_0 - i\mid\mathbf{p}\mid}\right). \tag{4.9}$$

As in the scalar case we now use the identity (2.13). Again the singular contributions proportional to $\varepsilon(\mathrm{Re}\ p_0)$ disappear after we average over the result with the shift and the result without the shift. The sums over n in eqs. (4.5) and (4.6) can then be written as.

$$\sum_{n=-\infty}^{\infty}\left[4\mathbf{p}^2 + 2\left(2k_0+p_0\right)^2\right] I(k_0,4) = \varepsilon(\mathrm{Re}\ p_0)\frac{p_0^2 + 2\mathbf{p}^2}{i\mid\mathbf{p}\mid}\ln\frac{p_0 + i\mid\mathbf{p}\mid}{p_0 - i\mid\mathbf{p}\mid}$$
$$-\frac{4\pi T}{3}\frac{p_0}{i\mid\mathbf{p}\mid}\ln\frac{i\mid\mathbf{p}\mid - p_0}{i\mid\mathbf{p}\mid + p_0}$$
$$+\frac{4\theta(\mathrm{Re}\ p_0)}{i\mid\mathbf{p}\mid}\sum_{n=1}^{\infty}(4k_0^2 + 4k_0p_0 + \mathbf{p}^2)\ln\frac{2k_0 + p_0 + i\mid\mathbf{p}\mid}{2k_0 + p_0 - i\mid\mathbf{p}\mid} + (p_0 \to -p_0) \tag{4.10}$$

and

$$\sum_{n=-\infty}^{\infty} I(k_0,4) = \varepsilon(\mathrm{Re}\ p_0)\frac{1}{2i\mid\mathbf{p}\mid}\ln\frac{p_0 + i\mid\mathbf{p}\mid}{p_0 - i\mid\mathbf{p}\mid}$$
$$+2\theta(\mathrm{Re}\ p_0)\frac{1}{i\mid\mathbf{p}\mid}\sum_{n=1}^{\infty}\ln\frac{2k_0 + p_0 + i\mid\mathbf{p}\mid}{2k_0 + p_0 - i\mid\mathbf{p}\mid} + p_0 \to -p_0, \tag{4.11}$$

where the contributions proportional to $\varepsilon(\mathrm{Re}\ p_0)$ come only from the $n = 0$ term. Expanding the logarithm in powers of k_0^{-1} and using the eqs. (3.4) and (1.2), we obtain

$$\sum_{n=-\infty}^{\infty}\left[4\mathbf{p}^2 + 2\left(k_0+p_0\right)^2\right] I(k_0, 4p_0) =$$
$$\varepsilon(\mathrm{Re}\ p_0)\left[\frac{2p_0^2 + 4\mathbf{p}^2}{i\mid\mathbf{p}\mid}\ln\frac{p_0 + i\mid\mathbf{p}\mid}{p_0 - i\mid\mathbf{p}\mid} - 8\sum_{j=0}^{\infty}\frac{V_{2j} + \mathbf{p}^2 W_{2j}}{(2\pi T)^{2j}}\zeta(2j)\right]$$
$$-\frac{8\pi T}{3}\left(1 + \frac{p_0}{i\mid\mathbf{p}\mid}\ln\frac{i\mid\mathbf{p}\mid - p_0}{i\mid\mathbf{p}\mid + p_0}\right) - 8\sum_{j=0}^{\infty}\frac{V_{2j+1} + \mathbf{p}^2 W_{2j+1}}{(2\pi T)^{2j+1}}\zeta(2j+1) \tag{4.12}$$

and

$$\sum_{n=-\infty}^{\infty} I(k_0,4) = \varepsilon(\mathrm{Re}\ p_0)\left[\frac{1}{i\mid\mathbf{p}\mid}\ln\frac{p_0 + i\mid\mathbf{p}\mid}{p_0 - i\mid\mathbf{p}\mid} - 4\sum_{j=1}^{\infty}\frac{W_{2j}}{(2\pi T)^{2j}}\zeta(2j)\right]$$
$$-4\sum_{j=0}^{\infty}\frac{W_{2j+1}}{(2\pi T)^{2j+1}}\zeta(2j+1), \tag{4.13}$$

where we have made use of the eq. (3.6) for V_k and of the same property obeyed by W_k. As in the scalar $\lambda\phi^3$ theory, there are singular contributions which are proportional to $\zeta(1)$. Inserting only the finite terms of eqs. (4.12) and (4.13) into eqs. (4.5) and (4.6) we obtain

$$(\tilde\Sigma_{00}^{ab})^T = N\delta^{ab}g^2\left\{\varepsilon(\mathrm{Re}\ p_0)\left[\frac{p_0^2 + 2\mathbf{p}^2}{8\pi i\mid\mathbf{p}\mid}\ln\frac{p_0 - i\mid\mathbf{p}\mid}{p_0 + i\mid\mathbf{p}\mid}T + \frac{1}{4\pi^2}\sum_{j=0}^{\infty}\frac{V_{2j} + \mathbf{p}^2 W_{2j}}{(2\pi T)^{2j-1}}\zeta(2j)\right]\right.$$
$$\left.+\frac{T^2}{3}\left(1 + \frac{p_0}{2i\mid\mathbf{p}\mid}\ln\frac{i\mid\mathbf{p}\mid - p_0}{i\mid\mathbf{p}\mid + p_0}\right) + \frac{1}{4\pi^2}\sum_{j=1}^{\infty}\frac{V_{2j+1} + \mathbf{p}^2 W_{2j+1}}{(2\pi T)^{2j}}\right\}\zeta(2j+1) \tag{4.14a}$$

and

$$(\tilde\Sigma_\mu^{ab\mu})^T = N\delta^{ab}g^2\left\{\varepsilon(\mathrm{Re}\ p_0)\left(p_0^2 + \mathbf{p}^2\right)\left[\frac{5}{8\pi i\mid\mathbf{p}\mid}\ln\frac{p_0 - i\mid\mathbf{p}\mid}{p_0 + i\mid\mathbf{p}\mid}T\right.\right.$$

$$+\frac{5}{4\pi^2}\sum_{j=1}^{\infty}\frac{W_{2j}}{(2\pi T)^{2j-1}}\zeta(2j)\Bigg] + \frac{T^2}{3} + \frac{5}{4\pi^2}\left(p_0^2+\mathbf{p}^2\right)\sum_{j=1}^{\infty}\frac{W_{2j+1}}{(2\pi T)^{2j}}\zeta(2j+1)\Bigg\} \qquad (4.14b)$$

In the same way as in the scalar theory, the divergent contributions to $\tilde{\Sigma}^{ab}_{\mu\nu}$ have to be regularized. This can be done keeping $\epsilon \neq 0$ in eqs. (4.5) and (4.6) and expanding the result in powers of k_0. The divergent contribution (in the limit $\epsilon \to 0$) is obtained selecting all the terms proportional to $k_0^{-1+2\epsilon}$.

It is common ground that the finite-temperature divergence have the same structure as the vacuum one [2] (this can be directly seen in the eq. (1.1)). Using dimensional regularization, the vacuum two-point function reads [10]

$$\tilde{\Sigma}^{ab\mu}_{\mu}\Big|_{vac} = N\delta^{ab}g^2\frac{p_0^2+\mathbf{p}^2}{8\pi^2}\frac{5}{2}\left(\ln\frac{p_0^2+\mathbf{p}^2}{\mu^2} + \ln\frac{1}{4\pi} + \gamma - \frac{21}{15} + \frac{1}{\epsilon}\right), \qquad (4.15a)$$

$$\tilde{\Sigma}^{ab}_{00}\Big|_{vac} = N\delta^{ab}g^2\frac{\mathbf{p}^2}{8\pi^2}\frac{5}{6}\left(\ln\frac{p_0^2+\mathbf{p}^2}{\mu^2} + \ln\frac{1}{4\pi} + \gamma - \frac{31}{15} + \frac{1}{\epsilon}\right). \qquad (4.15b)$$

The computation of the finite T divergent contribution follows exactly as in the scalar case. Using eqs. (3.8), (3.9) and (3.12) with $\epsilon \to \epsilon - 1$ and expanding the result in powers of k_0 we obtain

$$\begin{aligned} I(k_0, 4+2\epsilon) = {} & \frac{1}{i\mid\mathbf{p}\mid}\ln\frac{i\mid\mathbf{p}\mid - p_0}{i\mid\mathbf{p}\mid + p_0}\left(1+\frac{p_0^2}{\mathbf{p}^2}\right)\frac{\mid k_0\mid^{1+2\epsilon}}{k_0} \\ & + \left(1+\epsilon\frac{p_0}{i\mid\mathbf{p}\mid}\ln\frac{i\mid\mathbf{p}\mid - p_0}{i\mid\mathbf{p}\mid + p_0}\right)\mid k_0\mid^{-1+2\epsilon} \\ & + \left(\frac{\epsilon-1}{2}p_0 + \frac{\mathbf{p}^2-p_0^2}{2i\mid\mathbf{p}\mid}\ln\frac{i\mid\mathbf{p}\mid - p_0}{i\mid\mathbf{p}\mid + p_0}\right)\frac{\mid k_0\mid^{-1+2\epsilon}}{k_0} \\ & + \left\{\frac{1}{12}\left[\mathbf{p}^2\left((3-\epsilon)\frac{p_0^2}{\mathbf{p}^2} - 1 - \epsilon\right) + \epsilon p_0\frac{p_0^2-3\mathbf{p}^2}{2i\mid\mathbf{p}\mid}\ln\frac{i\mid\mathbf{p}\mid - p_0}{i\mid\mathbf{p}\mid + p_0}\right]\right. \\ & \left. + \frac{\epsilon}{4}\left(\mathbf{p}^2 - p_0^2\right)\right\}\mid k_0\mid^{-3+2\epsilon}. \end{aligned} \qquad (4.16)$$

Inserting eq. (4.16) into eqs. (4.5) and (4.6) and using eq. (3.20) we obtain

$$(\tilde{\Sigma}^{ab\mu}_{\mu})^{\epsilon} = N\delta^{ab}g^2\frac{p_0^2+\mathbf{p}^2}{8\pi^2}\frac{5}{2}\left(\ln\frac{4\pi T^2}{\mu^2} + \frac{p_0}{i\mid\mathbf{p}\mid}\ln\frac{i\mid\mathbf{p}\mid - p_0}{i\mid\mathbf{p}\mid + p_0} - \gamma + \frac{3}{5} + \frac{1}{\epsilon}\right) \qquad (4.17a)$$

and

$$\begin{aligned} (\tilde{\Sigma}^{ab}_{00})^{\epsilon} = {} & N\delta^{ab}g^2\left\{\frac{\mathbf{p}^2}{8\pi^2}\frac{5}{6}\left(\ln\frac{4\pi T^2}{\mu^2} - \gamma + \frac{1}{5} + \frac{1}{\epsilon}\right)\right. \\ & \left. + \frac{p_0^2}{24\pi^2}\left(1+\frac{p_0}{2i\mid\mathbf{p}\mid}\ln\frac{i\mid\mathbf{p}\mid - p_0}{i\mid\mathbf{p}\mid + p_0}\right) + \frac{p_0\mid\mathbf{p}\mid}{8\pi^2 i}\ln\frac{i\mid\mathbf{p}\mid - p_0}{i\mid\mathbf{p}\mid + p_0}\right\}. \end{aligned} \qquad (4.17b)$$

Comparing the eqs. (4.15) with eqs. (4.17) we note that the contributions proportional to $1/\epsilon$ are indeed identical. If we define the finite part of the vacuum two-point function as the first terms in eq. (4.15) so that

$$(\tilde{\Sigma}^{ab\mu}_{\mu})^{f}\Big|_{vac} = N\delta^{ab}g^2\frac{p_0^2+\mathbf{p}^2}{8\pi^2}\frac{5}{2}\ln\frac{p_0^2+\mathbf{p}^2}{\mu^2}, \qquad (4.18a)$$

$$(\tilde{\Sigma}^{ab}_{00})^{f}\Big|_{vac} = N\delta^{ab}g^2\frac{\mathbf{p}^2}{8\pi^2}\frac{5}{6}\ln\frac{\mathbf{p}^2}{\mu^2}, \qquad (4.18b)$$

and use the same scheme of subtraction for the finite-temperature two-point function, then from the eqs. (4.17) we obtain

$$(\tilde{\Sigma}^{ab\mu}_{\mu})^{f} = N\delta^{ab}g^2\frac{p_0^2+\mathbf{p}^2}{8\pi^2}5\left(\ln\frac{4\pi T}{\mu} - \gamma + 1 + \frac{p_0}{2i\mid\mathbf{p}\mid}\ln\frac{i\mid\mathbf{p}\mid - p_0}{i\mid\mathbf{p}\mid + p_0}\right), \qquad (4.19a)$$

$$\begin{aligned} (\tilde{\Sigma}^{ab}_{00})^{f} = {} & N\delta^{ab}g^2\left\{\frac{\mathbf{p}^2}{8\pi^2}\frac{5}{3}\left(\ln\frac{4\pi T}{\mu} - \gamma + \frac{17}{15}\right)\right. \\ & \left. + \frac{p_0^2}{24\pi^2}\left(1+\frac{p_0}{2i\mid\mathbf{p}\mid}\ln\frac{i\mid\mathbf{p}\mid - p_0}{i\mid\mathbf{p}\mid + p_0}\right) + \frac{p_0\mid\mathbf{p}\mid}{8\pi^2 i}\ln\frac{i\mid\mathbf{p}\mid - p_0}{i\mid\mathbf{p}\mid + p_0}\right\}. \end{aligned} \qquad (4.19b)$$

The renormalized two-point function for the finite-temperature Yang-Mills theory may be finally written as

$$\left.\begin{aligned} \tilde{\Sigma}^{ab\mu}_{\mu} &= (\tilde{\Sigma}^{ab\mu}_{\mu})^{T} + (\tilde{\Sigma}^{ab\mu}_{\mu})^{f} \\ \tilde{\Sigma}^{ab}_{00} &= (\tilde{\Sigma}^{ab}_{00})^{T} + (\tilde{\Sigma}^{ab}_{00})^{f} \end{aligned}\right\}, \qquad (4.20)$$

where the two terms on the right hand side are given by the eqs. (4.14) and (4.19).

We may now consider any particular direction in the complex p_0-plane. A case of special interest is when p_0 becomes a pure imaginary quantity. This corresponds to the real-time limit of the Green functions (we recall that our result has been obtained in the Euclidean space). In order to obtain the real-time limit from our analytically continued imaginary-time result, let us substitute in the eqs. (4.20) $p_0 = iq_0 + q_0\epsilon$, where $\epsilon \to 0^+$ and q_0 is real. Using the identities

$$\frac{\varepsilon(\mathrm{Re}\, p_0)}{\pi} \ln \frac{p_0 - i \mid \mathbf{p} \mid}{p_0 + i \mid \mathbf{p} \mid} = \frac{\varepsilon(q_0)}{\pi} \ln \left| \frac{q_0 - \mid \mathbf{p} \mid}{q_0 + \mid \mathbf{p} \mid} \right| - i\theta(-q_0^2 + \mathbf{p}^2), \qquad (4.21a)$$

$$\ln \frac{i \mid \mathbf{p} \mid - p_0}{i \mid \mathbf{p} \mid + p_0} = \ln \left| \frac{\mid \mathbf{p} \mid - q_0}{\mid \mathbf{p} \mid + q_0} \right| + i\pi\varepsilon(q_0)\theta(q_0^2 - \mathbf{p}^2) \qquad (4.21b)$$

and

$$\mathrm{Re}\, W_{2j}(p_0 = iq_0) = \mathrm{Im}\, W_{2j+1}(p_0 = iq_0) = 0, \qquad (4.21c)$$

one can easily verify that the real-time limit of our analytically continued imaginary-time result, which yields $\bar{\Sigma}_{\mathrm{F}\,\mu\nu}^{\;ab}$ [9], has a real part which is in agreement with the result obtained by Weldon [2], and an imaginary part which can be explicitly written as

$$\mathrm{Im}\, \bar{\Sigma}_{\mathrm{F}\,\mu}^{\;ab\mu} = N\delta^{ab} g^2 \varepsilon(q_0) \frac{5(\mathbf{p}^2 - q_0^2)}{4\pi} \left\{ \frac{1}{2 \mid \mathbf{p} \mid} \ln \left| \frac{q_0 + \mid \mathbf{p} \mid}{q_0 - \mid \mathbf{p} \mid} \right| T + \frac{q_0}{4 \mid \mathbf{p} \mid} \theta(q_0^2 - \mathbf{p}^2) \right.$$
$$\left. + \frac{1}{\pi i} \sum_{j=1}^{\infty} \frac{W_{2j}(p_0 = iq_0)}{(2\pi T)^{2j-1}} \zeta(2j) \right\}, \qquad (4.22a)$$

$$\mathrm{Im}\, \bar{\Sigma}_{\mathrm{F}\,00}^{\;ab} = N\delta^{ab} g^2 \varepsilon(q_0) \left\{ \frac{q_0}{6 \mid \mathbf{p} \mid} \theta(q_0^2 - \mathbf{p}^2)\, T^2 + \left(\frac{2\mathbf{p}^2 - q_0^2}{2 \mid \mathbf{p} \mid} \ln \left| \frac{q_0 + \mid \mathbf{p} \mid}{q_0 - \mid \mathbf{p} \mid} \right| + q_0 \right) \frac{T}{4\pi} \right.$$
$$\left. + \frac{q_0}{48\pi \mid \mathbf{p} \mid} (6\mathbf{p}^2 - q_0^2) \theta(q_0^2 - \mathbf{p}^2) + \frac{1}{4\pi^2 i} \sum_{j=1}^{\infty} \frac{V_{2j}(p_0 = iq_0) + \mathbf{p}^2 W_{2j}(p_0 = iq_0)}{(2\pi T)^{2j-1}} \zeta(2j) \right\}. \qquad (4.22b)$$

In order to relate our real-time limit with the real-time formalism, one can use for instance the eqs. (3.2.18) to (3.2.21) of ref. [9]. In this way, the real and the imaginary parts of $\bar{\Sigma}_{\mathrm{F}\,\mu\nu}^{\;ab}$ yields the four functions $(\bar{\Sigma}_{\mu\nu}^{ab})_{ij}$ of the real-time formalism.

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Figure captions

Fig. 1– Diagram contributing to the two-point function in the $\lambda\phi^3$ theory.

Fig. 2– The three diagrams contributing to the two-point function in the Yang-Mills theory. Wavy lines denote gluons and broken lines denote ghosts.

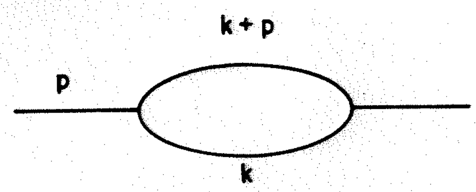

Fig. 1

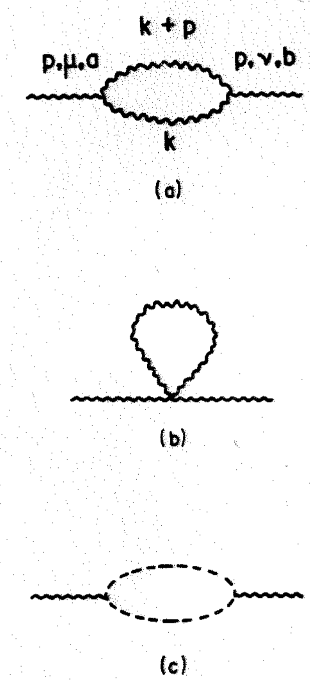

Fig. 2