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SEMI MICROSCOPIC DESCRIPTION OF THE ^{90}Y, ^{90}Zr AND ^{91}Zr NUCLEI

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ABSTRACT

The structure of the low-lying levels of the ^{90}Y, ^{90}Zr and ^{91}Zr nuclei is discussed in a framework of the cluster-phonon coupling model. In order to describe simultaneously positive and negative parity states, octupole as well quadrupole vibrations of the ^{88}Sr core are allowed. The cluster states include single neutron coupled to single proton (^{90}Y), two single proton (^{90}Zr) and two single neutrons coupled to single proton (^{91}Zr). The residual interaction among the cluster particles is assumed to be the modified surface δ-interaction. For each case, energy levels and electromagnetic properties are calculated and compared with the experimental data.

NUCLEAR STRUCTURE: ^{90}Y, ^{90}Zr, and ^{91}Zr; calculated levels J, π, Q, μ, τ, $B(E\lambda)$, and $\delta(E2/M1)$. Cluster-phonon coupling model. Modified surface δ-interaction

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I - INTRODUCTION

The isotopes ^{90}Zr and ^{91}Zr have been the subject of many experimental and theoretical investigations in the past and many features of the experimental data have been found to be readily explicable in terms of fairly simple shell model descriptions(1-7). For the N=50 isotones the use of the $(2p_{1/2}, 1g_{9/2})^n$ model space with an inert ^{88}Sr core in the shell model calculation schemes has been quite successful in reproducing energy levels, life-times and decay rates, nuclear moments and other parameters. However in the case of ^{90}Zr, most of the low lying negative parity states as well as a few odd angular momentum positive parity states cannot be explained with this model. The collective structure of the 3^- state in ^{90}Zr is fairly well established(8) and the exact structures of other negative parity states in this nucleus have not been determined. To explain these states therefore, one needs to consider another, and perhaps a larger model space as well as the possibility that these states are due to core excitations. In the case of ^{91}Zr nucleus previous calculations(4-6) included besides the well known proton interaction determined from N=50 calculation, the coupling of neutron degrees of freedom ($2d_{5/2}$, $3s_{1/2}$, $2d_{3/2}$ and $1g_{7/2}$ orbits). In this model space the importance of core excitations has never been investigated.

The main aim of the present work is to describe simultaneously positive and negative parity states of ^{90}Zr and positive parity states of ^{91}Zr, using a model in which quadrupole as well as octupole vibrations of the ^{88}Sr core are allowed. Recent calculations(8,9) performed for nuclei with A ~ 90 using the cluster (quadrupole-octupole) phonon model indicate that this model represents a good starting point to describe the general trend of experimental data in ^{89}Y, ^{90}Y, ^{87}Sr and ^{89}Sr.

In order to test the parametrization of the residual proton-neutron interaction and other parameters of the cluster phonon model, the available experimental data for the ^{90}Y nucleus were discussed.

II - THE NUCLEAR MODEL

A detailed description of the cluster vibrator model is given in Refs. 11 and 12. He we only sketch the main formulas in order to establish the notation. The total Hamiltonian is

$$H = H_o + H_{res} + H_{int} \qquad (1)$$

where H_o represents the energy of the unperturbed system consisting of quadrupole and octupole vibrational fields and valence particles in a central field. The effective proton-proton and proton-neutron residual interaction among the particles in the shell-model cluster, H_{res}, only includes explicity the modified surface delta interaction (MSDI). This two-body force is expressed in the form(13),

$$H_{res} = -4\pi A_T \ \delta(\vec{r}(1)-\vec{r}(2)) \ \delta(r(1)-R_o)+B\left[\vec{\tau}(1) \cdot \vec{\tau}(2)\right] , \qquad (2)$$

where $\vec{r}$ and $\vec{\tau}$ are the position and isospin vector operators of the interacting particles and R_o the nuclear radius. The isospin T is used to label the strength parameters A_1, A_o for T=1 and T=0, respectively. The last term contributes only to the diagonal matrix elements, and is necessary only for ^{91}Zr calculations. The interaction between the p-particle cluster and the vibrational fields is given by the expression

$$H_{int} = \sum_{\lambda=2}^{3} \frac{\beta_\lambda}{(2 +1)^{1/2}} \sum_{\mu=-\lambda}^{\lambda} \left[b_\lambda^{\mu+}+(-)^{\lambda-\mu} b_\lambda^{-\mu}\right] \sum_{i=1}^{p} k(r_i) Y^{*}_{\lambda\mu}(\theta_i,\phi_i) \qquad (3)$$

where $k(r_i)$ is the interaction intensity, β_λ are the deformation parameters and all other symbols have the standard meaning.

The matrix elements of H_{int} are parametrized by the coupling constants a_λ defined as

$$a_\lambda = \frac{\langle k\rangle}{\sqrt{4\pi}} \frac{\beta_\lambda}{\sqrt{2\lambda+1}}$$

where <k> is the mean value of the radial matrix element of the interaction.

The eingenvalue problem (1) is solved in the basis

$$|(j_1 j_2)J_{12}, R; I\rangle$$

for the A=90 nuclei (p=2) and in the basis

$$|(j_1j_2)J_{12},j_3|\; J,R;I\rangle$$

for the A=91 nucleus (p=3). Here j=(nlj) stands for the quantum numbers of the single proton or neutron state, J the total angular momentum of the cluster, R represent the quantum numbers $\{N_2R_2, N_3R_3, R\}$ where N_λ is the number of λ-pole phonons of angular momentum R_λ , $\vec{R} = \vec{R}_2 + \vec{R}_3$, and I is the total angular momentum.

The electric and magnetic operators consist of a particle and a collective part

$$\mathcal{M}(E\lambda,\mu) = \sum_{i=1}^{p} e^{eff}(i)\; r^\lambda(i)\; Y_{\lambda\mu}(\theta_i,\phi_i) + \frac{3}{4\pi}\, e_v^{eff}\; R_o^\lambda\left[b_\lambda^{\mu+} + (-)^{\mu-\lambda} b_\lambda^{-\mu}\right] , \tag{5}$$

$$\mathcal{M}(M1,\mu) = \left(\frac{3}{4\pi}\right)^{1/2}\left[g_R\, R_\mu + \sum_{i=1}^{p}(g_s(i)S_\mu(i)+g_\ell(i)L_\mu(i))\right]\mu_N, \tag{6}$$

where e_p^{eff} is the effective particle charge, $e_v^{eff} = Ze\beta_\lambda/\sqrt{2\lambda+1}$ is the effective vibrator charge, and g_R, g_ℓ and g_s are, respectively, the collective, orbital and spin gyromagnetic ratios.

The mixing ratio $\delta(E2/M1)$ is given by

$$\delta(E2/M1) = 0.835\; (E_\lambda/\mathrm{MeV})\,(\mathcal{D}\;/eb\;\mu_N^{-1}) \tag{7}$$

with

$$\mathcal{D} = \frac{\langle I_i \| \;\mathcal{M}(E2)\| \; I_f\rangle}{\langle I_i \| \;\mathcal{M}(M1)\| \; I_f\rangle} , \tag{8}$$

and $E_\gamma = E_i - E_f$ is the transition energy.

The reduced transition probabilities are

$$B(\bar{\omega}L;I_i \to I_f) = \frac{\langle I_i \| \;\mathcal{M}(\bar{\omega}L)\| \; I_f\rangle^2}{2J_i+1} \tag{9}$$

with $\bar{\omega}L$ = E2,M1 for electric and magnetic cases, respectively. The matrix elements of E2 and M1 operators are expressed in the forms

$$\langle I_i \| \;\mathcal{M}(E\lambda)\| \; I_f\rangle = (e_p^{eff}\; A + e_n^{eff}\; A' + e_v^{eff}\; B)\; e\; fm^\lambda \tag{10a}$$

$$\langle I_i \| \;\mathcal{M}(M1)\| \; I_f\rangle = (g_{s_p}^{eff}\; C + g_{s_n}^{eff}\; C' + g_{\ell_p}\; D + g_{\ell_n}\; D' + g_R E)\mu_N \tag{10b}$$

and the quantities A, A′, B, C, C′, D, D′ and E are calculated from the model wave functions.

The mean lifetime corresponding to the transition probability T of decaying state I_i to I_f is

$$\tau(\bar{\omega}L;I_i \to I_f) = 1/T(\bar{\omega}L;I_i \to I_f) \tag{11}$$

with

$$T(\bar{\omega}L;I_i \to I_f) = \frac{8\pi(L+1)}{L\left[(2L+1)!!\right]^2} \frac{3\times10^{23}s^{-1}}{137} \left(\frac{E_\gamma}{197\ \text{MeV}}\right)^{2L+1} \frac{B(\bar{\omega}L;I_i \to I_f)}{e^2\ \text{fm}^{2L}} .$$

III - PARAMETERS

The clusters are assumed to consist of one proton and one neutron, two protons, and two protons and one neutron for ^{90}Y, ^{90}Zr and ^{91}Zr, respectively. The configuration space is generated by the valence particles distributed among the single-proton states: $2p_{1/2}$, $1g_{9/2}$ and the single-neutron states: $2d_{5/2}$, $3s_{1/2}$, $2d_{3/2}$, and $1g_{7/2}$, and coupled to N=0, 1, 2 quadrupole phonon and N=0,1 octupole phonon.

The Hamiltonian was diagonalized with the following set of parameters:

(a) single particle energies $\varepsilon p_{1/2} = 0$, $\varepsilon g_{9/2} = 0{,}839$ MeV, $\varepsilon d_{5/2} = 0$, $\varepsilon s_{1/2} = 1.08$ MeV, $\varepsilon d_{3/2} = 2.00$ MeV, and $\varepsilon g_{7/2} = 2.34$ MeV, were taken from the work of Chuu[5];

(b) phonon energies $\hbar\omega_2 = 1.836$ MeV and $\hbar\omega_3 = 2.734$ MeV, are the experimental energies of 2_1^+ and 3_1^- states in the ^{88}Sr nucleus;

(c) MSDI strengths $A_o = 0.4$ MeV, is fixed for ^{90}Y and ^{91}Zr, $A_1 = 0.3$ MeV, 0.4 MeV and 0.5 MeV, are used for ^{90}Y, ^{90}Zr, and ^{91}Zr, respectively, and B = 0.35 MeV is taken for ^{91}Zr.

(d) particle-vibration coupling constants $a_2 = 0.5$ MeV and $a_3 = 0.4$ MeV are fixed for the three nuclei.

The electromagnetic properties were obtained with the usual values of the effective electric charge and effective gyromagnetic ratios, namely

$$\left.\begin{array}{l} \text{Set I} : e_p^{eff} = e \\ \text{Set II} : e_p^{eff} = 2e \end{array}\right\} , \quad e_v^{eff} = \frac{Ze\beta_\lambda}{\sqrt{2\lambda+1}} , \quad e_n^{eff} = 0.5e,$$

for the electric transitions, and

$$\left.\begin{array}{l} \text{SET A} : g_{s_p} = g_{s_p}^{free} , \quad g_{s_n} = g_{s_n}^{free} \\ \text{SET B} : g_{s_p} = 0.7g_{s_p}^{free} , \quad g_{s_n} = 0.7g_{s_n}^{free} \end{array}\right\} , \quad g_{\ell_p} = 1,\ g_{\ell_n} = 0,\ g_R = Z/A$$

In the calculations of electric properties we used for the radial matrix elements $\langle j_a | r^\lambda | j_b \rangle$ the usual estimate $\langle r^\lambda \rangle = (3/(\lambda+3)) R_o^\lambda$ with a nuclear radius $R_o = 1.30\ A^{1/3}$, and the fixed value $\langle k \rangle = 50$ MeV[14].

The theoretical mixing ratios $\delta(E2/M1)$ and mean lifetimes τ were evaluated assuming the ordering and excitation energies of the experimental levels.

IV - RESULTS AND DISCUSSION

A. ^{90}Y Nucleus

The goal of the present calculation for this nucleus is to test the parametrization quoted in the previous section, in particular, to adjust the proton-neutron interaction strength parameters. The experimental[15] and theoretical spectra of ^{90}Y are compared in Fig. 1.

As an example, in fig. 1 the calculated spectrum for $A_1 = 0.2$ MeV is also exhibited. It should be noted that the agreement between the calculated and measured energy spectra for ^{90}Y nucleus can be improved by lowering the MSDI strength parameter A_1, with $A_o = 0.4$ MeV. The results for the electromagnetic properties of ^{90}Y are shown in Table 1. Our calculations reproduce the electric quadrupole moment of the ground state. The magnetic dipole moment of the ground state and the lifetime of the 3_1^- state obtained in Ref. 10 are in good agreement with experimental data. These values were calculated by means of wave functions, effective charges and gyromagnetic ratios similar to those employed in our calculations. Only the effective core gyromagnetic ratio $g_R = 0$ value used[10] is quite different.

B. ^{90}Zr Nucleus

The experimental[15] and theoretical spectra for ^{90}Zr are compared in fig. 2. The agreement between calculated and measured positive parity levels is acceptable. Although the single neutron $1h_{11/2}$ state is not included in the calculations, particularly for negative parity levels, there is good agreement between experiment and theory.

The configurations and amplitudes of the wave functions of low-lying states in ^{90}Zr which contribute more than 4% are listed in Table 2. Only the ground state is a pure two proton state. The 2_1^+ and 3_1^- appear to have collective characteristics. The remaining states are based on the two-proton $(g_{9/2})^2$ and $(p_{1/2}, g_{9/2})$ configurations for positive and negative parity levels, respectively. In these states there is a small mixing of one quadrupolar phonon configuration.

Experimental information on the electromagnetic properties are displayed in Table 3. We also show theoretical results. The mean lifetimes were obtained by using the experimental excitation energies 1.761 MeV, 2.186 MeV, 2.319 MeV, 2.739 MeV, 2.750 MeV, 3.077 MeV, 3.309 MeV, 3.448 MeV, 3.589 MeV and 3.843 MeV, for the 0_2^+, 2_1^+, 5_1^-, 4_1^-, 3_1^-, 4_1^+, 2_2^+, 6_1^+, 8_1^+ and 2_3^+ states, respectively. By inspecting the experimental and theoretical results one sees that the measured electric quadrupole and magnetic dipole moments for the 8_1^+ state are reproduced by the calculation. The agreement between calculated

(e_p = 2e) and experimental B(Eλ) and mean lifetimes values is, in general, reasonable. Our calculation reproduces the measured $\tau\,(2_1^+)$[15] and B(E2; $0_1^+ \rightarrow 2_2^+$)[16] values.

c. ^{91}Zr Nucleus

The calculated energy spectra for positive parity levels is compared with experiment in fig. 3, and there is good agreement between them. The experimental sequence for the first six levels is reproduced by the theory. For these levels, the components of the corresponding wave functions which contribute more than 4% are listed in Table 4. It appears that the $5/2_1^+$ and $5/2_2^+$ states have mainly a three-particle configuration, while the others have mixing characteristics.

Experimental data on the electromagnetic properties are presented and compared with the calculated values in Table 5. The calculated E2/M1 mixing ratios and mean lifetimes were obtained by using for the excitation energies of the $1/2_1^+$, $5/2_2^+$, $7/2_1^+$, $3/2_1^+$ and $9/2_1^+$ states the measured values 1.205 MeV, 1.467 MeV, 1.882 MeV, 2.041 MeV and 2.131 MeV, respectively. The measured values for the quadrupole and dipole moments of the ground state[17], the mean lifes $\tau(1/2_1^+)$[19], $\tau(5/2_1^+)$[7,18,19], $\tau(7/2_1^+)$[7,19], $\tau(3/2_1^+)$[18] and $\tau(9/2_1^+)$[19], and the B(E2; $5/2_1^+ \rightarrow 1/2_1^+$)[17,21] are well reproduced by the theory. Only for the values involving the $1/2_1^+$ state, i.e. the $\tau(1/2_1^+)$ and B(E2: $5/2_1^+ \rightarrow 1/2_1^+$), we needed to use the effective neutron charge, e_n^{eff} = 1.5e, taked from Gloeckner's work[3].

The calculated signs and magnitudes of the mixing ratios $\tau(7/2_1^+ \rightarrow 5/2_1^+)$ and $\delta(3/2_1^+ \rightarrow 5/2_1^+)$ are in good agreement with the available experimental data in Refs. 19 and 20, respectively.

V - SUMMARY AND CONCLUSIONS

The properties of the ^{90}Y, ^{90}Zr and ^{91}Zr nuclei were calculated within the framework of two-particle and tree-particle cluster core coupling model, respectively. The cluster of particles is coupled to the quadrupole and octupole vibrations field of the ^{88}Sr core. In the calculations only the intensity strength $A_{T=0}$ were adjustable with different value for each nucleus.

The available data on the energy spectra, electric and magnetic moments, B(E λ) and B(M1) values, mixing eletromagnetic ratios and mean lifetimes were examined. The results reported here demonstrated that, in this picture, it was possible to give a reasonably accurate description of the low-lying level properties of these nuclei using the same residual interaction (MSDI) and a uniform set of parameters.

It seems that the correlation and excitation modes not included as particle-hole excitations, single-neutron $1h_{11/2}$ state and larger number of phonons, may affect in an appreciable way only the states above ~ 3 MeV of excitation energy.

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FIGURE CAPTIONS

Figure 1 - Comparison of experimental levels of ^{90}Y from Ref. 15. with the calculated spectra.

Figure 2 - Comparison of experimental levels of ^{90}Zr from Ref. 15 with the calculated spectra.

Figure 3 - Comparison of experimental levels of ^{91}Zr from Ref. 17 with the calculated spectra.

TABLE CAPTIONS

Table 1 - Comparison of experimental[15] and theoretical electric quadrupole and magnetic dipole moments and mean lifetimes for I_i^{π} state in ^{90}Y nucleus.

Table 2 - Wave functions of a few low-lying states in ^{90}Zr. Only those amplitudes which are larger than 4% are listed.

Table 3 - Comparison of the theoretical electromagnetic observables with the available experimental information for ^{90}Zr nucleus.

Table 4 - Wave functions of a few low-lying states in ^{91}Zr. Only those amplitudes which are larger than 4% are listed.

Table 5 - Comparison of the theoretical electromagnetic observables with the available experimental information for ^{91}Zr nucleus.

TABLE 1

I_i^{π}	Q(eb)			μ(nm)			τ(ps)		
	EXP	THEORY		EXP	THEORY		EXP	THEORY	
		I	II		A	B		IA	IB
2_1^-	-0.155 3	-0.13	-0.20	-1.630 8	6.66	5.63			
3_1^-							250 7	35	70

TABLE 2

I_i^π	j_a	j_b	j_{ab}	N	R	AMPLITUDE
0_1^+	$\frac{9}{2}$	$\frac{9}{2}$	0	0	0	0.777
	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	-0.598
0_2^+	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0.789
	$\frac{9}{2}$	$\frac{9}{2}$	0	0	0	0.533
	$\frac{9}{2}$	$\frac{9}{2}$	2	1	2	-0.279
2_1^+	$\frac{9}{2}$	$\frac{9}{2}$	0	1	2	-0.750
	$\frac{1}{2}$	$\frac{1}{2}$	0	1	2	0.516
	$\frac{9}{2}$	$\frac{9}{2}$	2	0	0	0.348
2_2^+	$\frac{9}{2}$	$\frac{9}{2}$	2	0	0	0.816
	$\frac{1}{2}$	$\frac{1}{2}$	0	1	2	-0.372
	$\frac{9}{2}$	$\frac{9}{2}$	4	1	2	-0.263
	$\frac{9}{2}$	$\frac{9}{2}$	2	1	2	0.261
3_1^-	$\frac{9}{2}$	$\frac{9}{2}$	0	1	3	0.756
	$\frac{1}{2}$	$\frac{1}{2}$	0	1	3	-0.655
4_1^-	$\frac{1}{2}$	$\frac{9}{2}$	4	0	0	0.957
	$\frac{1}{2}$	$\frac{9}{2}$	4	1	2	-0.273

TABLE 2 (continuation)

I_i^π	j_a	j_b	j_{ab}	N	R	AMPLITUDE
5_1^-	$\frac{1}{2}$	$\frac{9}{2}$	5	0	0	0.958
	$\frac{1}{2}$	$\frac{9}{2}$	5	1	2	-0.275
4_1^+	$\frac{9}{2}$	$\frac{9}{2}$	4	0	0	0.869
	$\frac{9}{2}$	$\frac{9}{2}$	2	1	2	-0.358
	$\frac{9}{2}$	$\frac{9}{2}$	6	1	2	-0.215
6_1^+	$\frac{9}{2}$	$\frac{9}{2}$	6	0	0	0.948
	$\frac{9}{2}$	$\frac{9}{2}$	4	1	2	-0.256
8_1^+	$\frac{9}{2}$	$\frac{9}{2}$	8	0	0	0.933
	$\frac{9}{2}$	$\frac{9}{2}$	8	1	2	-0.322

TABLE 3

I_i^π	Q(eb) EXP	Q(eb) THEORY I	Q(eb) THEORY II	μ(nm) EXP	μ(nm) THEORY A	μ(nm) THEORY B
8_1^+	10.91[a]	11.97	10.51	-0.51[a]	-0.49	-0.67

I_i^π	τ EXP	τ THEORY IA	τ THEORY IIA	λ	$B(E\lambda\uparrow)$ $(e^2fm^{2\lambda})$ EXP	$B(E\lambda\uparrow)$ THEORY I	$B(E\lambda\uparrow)$ THEORY II
2_1^+	84fs 7[b]	92fs	70fs				
$0_1^+ \rightarrow 2_1^+$				2	674 60[b]	882	1.170
5_1^-	0.81s 2[b]	7s	1.7s				
$0_1^+ \rightarrow 3_1^-$				3	108,000[b]	20,500	20,500
$0_1^+ \rightarrow 4_1^+$				4	340,000[b]	42,000	168,000
$0_1^+ \rightarrow 2_2^+$				2	70.7[c]	0.7	86
8_1^+	125ns 6[b]	647ns	210ns				
$6_1^+ \rightarrow 8_1^+$				2	275[a]	30	80
2_3^+	11.9fs 4[b]	9.4fs	9.1fs				

a - Ref. 6

b - Ref. 15

c - Ref. 16

TABLE 4

I_i^π	j_a	j_b	j_{ab}	J_c	j	N	R	AMPLITUDE
$1/2_1^+$	$\frac{9}{2}$	$\frac{9}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0.724
	$\frac{9}{2}$	$\frac{9}{2}$	0	$\frac{5}{2}$	$\frac{5}{2}$	1	2	-0.406
	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	-0.338
	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{5}{2}$	$\frac{5}{2}$	1	2	0.330
	$\frac{9}{2}$	$\frac{9}{2}$	2	$\frac{5}{2}$	$\frac{1}{2}$	0	0	-0.219
$3/2_1^+$	$\frac{9}{2}$	$\frac{9}{2}$	0	$\frac{3}{2}$	$\frac{3}{2}$	0	0	0.547
	$\frac{9}{2}$	$\frac{9}{2}$	0	$\frac{5}{2}$	$\frac{5}{2}$	1	2	0.416
	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{3}{2}$	$\frac{3}{2}$	0	0	-0.409
	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{5}{2}$	$\frac{5}{2}$	1	2	-0.350
	$\frac{9}{2}$	$\frac{9}{2}$	2	$\frac{5}{2}$	$\frac{3}{2}$	0	0	0.294
	$\frac{9}{2}$	$\frac{9}{2}$	4	$\frac{5}{2}$	$\frac{3}{2}$	0	0	0.273
$5/2_1^+$	$\frac{9}{2}$	$\frac{9}{2}$	0	$\frac{5}{2}$	$\frac{5}{2}$	0	0	-0.716
	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{5}{2}$	$\frac{5}{2}$	0	0	0.560
	$\frac{9}{2}$	$\frac{9}{2}$	0	$\frac{5}{2}$	$\frac{5}{2}$	1	2	0.206
	$\frac{9}{2}$	$\frac{9}{2}$	2	$\frac{5}{2}$	$\frac{5}{2}$	0	0	-0.204

TABLE 4 (continuation)

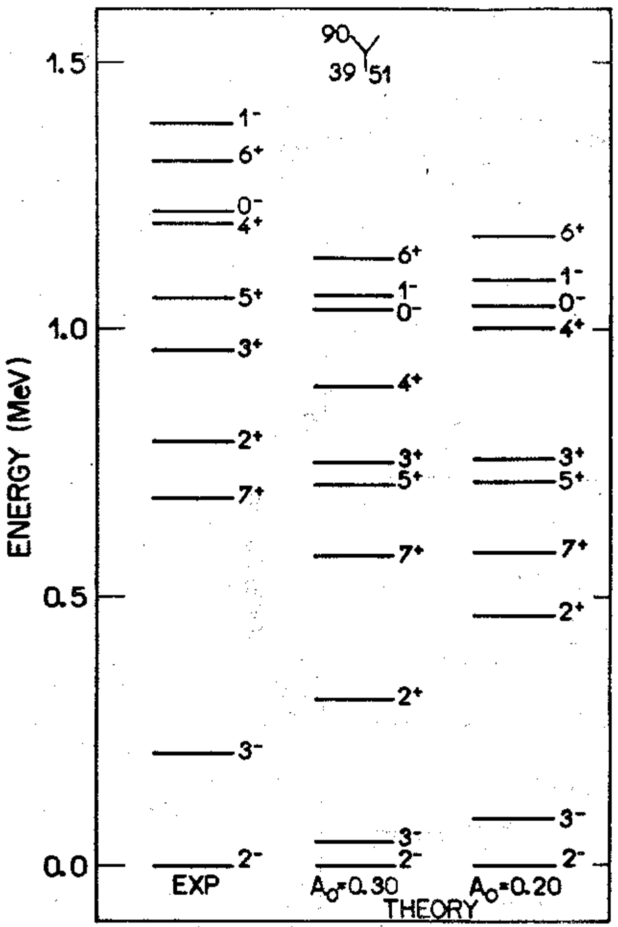
I_i^π	j_a	j_b	j_{ab}	J_c	j	N	R	AMPLITUDE
$5/2_2^+$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{5}{2}$	$\frac{5}{2}$	0	0	0.709
	$\frac{9}{2}$	$\frac{9}{2}$	2	$\frac{5}{2}$	$\frac{5}{2}$	0	0	0.489
	$\frac{9}{2}$	$\frac{9}{2}$	0	$\frac{5}{2}$	$\frac{5}{2}$	0	0	0.289
	$\frac{9}{2}$	$\frac{9}{2}$	0	$\frac{5}{2}$	$\frac{5}{2}$	1	2	0.206
$7/2_1^+$	$\frac{9}{2}$	$\frac{9}{2}$	2	$\frac{5}{2}$	$\frac{7}{2}$	0	0	−0.683
	$\frac{9}{2}$	$\frac{9}{2}$	4	$\frac{5}{2}$	$\frac{7}{2}$	0	0	−0.402
	$\frac{9}{2}$	$\frac{9}{2}$	0	$\frac{5}{2}$	$\frac{5}{2}$	1	2	−0.373
	$\frac{9}{2}$	$\frac{9}{2}$	2	$\frac{5}{2}$	$\frac{5}{2}$	1	2	−0.249
	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{5}{2}$	$\frac{5}{2}$	1	2	0.207
$9/2_1^+$	$\frac{9}{2}$	$\frac{9}{2}$	2	$\frac{5}{2}$	$\frac{9}{2}$	0	0	0.569
	$\frac{9}{2}$	$\frac{9}{2}$	4	$\frac{5}{2}$	$\frac{9}{2}$	0	0	0.447
	$\frac{9}{2}$	$\frac{9}{2}$	0	$\frac{5}{2}$	$\frac{5}{2}$	1	2	−0.408
	$\frac{9}{2}$	$\frac{9}{2}$	6	$\frac{5}{2}$	$\frac{9}{2}$	0	0	0.291
	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{5}{2}$	$\frac{5}{2}$	1	2	0.265

TABLE 5

I_i^π	Q(eb) EXP	Q(eb) THEORY I	Q(eb) THEORY II	μ(nm) EXP	μ(nm) THEORY A	μ(nm) THEORY B
$5/2_1^+$	−0.21 ± 0.02[a]	−0.21	−0.25	−1.303 ± 0.001[a]	−1.58	−0.10

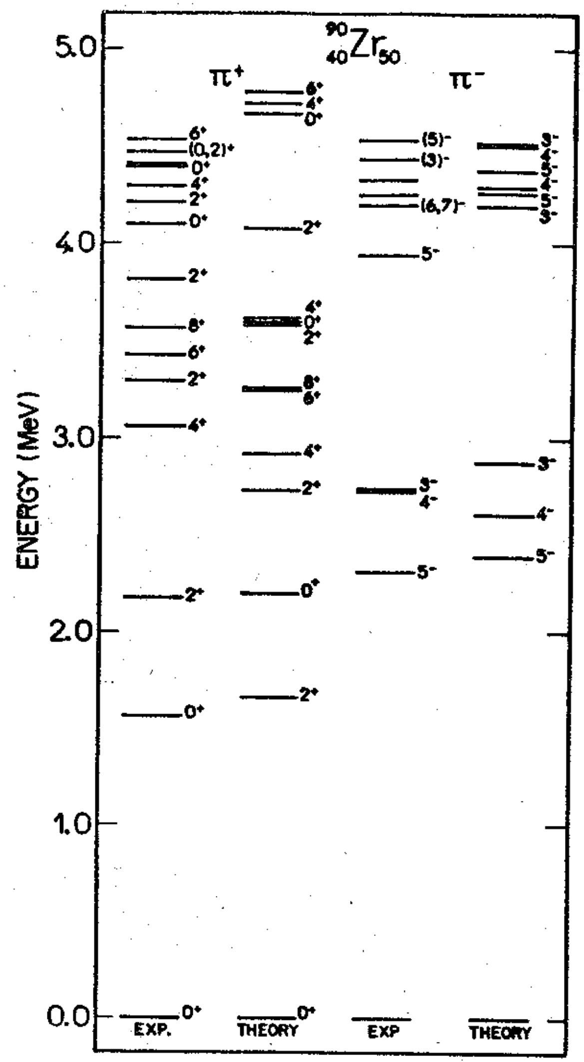
I_i^π	τ (ps) EXP	τ (ps) THEORY IB	τ (ps) THEORY IIB	δ(E2/M1) EXP	δ(E2/M1) THEORY IB	δ(E2/M1) THEORY IIB	B(E2↑) (e^2 fm^4) EXP	B(E2↑) THEORY I	B(E2↑) THEORY II
$1/2_1^+$	0.9 ± 0.2[b] 0.25 $^{+0.09}_{-0.07}$[c] 3.2 ± 5.0[d] 1.20 ± 0.55[f]	8.9	6.2*				120 ± 30[a] 412 ± $^{115}_{150}$[c] .34 ± 50[d] .90 ± 40[f]	108	153*
$5/2_1^+$	0.46 ± 0.15[b] 0.28 $^{+0.16}_{-0.11}$[c] 0,51 ± 0.16[d]	0.50	0.43						
$7/2_1^+$	0.24 ± 0.10[b] 0.11 ± 0.20[d]	0.70	0.06	−0.4 ± 0.1[d] or −1.25 ± 0.15[d] +1.0 $^{+2.7}_{-0.6}$[e]	−0.59	−0.83			
$3/2_1^+$	0.10 ± 0.05[b] <0.030[c] 0.016 ± 0.002[d]	0.007	0.007	$-10 < \delta < 0.1$[e]	+0.25	0.17			
$9/2_1^+$	0.35 ± 0.10[b] 0.17 ± 0.02[d]	0.18	0.11						

a - Ref. 17
b - Ref. 7
c - Ref. 18
d - Ref. 19
e - Ref. 20
f - Ref. 21
* using e_n = 1.5e



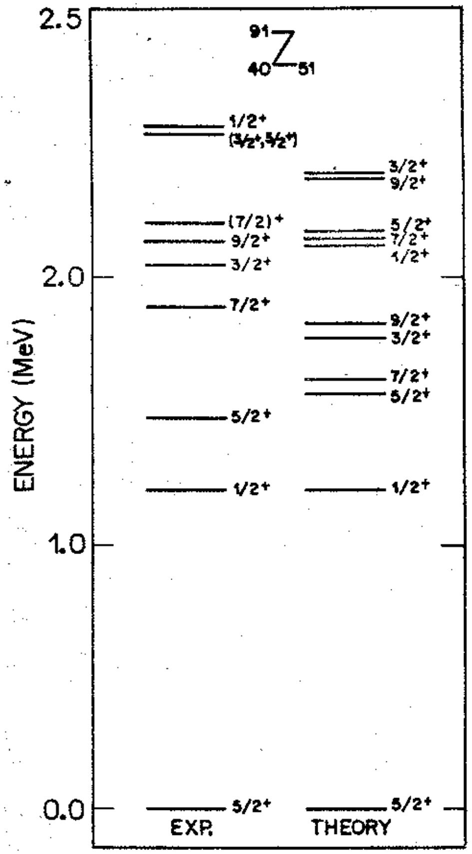
90 Y 39 51
1.5
1.0
0.5
0.0
ENERGY (MeV)
EXP
A_0=0.30
A_0=0.20
THEORY

FIG. 1



90 Zr 40 50
π+
π−
5.0
4.0
3.0
2.0
1.0
0.0
ENERGY (MeV)
EXP.
THEORY
EXP
THEORY

FIG. 2



2.5
2.0
1.0
0.0
ENERGY (MeV)
$^{91}_{40}Zr_{51}$
1/2+
(3/2+,5/2+)
(7/2)+
9/2+
3/2+
7/2+
5/2+
1/2+
5/2+
3/2+
9/2+
5/2+
7/2+
1/2+
9/2+
3/2+
7/2+
5/2+
1/2+
5/2+
EXP.
THEORY

FIG. 3