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Chiral solitons are usually obtained in the non–linear sigma model. Here we consider soliton solutions of an alternative chiral Lagrangian which is also compatible with PCAC. Theoretical predictions agree fairly well with experiment and include the value $f_\pi = 78$ MeV.

I. INTRODUCTION

Chiral solitons possessing a conserved topological quantum number were introduced by Skyrme[1] and revived by Adkins, Nappi and Witten[2]. Since then, they deserved a considerable attention in the literature, and both their static properties and interactions were widely studied. The usual approach to the problem is based on the non–linear σ–model, where the Lagrangian is invariant under axial transformations changing the pion field $\boldsymbol{\pi}$ into the function $\sigma = \sqrt{f_\pi^2 - \pi^2}$ and vice–versa. In addition to the kinetic energy, the Lagrangian must contain terms involving higher order derivatives so as to stabilize the soliton. Some versions also include a symmetry breaking term which endows the pion with a mass and is responsible for PCAC.

The predictions of the various versions of the usual model for the static baryon properties agree reasonably well with experiment, except for the values of f_π and g_A,

respectively the pion decay constant and the axial coupling constant. When masses are used as input, predictions for f_π and g_A lie typically around 2/3 of measured values. This disagreement does not change much when the pions are either massless[2] or massive[3], or when stabilization is achieved by means of the Skyrme term[2,3] or ω mesons[4], suggesting that it might be due to some intrinsic feature of the model. This could be associated with the fact that one is representing a system whose number of colours N is 3 by a $N \to \infty$ Lagrangian. However, before reaching conclusions about this subject, one would need to know to what extent results do depend on the use of the non–linear sigma model. The discussion of this point in the main purpose of this work.

Long ago Weinberg[5] has shown that the non–linear σ–model is just one among many other possible realizations of chiral symmetry. Thus, in order to test the dependence of results on the use of the σ–model, we construct a chiral soliton employing an alternative chiral Lagrangian, which is also compatible with PCAC.

2. COVARIANT DERIVATIVES, PCAC, HEDGEHOGS AND THE BARYONIC CURRENT

The most general axial transformation of the pion field $\boldsymbol{\pi}$ is generated by[5]

$$[X_a \, , \, \pi_b] \; = \; i \, (\delta_{ab} \, f + \pi_a \, \pi_b \, g) \tag{1}$$

where X_a is the axial charge, f is an arbitrary function of π^2 such that $f(0) = f_\pi$ and g is given by

$$g \; = \; \frac{1 + 2ff'}{f - 2\pi^2 f'} \; ; \tag{2}$$

primes indicate derivatives with respect to π^2 .

The pion covariant derivative is written as

$$D^{\mu} \boldsymbol{\pi} = f_{\pi} \left\{ \frac{\partial^{\mu} \pi}{\sqrt{f^2+\pi^2}} - \frac{2f'(f+\sqrt{f^2+\pi^2}) + 1}{2(f^2+\pi^2)(f+\sqrt{f^2+\pi^2})} \boldsymbol{\pi} \partial^{\mu} \pi^2 \right\} . \tag{3}$$

The chiral invariant part of the Lagrangian is obtained by constructing functions of $D^{\mu}\boldsymbol{\pi}$ which are simultaneously Lorentz invariant and isoscalar.

The symmetry breaking term of the Lagrangian endows the pion with a mass and is chosen so as to allow the divergence of the axial current A_{μ} to be written as

$$\partial^{\mu} A_{\mu} = i\left[X, \mathcal{L}_{SB}\right] \tag{4}$$

$$= f_{\pi} m_{\pi}^2 \boldsymbol{\pi} \tag{5}$$

Isospin invariance requires $\mathcal{L}_{SB}$ to be a function of π^2 only, and we have:

$$i\left[X, \mathcal{L}_{SB}\right] = 2i\, \mathcal{L}'_{SB}\, i\left[X, \pi_b\right] \pi_b$$

$$= -2\, \mathcal{L}'_{SB} (f + \pi^2 g)\, \boldsymbol{\pi} . \tag{6}$$

Comparison with (5) yields:

$$\mathcal{L}'_{SB} = -\frac{f_{\pi} m_{\pi}^2}{2} (f + \pi^2 g)^{-1} . \tag{7}$$

The form of the function f can be determined when one characterizes the symmetry breaking part of the Lagrangian in terms of its transformation properties. The assumption that $\mathcal{L}_{SB}$ transforms according to the (N/2 , N/2) representation of SU(2) × SU(2) allows us to write[5]

$$\sum_a \left[X_a, \left[X_a, \mathcal{L}_{SB}(N)\right]\right] = N(N+2)\, \mathcal{L}_{SB}(N) . \tag{8}$$

This relation, when combined with (4) and (5) produces

$$\mathcal{L}_{SB}(N) = \frac{f_{\pi}\, m_{\pi}^2}{N(N+2)} (3f + \pi^2 g) . \tag{9}$$

Deriving this $\mathcal{L}_{SB}(N)$ and comparing with (7), we obtain a differential equation for f (recall that $g = g(f)$)

$$\frac{1}{N(N+2)} (3f' + g + \pi^2 g') = -\frac{1}{2} (f + \pi^2 g)^{-1} . \tag{10}$$

In this work we consider two solutions f(N) of this equation, namely

$$f(1) = \sqrt{f_{\pi}^2 - \pi^2} \tag{11}$$

and

$$f(2) = \frac{1}{2} \left(f_{\pi} + \sqrt{f_{\pi}^2 - 4\pi^2} \right) , \tag{12}$$

which are associated with the following symmetry breaking terms:

$$\mathcal{L}_{SB}(1) = f_{\pi} m_{\pi}^2 \sqrt{f_{\pi}^2 - \pi^2} \tag{13}$$

and

$$\mathcal{L}_{SB}(2) = f_{\pi} m_{\pi}^2 \frac{1}{8} \left(f_{\pi} + 2\sqrt{f_{\pi}^2 - 4\pi^2} \right) . \tag{14}$$

The solution with N = 1 corresponds to the usual σ–model.

The soliton fields in both N = 1 and N = 2 representations are assumed to be of the hedgehog form

$$\vec{\pi}(1) = f_\pi \text{ sen } F\, \hat{r} \quad (15)$$

and

$$\vec{\pi}(2) = \frac{f_\pi}{2} \text{ sen } G\, \hat{r} \quad (16)$$

where F and G represent chiral angles in the σ model and in the alternative version. The scale of the fields are determined by the requirement of positivity of the square roots in eqs. (11) and (12).

These ansätze allow the spatial components of the covariant derivatives to be expressed as

$$D_k \pi_a(1) = f_\pi \left\{ \frac{\sin F}{r} \delta_{ka} + \left(F' - \frac{\sin F}{r}\right) \hat{r}_k \hat{r}_a \right\} \quad (17)$$

and

$$D_k \pi_a(2) = f_\pi \left\{ \frac{\sin G/2}{r} \delta_{ka} + \left(\frac{G'}{2} - \frac{\sin G/2}{r}\right) \hat{r}_k \hat{r}_a \right\} . \quad (18)$$

This means that $D_k \vec{\pi}(2)$ can be obtained from $D_k \vec{\pi}(1)$ by means of the replacement: $F \to G/2$.

In both cases the conserved baryonic current can be written as

$$B^\mu(N) = \frac{K(N)}{f_\pi^3} \varepsilon^{\mu\alpha\beta\gamma} D_\alpha \vec{\pi}(N) \cdot D_\beta \vec{\pi}(N) \times D_\gamma \vec{\pi}(N) . \quad (19)$$

This definition represents an improvement over the usual one, since it produces a B^μ which is chiral invariant. Its components are given by

$$B^0(i) = 6\, K(i) \frac{\sin^2 \Theta(i)\, \Theta'(i)}{r^2} \quad (20)$$

and

$$B^j(i) = -6K(i) \frac{\sin^2 \Theta(i)}{r} \Theta'(i)\, D_{aj}\, \dot{D}_{ak}\, \hat{r}_k . \quad (21)$$

where $\Theta(1) = F$, $\Theta(2) = G/2$ and the D_{aj} are the usual matrices containing the collective coordinates used in the quantization of the soliton; their expectation value in nucleon states is related to the usual Pauli matrices by $<N'|D_{aj}|N> = -1/3 <N'|\tau_a \sigma_j|N>$. The values of the constants K(N) are obtained by using the boundary condition $F(0) = G(0) = \pi$, $F(\infty) = G(\infty) = 0$ and imposing the lowest non–vanishing baryon number to be 1. Thus we get $K(1) = -1/(12\pi^2)$ and $K(2) = = -1/(6\pi^2)$.

3. THE MODEL

In this work we follow refs. (4) and (6), and adopt an stabilizing term proportional to $B_\mu B^\mu$. We do not work with the usual Skyrme term[(2,3)] because it does not allow a solution with baryon number $B = 1$ in the $N = 2$ representation. The Lagrangian density is written as:

$$\mathcal{L}(N) = \frac{1}{2} D_\mu \vec{\pi}(N) \cdot D^\mu \vec{\pi}(N) - b(N)\, B_\mu(N)\, B^\mu(N) + \mathcal{L}_{SB}(N) . \quad (22)$$

Using the hedgehog ansätze, eqs. (15–16), we obtain the following forms for the classical Lagrangians

$$L(1) = -f_\pi 2\pi \int dx\, x^2 \left[\frac{1}{2} \left[F'^2 + \frac{2\sin^2 F}{x^2} \right] + \beta(1) \frac{\sin^4 F}{x^4} F'^2 - \frac{m_\pi^2}{4 f_\pi^2} (\cos F - 1) \right] \quad (23)$$

$$L(2) = -f_\pi 2\pi \int dx\, x^2 \left[\frac{1}{2} \left[\frac{1}{4} G'^2 + \frac{2\sin^2 G/2}{x^2} \right] + \beta(2) \frac{\sin^4 G/2}{x^4} \frac{G'^2}{4} \right.$$

$$\left. - \frac{m_\pi^2}{16 f_\pi^2} (\cos G - 1) \right] \tag{24}$$

where $x = 2f_\pi r$, $\beta(1) = \frac{4f_\pi^2}{\pi^4} b(1)$, $\beta(2) = \frac{16 f_\pi^2}{\pi^4} b(2)$. Note that for the symmetry breaking contribution does not hold the rule $F \leftrightarrow G/2$.

These results yield the following differential equations for F and G

$$\left[\frac{x^2}{4} + \frac{\beta(1)}{2} \frac{\sin^4 F}{x^2} \right] F'' + \left[\frac{1}{2} - \beta(1) \frac{\sin^4 F}{x^4} \right] xF' - \frac{\sin 2F}{4} + \frac{\beta(1)}{2} \frac{\sin^2 F}{x^2} \sin 2F\, F'^2$$

$$- \frac{m_\pi^2}{16 f_\pi^2} x^2 \sin F = 0 \tag{25}$$

$$\left[\frac{x^2}{4} + \frac{\beta(2)}{2} \frac{\sin^4 G/2}{x^2} \right] \frac{G''}{2} + \left[\frac{1}{2} - \beta(2) \frac{\sin^4 G/2}{x^4} \right] x \frac{G'}{2} - \frac{\sin G}{4}$$

$$+ \frac{\beta(2)}{2} \frac{\sin^2 G/2}{x^2} \sin G \frac{G'^2}{4} - \frac{m_\pi^2}{16 f_\pi^2} x^2 \sin G = 0 . \tag{26}$$

The usual quantization procedure[2] allows the baryon masses to the written as

$$M(i) = -L(i) + J^2/2\lambda(i) \tag{27}$$

where J is the angular momentum operator and the $\lambda(i)$ are given by

$$\lambda(i) = \frac{1}{2f_\pi} \frac{4\pi}{3} \int dx\, x^2 \left[\frac{1}{2} \operatorname{sen}^2 \Theta(i) + \beta(i) \frac{\sin^4 \Theta(i)}{x^2} (\Theta'(i))^2 \right] . \tag{28}$$

In order to compute the static properties of nucleons we evaluate the vector and axial currents, which are given by:

$$V^{a,0}(i) = -\frac{1}{3} \sin^2 \Theta(i) \left\{ f_\pi^2 + 2b(i)\,(6K(i))^2 \frac{\sin^2\Theta(i)}{r^2} (\Theta'(i))^2 \right\} \varepsilon_{amn} D_{me} \dot{D}_{ne} \tag{29}$$

$$V^{a,j}(i) = \frac{\sin^2\Theta(i)}{r} \left\{ f_\pi^2 + 2b(i)\,(6K(i))^2 \frac{\sin^2\Theta(i)}{r^2} (\Theta'(i))^2 \right\} \varepsilon_{jmn} \hat{r}_m D_{ai} \tag{30}$$

$$A^{a,j}(i) = -\left\{ \frac{f_\pi^2}{3} \left(2 \frac{\sin\Theta(i) \cos\Theta(i)}{r} + \Theta'(i) \right) + \frac{2}{3} b(i)\,(6K(i))^2 \left(\frac{\sin\Theta(i)}{r} \right. \right.$$

$$\left. \left. + 2\cos\Theta(i)\, \Theta'(i) \right) \frac{\sin^3\Theta(i)}{r^3} \Theta'(i) \right\} D_{aj} \tag{31}$$

In writing equations (29–31) we have neglected the terms involving two time derivatives because they are much smaller than those displayed.

4. RESULTS AND CONCLUSIONS

Following refs. (2–4), we have adjusted the parameters f_π and b so as to reproduce the observed nucleon and delta masses. This procedure yields the soliton shape functions shown in fig. 1 and the static baryon properties displayed in table I. In the latter, for the sake of comparison, we also include the values of Adkins and Nappi[4], who stabilized the soliton via the ω meson.

Our results for the case $N = 1$ can be understood as being just a reproduction of those of ref. (4). The model with $N = 2$, on the other hand, possesses two new and remarkable features. Firstly, it predicts a better value of f_π and secondly, as it can be seen from fig. 1, it gives a shape function which is rather smooth at the origin. Indeed the behavior of the function G suggests the existence of a "plateau" for $0 \leq r \leq 0.3$ fm where G is approximately constant and takes values around π, implying the vanishing of the pion field inside this region. This might be in agreement with the picture of the hybrid model[7], in which there are no mesonic fields inside a small volume V where perturbative QCD should be valid.

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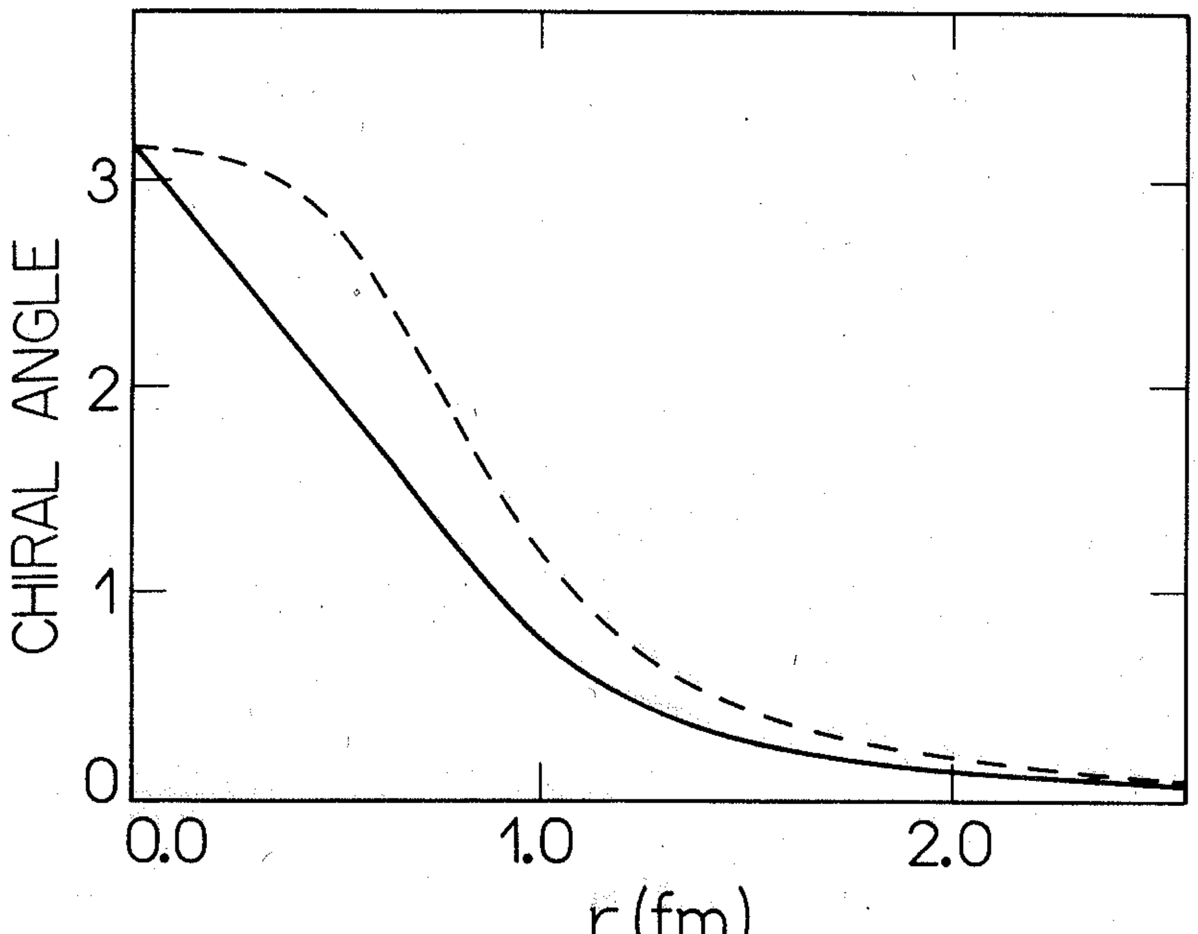
FIGURE CAPTIONS

FIGURE 1: The chiral angle as a function of the radial distance. The continuous curve represents F (N = 1) and the dashed curve represents G (N = 2).

TABLE I

Quantity	Adkins and Nappi	This work N = 1	This work N = 2	Experiment
m_N (MeV)	input	input	input	938.9
m_π (MeV)	input	input	input	138
f_π (MeV)	62	65.1	77.8	93
β	–	0.019	0.103	–
$\left\langle r^2 \right\rangle^{1/2}_{I=0}$ (fm)	0.74	0.72	0.71	0.72
$\left\langle r^2 \right\rangle^{1/2}_{I=1}$ (fm)	1.06	1.03	0.97	0.88
$\left\langle r^2 \right\rangle^{1/2}_{M,I=0}$ (fm)	0.92	0.91	0.89	0.82
$\left\langle r^2 \right\rangle^{1/2}_{M,I=1}$ (fm)	1.02	1.03	0.97	0.80
μ_p	2.34	2.00	1.99	2.79
μ_n	− 1.46	− 1.20	− 1.20	− 1.91
g_A	0.82	0.83	0.83	1.23
$g_{\pi NN}$	13.0	12.6	11.1	13.5
$g_{\pi N\Delta}$	19.5	18.9	16.6	20.4
$\mu_{N\Delta}$	2.7	2.26	2.26	3.3



CHIRAL ANGLE
3
2
1
0
0.0
1.0
2.0
r (fm)