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A VIABLE ROUTE TO THE PRODUCTION OF SUPERHEAVY ELEMENTS: LOW ENERGY FUSION OF VERY NEUTRON-RICH NUCLEI

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ABSTRACT

It is suggested that the inclusion of the virtual excitation of the soft giant dipole (pymgy) resonance in the calculation of the cross–section for very neutron–rich radioactive beam–induced fusion reactions may enhance the formation probability of the heavy compound nucleus produced at low excitation energy. Both spherical and deformed target nuclei are considered.

[THIS WORK IS DEDICATED TO THE PEOPLE OF IRAQ]

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1. INTRODUCTION

The quest for superheavy elements has been going on for some time by now and the overall conclusion so far reached is that no elements beyond charge number 109 were discovered and elements with $Z < 109$ became accessible through reactions involving Pb and Bi targets[1)].

Recently, it has been suggested[2)] that the beams of neutron–rich radioactive nuclei offer a rather unique possibility for synthesizingm both the superheavy nuclei lying around the magic neutron and proton numbers $N = 184$ and $Z = 114$ and the heavy isotopes with $N \geq 160$ of new elements. Owing to the larger N/Z ratio of these exotic nuclei the effective Coulomb barrier is basically lowered, permitting the appreciable formation of not so excited compound nuclei at low energies. These cold compound nuclei have lower fission probability, thus increasing the possibility of observing them.

The theoretical calculation of the survival probability of heavy elements using radioactive neutron–rich beams has been done using the macroscopic model of extra–extra push of Swiatecki[3)]. Substantial lowering of the effective fissility and, consequently a lower effective fusion barrier is obtained. The degree of lowering of these physical parameters has, however, been recently questioned[4)].

In the present paper, we address ourselves to another, dynamical effect involving neutron–rich nuclei. It has been theoretically established that nuclei in the neutron drip region exhibit appreciable collective behaviour at quite low excitation energies. In particular, the soft giant dipole resonance, in nuclei such as ^{11}Li, is predicted to be situated in the 1–2 MeV energy region, exhausting about 12% of the classical dipole sum rule and accounting for about 90%, of the observed fragmentation cross–section[5,6,7)]. We shall demonstrate here that the coupling to the "pygmy resonance" could enhance the fusion probability of neutron–rich nuclei by as much as a factor 50 or more.

As figure 1 shows, when a neutron–rich projectile approaches a heavy deformed target nucleus, the interaction sets in a dipole oscillation of the excess neutrons with respect to the core in the projectile, allowing a closer nuclear contact with the target and thus increasing the fusion probability. This dynamic effect should be considered in conjunction with the static one related to the larger N/Z ratio and accordingly the lower static Coulomb barrier. We base our discussion on known facts about ^{11}Li and make reasonable extrapolations to the Fe isotopes induced fusion considered in reference 2).

The paper is organized as follows. In Section 2 we discuss the salient features of the soft giant dipole resonance (pygmy resonance) in neutron–rich nuclei. In Section 3 we develop the theory of the fusion cross–section in the presence of the pygmy resonance for a reaction involving a spherical target nucleus, and apply the theory to $^{A}Fe+^{208}Pb$. In Section 4 we extend our discussion to deformed target nuclei. In section 5 we present our numerical results and finally, in section 6 the concluding remarks are given.

2. THE PYGMY RESONANCE

In nuclei such as ^{11}Li, it has been suggested that the two neutrons in the $p_{1/2}$ level, form a "halo", and as such is very distanced from the ^{9}Li core. When discussing the collective dipole excitation of such a loosely bound system, one is bound to consider two types of vibrations: the usual ($E^* \approx 20$ MeV) isovector proton vs. neutron vibration in the core, with the halo neutrons taken as mere spectators, and the oscillation of the whole core nucleus against the halo neutrons (the pygmy resonance). In this latter case the rather extended distribution of the halo results in a weak restoring force, and consequently a low excitation energy of the pygmy resonance (also known as the soft giant dipole resonance).

Recent microscopic calculation[5–7] of the structure of neutron–rich nuclei clearly confirmed the above qualitative picture. For the present paper, however we

shall use macroscopic, Steinwedel–Jensen, modeling guided with appropriate sum rule arguments to discuss the pygmy resonance in the Fe isotopes.

In a recent letter, Suzuki, Ikeda and Sato[8] predicted the following excitation energy of the pygmy resonance, using the S–J model

$$E^*_{pR} = \left[\frac{Z(N-N_c)}{N(Z+N_c)}\right]^{1/2} E^*_{GDR} \tag{1}$$

where $\hbar\omega_{GDR}$ is the excitation energy of the usual giant dipole resonance $\left[\approx \frac{80}{A^{1/3}}\,\mathrm{MeV}\right]$ and N_c refers to the neutron number of the core. N and Z are the neutron and proton numbers of the whole nucleus. Thus for the ^{A}Fe isotopes with $A = 56, \ldots, 70$, we have, e.g., $E^*_{pR}(70) = 0.38\, E^*_{GDR}$. This shows that in ^{70}Fe, the pygmy resonance occurs at ≈ 5 MeV. This value could very well be lower if the separation energy of the excess neutrons is small as e.g. the case in ^{11}Li. The pygmy resonance in this latter nucleus is found to occur at ≈ 2 MeV [6,7].

A pure cluster model supplies slightly different results from those of reference 8). Within this model, the dipole strength is distributed according to[9]

$$\frac{dB(E1)}{dE} = \frac{3\hbar^2 e^2}{\pi^2}\,\frac{Z^2 \Delta N}{AA_c}\,\frac{\sqrt{\varepsilon}\,(E^*-\varepsilon)^{3/2}}{E^{*4}} \tag{2}$$

when ΔN refers to the excess neutrons treated as a cluster and ε is the binding energy (separation energy) of this excess neutron clusters. The position of the maximum of $\frac{dB(E1)}{dE^*}$ is just the energy of the pygmy resonance and is easily calculated to be

$$E^*_{pR} = \frac{8}{5}\varepsilon\ , \tag{3}$$

thus the smaller ε is the lower E_{pR} will be. In a nucleus such as ^{70}Fe, ε could very well be in the few keV region. Of course an ambiguity remains as to what should be the core. However, equations (1) and (2) should serve our purposes of supplying estimates.

From the above discussion one may safely assume that the pygmy resonance may occur in the 0.2 – 2 MeV excitation energy region in the Fe isotopes.

In discussing the energy weighted sum rule for neutron–rich nuclei one may consider the usual classical sum rule which reads

$$S(E1) = 14.8 \frac{NZ}{A} [\mathrm{MeV\ fm^2 e^2}] \tag{4}$$

and the dipole cluster sum rule for the core plus excess neutrons system[10]

$$S_c(E1) = S(E1) - S_{cluster}(E1) - S_{excess}(E1)$$

$$= 14.8 \left[\frac{NZ}{A} - \frac{N_c Z_c}{A_c}\right] \left[\mathrm{MeV\ fm^2\ e^2}\right] . \tag{5}$$

It is usually found that the pygmy resonance exhausts about 10% of the classical rule and 80% of the cluster sum rule.

3. THE FUSION OF $^{A}Fe + ^{208}Pb$

Aside from the static, barrier penetration effects considered in reference (1), there are dynamic effects arising from the virtual excitation of giant resonances. Here we consider the effects of the pymgy resonances. It has been shown in the last few years that

the fusion cross section at low energies is appreciably increased over the static value, when channel coupling effects are taken into consideration[11]. The enhancement is largest when the Q–value of the non–elastic channel is lowest. We show below how the excitation of the pygmy resonance may help increase the fusion cross–seftion of the system $^{A}Fe + ^{208}Pb$. Although the theoretical description of coupied channels effect in the fusion of heavy ions is well developed we opt here for a simple two channel model that can be solved exactly [12,13]. Calling H_0+V_0 the entrance channel Hamiltonian, $H_{pR}+V_{pR}$ the pygmy resonance channel Hamiltonian, V_c the coupling Hamiltonian and Q_{pR} the Q–value of the pygmy resonance channel, the two–channel Schrödinger equation then reads,

$$\begin{bmatrix} H_0+V_0 & V_c \\ V_c & H_{pR}+V_{pR}+Q_{pR} \end{bmatrix} \psi = E\psi \tag{6}$$

$$Q_{pR} < 0$$

From the previous discussion we know that $|Q_{pR}|$ is small and we neglect it in what follows (the c.m. energy is much larger than Q_{pR}). Further $H_{pR}+V_{pR}$ describes the relative motion of the excited ^{A}Fe nucleus with respect to ^{208}Pb. We are safe in taking this Hamiltonian to be equal to H_0+V_0. We thus have

$$(H_0 + V_0 + V_c\sigma_x)\,\psi = E\psi \tag{7}$$

where σ_x is a Pauli spin matrix which is introduced here for notational convenience. Fusion with no coupling is accounted for by the complex bare optical potential V_0. The corresponding cross–section is

$$\overset{\circ}{\sigma}_F = \frac{k}{E} < \psi_0^{(+)} | - \text{Im } V_0 | \psi_0^{(+)} > = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell+1) \overset{\circ}{T}_\ell \; . \tag{8}$$

Taking into account the coupling interaction to all orders amounts to replacing $\overset{\circ}{\sigma}_F$ above by

$$\sigma_F = \frac{k}{E} < \psi^{(+)} | - \text{Im } V_0 | \psi^{(+)} > - \sigma_{pR} \tag{9}$$

where $\psi^{(+)}$ is the spinor $\begin{bmatrix} \psi_0^{(+)} \\ \psi_{pR} \end{bmatrix}$ and σ_{pR} is the angle integrated inelastic cross–section for the direct transition $0 \to pR$. The fusion cross–section σ_F can be written in closed form after performing a convenient transformation that diagonalizes σ_x. The result of σ_F is

$$\sigma_F = \frac{1}{2} \left[\sigma_R(V_c) + \sigma_R(-V_c) \right] \tag{10}$$

where $\sigma_R(V_c)$ is the total reaction cross section obtained from the Hamiltonian $H_0+V_0+V_c$ and $\sigma_R(-V_c)$ from $H_0+V_0-V_c$. We should stress, that in all our discussion above we have disregarded the angular momentum (1) of the pygmy resonance, which is quite valid considering the great values of the orbital angular momentum involved. Equation (10) has been previously derived in a slightly different manner, by Dasso et. al. and Lindsay and Rowley[13].

In calculating the enhancement of σ_F we use the Wong formula[14], which reads

$$\overset{\circ}{\sigma}_F = \sigma_F(V_c{=}0) = \frac{\hbar\omega R_B^2}{2E} \ln \left[1 + \exp\left[\frac{E-V_B}{\frac{\hbar\omega}{2\pi}} \right] \right] \tag{11}$$

were $\hbar\omega$ measures the curvature of the Coulomb barrier and V_B is its height. Here the Coulomb barrier is obtained from $V_0(r) + \frac{\hbar^2 \ell(\ell+1)}{2\mu^2}$, and the Coulomb interaction is

contained in V_0. We define the enhancement factor $\mathbf{E}(V_c)$ as

$$\mathbf{E}(V_c) = \frac{\sigma_F(V_c)}{\sigma_F(V_c{=}0)} =$$

$$= \frac{\ln\left\{1 + \exp\left[\left[\frac{E-V_B-V_c}{\frac{\hbar\omega}{2\pi}}\right]\right]\right\} + \ln\left\{1 + \exp\left[\left[\frac{E-V_B+V_c}{\frac{\hbar\omega}{2\pi}}\right]\right]\right\}}{2\ln\left\{1 + \exp\left[\left[\frac{E-V_B}{\frac{\hbar\omega}{2\pi}}\right]\right]\right\}} \tag{12}$$

At the barrier, $E = V_B$, one has

$$\mathbf{E}(V_c) = \frac{\ln\left\{1 + \exp\left[-\frac{V_c}{\frac{\hbar\omega}{2\pi}}\right]\right\} + \ln\left\{1 + \exp\left[\frac{V_c}{\frac{\hbar\omega}{2\pi}}\right]\right\}}{2 \ln 2} \tag{13}$$

We now write fully the structure of V_c, guided with the results for stable nuclei supplied by the collective model

$$V_c = C_1 \left(B_{pR}(E1)\right)^{1/2} F(r) + V_c^{\text{Coulomb}} \tag{14}$$

where C_1 is a strength which may be calculatd within the Tassie model, $F(r)$ is the radial form factor given by $\int \frac{d}{dr'} \rho_{Fe}(r')\rho_{Pb}(r'-r)dr'$, and V_c^{Coulomb} is the Coulomb piece of V_c which is also proportional to $(B_{pR}(E1))^{1/2}$. Thus

$$V_c = \mathbb{F}(r)(B_{pR}(E1))^{1/2} \; . \tag{15}$$

In Eqs. (14) and (15) $B_{pR}(E1)$ is the B(E1) value of the pygmy resonance, which in a cluster model (core + excess neutrons) can be written as[9] (by integrating Eq. (2) over E^*)

$$B_{pR}(E1) = \frac{3\hbar^2 e^2}{16\pi}\left(\frac{Z^2 \Delta N}{AA_c}\right)\frac{1}{\varepsilon} \tag{16}$$

where ε is the binding energy of the excess neutron cluster with respect to the core. It is obvious that ε is the determining facvtor in the degree of enhancement of σ_{fusion}. Thus, we obtain the final explicit form of the enhancement factor E (at $E_{c.m.} = V_B$) showing its dependence on the relevant physical parameters that characterize the exotic neutron–rich nucleus ^{A}Fe, with $A = A_c + \Delta n$, and using Eq. (3)

$$\mathbf{E} = \frac{1}{2\ln 2}\ln\left\{2\left[1+\cosh\left[\frac{\mathbb{F}(R_B)}{\frac{\hbar\omega}{2\pi}}(B_{pR}(E1))^{1/2}\right]\right]\right\} \tag{17}$$

where F(r) of Eq. (15) is evaluated at the barrier $r = r_B$. Since $\frac{Z^2}{A_c}$, in Eq. (16), is fixed once the core is decided upon, the quantity that varies, as more neutrons are added is $\frac{\Delta N}{A\varepsilon}$. The argument of the cosh could become very large for very neutron–rich isotopes such as ^{70}Fe, where $B_{pR}(E1)$ is expected to be large, rendering E to obtain great values.

In fact, if as a reference, we take for the factor $X \equiv \frac{\mathbb{F}(R_B)(B_{pR}(E1))^{1/2}}{\hbar\omega}$ the value 0.1 for ^{56}Fe then $E(^{56}Fe) \simeq 1$. Using Eqs. (16) and (3) to obtain a rough estimate of B(E1) for ^{70}Fe and assuming $E^*_{pR}(^{70}Fe) \sim 1$ MeV and $E^*_{pR}(^{56}Fe) \sim {}^*E_{GDR}(^{56}Fe) \sim$ 20 MeV, we have $X(^{70}Fe) \simeq 10$ and thus we get

$$E(^{70}Fe) \simeq \frac{10}{2\ln 2} = 7.2$$

At $E \ll V_B$, the fusion cross–section, Eq. (10), can be approximated by taking only the term with the lowest effective barrier (assuming $V_c > 0$).

$$\sigma_F \sim \frac{\hbar\omega R_B^2}{4E}\ln\left[1+\exp\left[2\pi\left[\frac{E-V_B+V_c}{\hbar\omega}\right]\right]\right],$$

$$\approx \frac{\hbar\omega R_B^2}{4E}\exp\left[\frac{2\pi}{\hbar\omega}(E-V_B+V_c)\right]. \tag{18}$$

Thus compared to the no–coupling fusion, the effect of the pygmy resonance at low center of mass energies can be represented by an effective increase in the center of mass energy $E \to E+V_c$.

With Eq. (18), the enhancement factor E attains the very simple energy independent form

$$\mathbf{E} = \frac{1}{2}\exp\left(2\pi\frac{V_c}{\hbar\omega}\right), \tag{19}$$

which, with the estimate given earlier, namely $X = 2\pi\frac{V_c}{\hbar\omega} = 10$ for ^{70}Fe, we obtain $E(^{70}Fe) \sim 10^4$! At higher energies this factor is of course reduced.

4. EFFECTS OF DEFORMATION

When considering the fusion involving deformed nuclei, one has to take into account the coupling to the rotational spectrum. A rather simple way of doing this is through the equivalent sphere method based on the adiabatic approximation. If $\sigma_F(\theta)$ is the fusion cross–section for a fixed orientation angle then the observed fusion cross section reads

$$\sigma_F = \int_0^{\pi/2} d\theta \sin\theta\, \sigma_F(\theta) \tag{20}$$

Eq. (20) is easily calculable. However, we aim here to give analytical estimate of the enhancement factor and thus resort to the treatment of Lindsay and Rowley[12] and, more recently, Nagarajan et al.[16]. According to Ref. (16), Eq. (20) can be written in an equivalent form (if a finite number, N , of rotor states are included)

$$\sigma_F = \sum_{\alpha=1}^{N} \omega_\alpha\, \sigma_F(\alpha) \quad , \quad \sum_{\alpha=1}^{N} \omega_\alpha = 1 \tag{21}$$

where $\sigma_F(\alpha)$ is the fusion cross–setion in eigenchannel α, where the real potential is given by $V+\lambda_\alpha\beta_2 f(R)$ with f(B) being the coupling form factor evaluated at R_B and is given by $\frac{1}{\sqrt{4\pi}} V_B \frac{R_2}{R_B}\left[1-\frac{3}{5}\frac{R_2}{R_B}\right]$, and β_2 the quadrupole deformation parameter. The radius of the deformed nucleus is given by $R_2(\theta) = R_2\left[1 + \sqrt{\frac{5}{16\pi}}\,\beta_2\, R_2\, P_2(\cos\theta)\right]$. In fact it was shown in 16) that λ_α can be obtained from an eigenvalue equation

$$\hat{P}_2(\cos\theta)\, f_\alpha(\theta) = \lambda_\alpha f_\alpha(\theta) \quad . \tag{22}$$

If two states (0^+ and 2^+) of the rotor are taken into account then Eq. (21) becomes[13,15]

$$\sigma_F = 0.652\, \sigma_R(0.73\, \beta_2\, f(R_B)) + 0.348\, \sigma_R(-1.37\, \beta_2\, f(R_B)) \tag{23}$$

where $\sigma_R(X)$ is the one–channel reaction cross section with the barrier height shifted by X. Note that $f(R_B)$, which is the sum of nuclear and Coulomb form factors, is positive.

If the projectile nucleus that fuses with the deformed target, is neutron rich and has

appreciable dipole strength at low excitation energy, then the previous discussion implies for the fusion cross section the following rather simple form (see Eq. (10))

$$\sigma_F = \frac{1}{2}\Big\{0.652\,\big(\sigma_R\,(V_c + 0.73\,\beta_2\, f(R)\big) + \sigma_R\big(-V_c + 0.73\,\beta_2\, f(R)\big)$$
$$+ 0.348\,\big(\sigma_R\,(V_c - 1.37\,\beta_2\, f(R)\big) + \sigma_R\big(-V_c - 1.37\,\beta_2\, f(R)\big)\Big\} \quad . \tag{24}$$

At very low energies the dominant term becomes, the Wong formula

$$\sigma_F \underset{E\ll V_B}{\approx} \frac{\hbar\omega R_B^2}{4E}\, 0.348 \ln\left[1 + \exp 2\pi\left[\frac{E-V_B+V_c+1.37\,\beta_2\, f(R)}{\hbar\omega}\right]\right]$$
$$\approx \frac{\hbar\omega R_B^2}{4E}\, 0.348 \exp\left[\frac{2\pi}{\hbar\omega}\right]\left[E - V_B + V_c + 1.37\,\beta_2\, f(R_B)\right] \quad , \tag{25}$$

which gives for the enhancement factor,

$$E = \frac{0.348}{2} \exp\left[\frac{2\pi}{\hbar\omega}\big(V_c + 1.37\,\beta_2\, f(R_B)\big)\right] \quad . \tag{26}$$

Thus, again, at very low energies the effects of both the pygmy resonance of the projectile and the deformation of the target (taking only the 0^+ and 2^+ states of the rotor) can be represented by an effective increase in the center of mass energy $E \longrightarrow E + V_c(R_B) + 1.37\,\beta_2 f(R_B)$. For a strongly deformed target nucleus like ^{238}U and a radioactive projectile nucleus that exhibit a very soft giant dipole resonance with $E_{pR} < 1$ MeV, the above effective increase in the center of mass energy can be quite large. The resulting enhancement of σ_F can be several orders of magnitude.

Before turning to the next section, we comment on the difference between the constant enhancement factor at very low energies obtained from Eq.(25) and that which results from an approximate evaluation of the integral in Eq.(20) which would represent the inclusion of all states of the rotor, taken of course as degenerate. Using the Wong formula for $\sigma_F(\theta)$ with θ–dependent Coulomb barrier, it is easy to show that after performing the θ–integration, in Eq.(20), the enhancement factor within the equivalent sphere approximation and at very low energies is

$$\mathrm{E} \simeq \exp\left[\frac{2\xi^2}{3}\right] \Big/ 2\xi^2$$

where

$$\xi^2 = \frac{3}{2}\sqrt{5\pi}\,\frac{R_2}{R_B}\,\frac{V_B}{\hbar\omega}\,\beta_2\left[1 - \frac{3\mu\,\omega^2\,R_B^2}{V_B[2 + \mu\,\omega^2\,R_B^3/Z_1\,Z_2\,e^2]}\right]^2 . \qquad (27)$$

Eq.(27) should be compared to the more realistic expression, Eq.(26), when V_c is dropped (the factor that multiplies the exponential becomes 0.348). As anticipated, the equivalent sphere enhancement factor, Eq.(27) is about one order of magnitude larger than that of Eq.(26). In what follows we shall use, for the evaluation of E, Equations (12) for the spherical case and the expression that results from Eq.(24) (by dividing over ϑ_F) for the deformed case.

5. NUMERICAL RESULTS

In this section we present the numerical results for E for two systems. The first is Fe + ^{208}Pb as a representation of a spherical system. The second is the fusion of Fe

isotopes with the strongly deformed nucleus ^{238}U. Since the interaction potential is not known owing to the lack of data, we use as a reference the proximity potential which is given by

$$V_{prox}(r) = V_0[1 + \exp(-\tau/0.75)]^{-1}$$

$$\tau = S/b \;, \quad S = r - C_1 - C_2$$

$$C_i = R_i[1 - (b/R_i) + \ldots] \;, \quad R_i = r_0\,A_i^{1/3}$$

$$N_0 = -3.437(4\pi\,\gamma\,\bar{C}) \qquad (28)$$

$$\bar{C} = C_1 C_2/(C_1 + C_2)$$

$$\gamma = 0.9517[1 - 1.7826\,I^2]$$

$$I = (N_1 - N_2 - Z_1 - Z_2)/A_1 + A_2 \;.$$

In our calculation of the barrier parameters R_B, V_B and $\hbar\omega$, we use $b \simeq 1$ fm and $r_0 = 1.15$ fm. We found for the system ^{70}Fe + ^{208}Pb, $\frac{\hbar\omega}{2\pi} \sim 0.74$ Mev and $V_B = 213$, $R_B = 10.2$ fm. For the other Fe isotopes, these parameters vary slightly. For ^{70}Fe + ^{238}U, we obtain $\frac{\hbar\omega}{2\pi} \sim 0.78$ Mev, $V_B = 232$ and $R_B = 10.6$ fm. Also small variation in the values of these parameters was found for the other Fe isotopes.

When calculation the fusion cross section, Eq.(10), for ^{A}Fe + ^{208}Pb, we have taken several values for the pygmy resonance coupling strength V_c. Figure 2 shows σ_F vis $E_{c.m.}$ for $V_c = 0$, $V_c = 2.0$ MeV and a rather exagerated case $V_c = 10.0$ MeV. The great enhancement at sub-barrier energies is clearly seen. This means that at c.m. energies of about 15 MeV below the barrier, σ_F, for a reasonable value of V_c is still in the measurable range of a few millibarns. The fusion cross-section for $V_c = 0$ at the same energy is about 10^{-4} millibarns.

The over all features of the enhancement factor E, Eq.(12), is shown in figure 3 for the same spherical system. The saturation to the value given by Eq.(19) at very low energies is clearly seen. It is seen that a slight increase in V_c results in a great increase in E at low energies. An interesting feature of the results shown in figure 2 is that E vis E resembles very much a Fermi function. In fact an excellent representation for E for energies lower than the barrier V_B is

$$E(E) = 1 + \frac{E_0}{1 + \exp\left[\frac{E - E_0}{\Delta}\right]} \tag{29}$$

where E_0 is the saturation value of E at very low energies. The parameters E_0, E_0 and Δ, of course depend on V_c.

We now turn to the deformed target case namely $Fe + {}^{238}U$. The deformation parameter β_2 of ${}^{238}U$ is about 0.27, and thus $\beta_2\,f(R)$, given the type of interaction used, Eq.(26) is about 7.2 MeV. Again since very little information concerning the density distribution of, e.g., ${}^{70}Fe$ is known, we allow a variation of the coupling $\beta_2\,f(R)$. In figure 4 we show E for $V_c = 4$ MeV, and taking for $\beta_2\,f(R)$ the values 0.0, 3.0 MeV and 7.2 MeV. It is clear that the effect of deformation is to enhance much further the value of E.

We should mention here that the deformation results in a greater value of E than an equivalent soft vibration coupling. To exhibit this fact we show in figure 5 the results for E when taking $V_c = 6.0$ MeV, $\beta_2\,f(R) = 0.0$ MeV compared to $V_c = 0.0$ MeV, $\beta_2\,f(R) = 6.0$ MeV. At low energies the pure deformation enhancement is about a factor

16 larger than the pure soft vibration case. This is easily understood from Eqs.(18) and (25):

$$\frac{E(\text{deformation})}{E(\text{vibration})} \simeq \frac{0.348\ \exp\left[\frac{2\pi}{\hbar\omega}\ 1.37\ \beta_2\ f(R_B)\right]}{0.5\ \exp\left[\frac{2\pi}{\hbar\omega}\ V_c\right]} . \tag{30}$$

Then if $\beta_2\,f(R) = V_c$, we have

$$\frac{E(\text{deformation})}{E(\text{vibration})} = 0.696\ \exp\left[\frac{2\pi}{\hbar\omega}\,0.37\ \beta_2\ f(R_B)\right] , \tag{31}$$

which gives 16.6, for $\beta_2\,f(R) = V_c = 6$ MeV.

6. CONCLUSIONS

Before presenting our concluding remarks, we warn the reader that our formula for E, equation (17) was derived in the sudden limite ($Q_{pR} = 0$) and using the cluster model for the pymgy resonance. The validity of the sudden approximation becomes suspect for small ΔN and one has to consider equation (17) as a great overestimation. In fact, in such cases, namely large values of E^*_{pR}, a more valid approximation is to simulate the excitation of the PR through an attractive local energy-independent polarization potential[18]. As far as the cluster model of the PR is concerned it has recently been demonstrated that this model overestimates the Coulomb fragmentation cross-section of ${}^{11}Li + {}^{208}Pb$ at $E_{Lab} = 800$ MeV$\cdot$A[9,19] and greatly overestimates the cross-section at lower energies[20]. However the analytic simplicity of the cluster model justifies its use here for the obtention of the simple estimate for E.

In conclusion, we have considered in this paper the influence of the excitation of the soft giant dipole resonance on the fusion of neutron–rich nuclei with heavy targets. The enhancement over the static fusion calculation, exemplified by Eq. (17), shows clearly that the determining factor is the smallness of the excitation energy E^*_{pR} or, more precisely the large value of B(E1) as the number $\frac{N}{Z}$ is increased. Any static fusion calculation of the type discussed in reference 1 must be ammended by the multiplication with **E** of Eq. (17) or, even better, by a detailed coupled channel calculations. We also investigate the enhancement resulting from the fusion of a neutron–rich projectile with a strongly deformed target. We found that the deformation supplies even greater enhancement at low energies.

Of course several questions have to be answered before a definite conclusion concerning the value of **E** can be reached. The most important of these questions is the precise value of $B_{pR}(E1)$ and $\mathbb{F}(r)$, which can only be settled through detailed measurement and analysis of the elastic scattering and break–up of these exotic neutron–rich nuclei.

FIGURE CAPTIONS

Figure 1. The figure shows that when a neutron–rich projectile approaches a heavy deformed nucleus, the interaction sets in a dipole oscillation of the excess neutron with respect the core, allowing a closer nuclear contact with the target.

Figure 2. The fusion cross–section for $^{70}Fe + ^{208}Pb$ for several values of the pygmy resonance coupling strength V_c. The arrow indicates the Coulomb barrier see text for details.

Figure 3. The enhancement factor **E** for $^{70}Fe + ^{208}Pb$ for several values of V_c. The arrow indicates the Coulomb barrier.

Figure 4. The enhancement factor **E** for the deformed system $^{70}Fe + ^{238}U$, taking for $V_c = 4.0$ MeV and for several values of the deformation strength potential $\beta_2 f(R)$. The arrow indicates the Coulomb barrier.

Figure 5. The enhancement factor **E** for $^{70}Fe + ^{238}U$, for the pure deformation case, $V_c = 0$, $\beta_2 f(R) = 6.0$ MeV (full line) and pure vibration case $V_c = 6.0$ MeV, $\beta_2 f(R) = 0.0$ (dashed line). The arrow indicates the Coulomb barrier.

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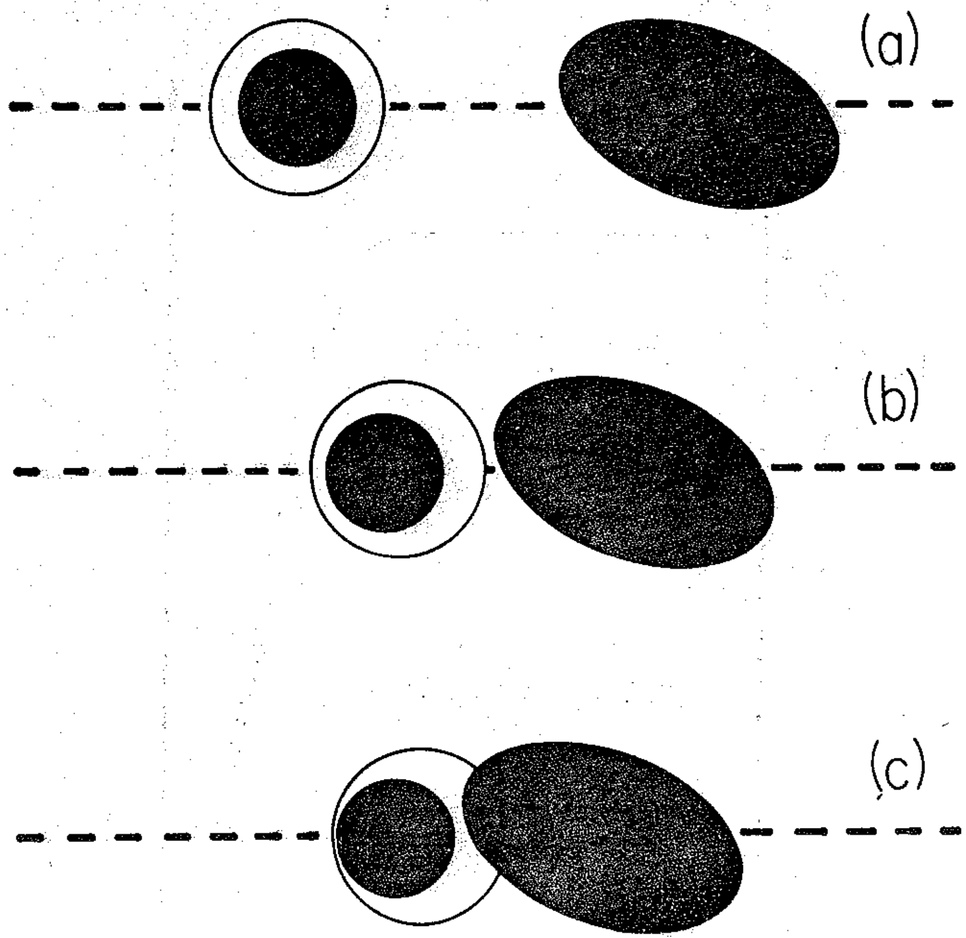
(a)
(b)
(c)

Fig. 1

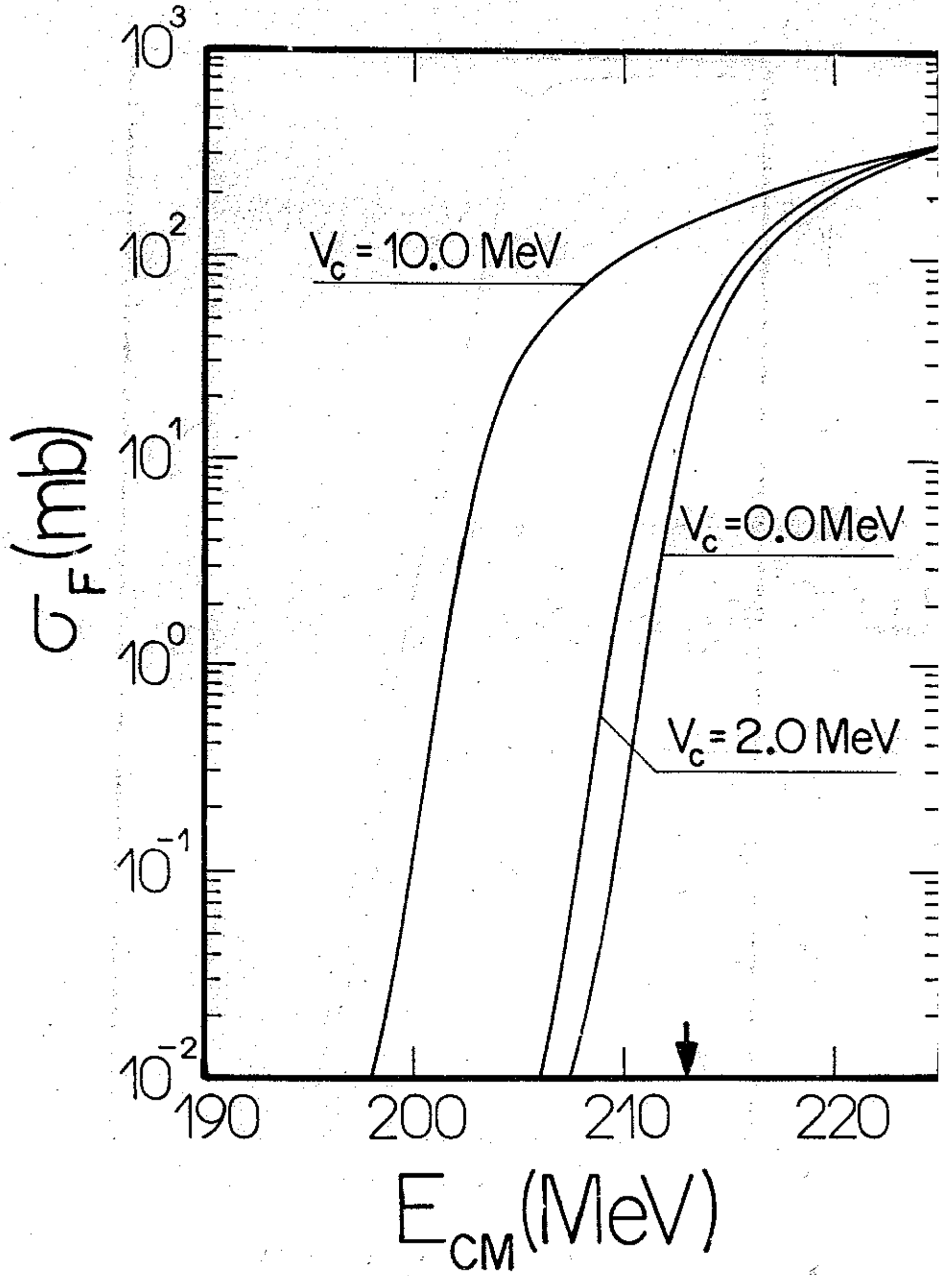
V_c = 10.0 MeV
V_c = 0.0 MeV
V_c = 2.0 MeV
σ_F(mb)
E_{CM}(MeV)
190
200
210
220

Fig. 2

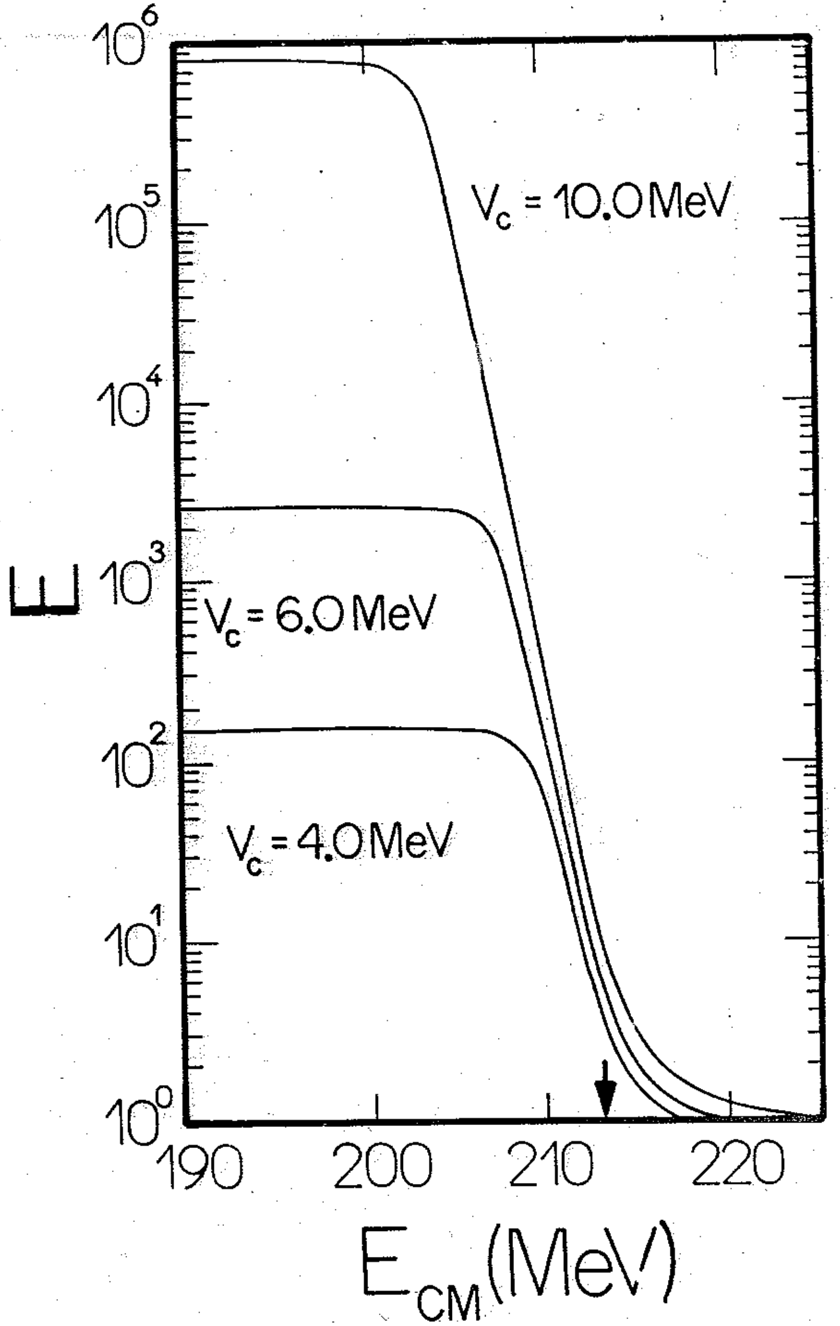
V_c = 10.0 MeV
V_c = 6.0 MeV
V_c = 4.0 MeV
E
E_{CM}(MeV)

Fig. 3

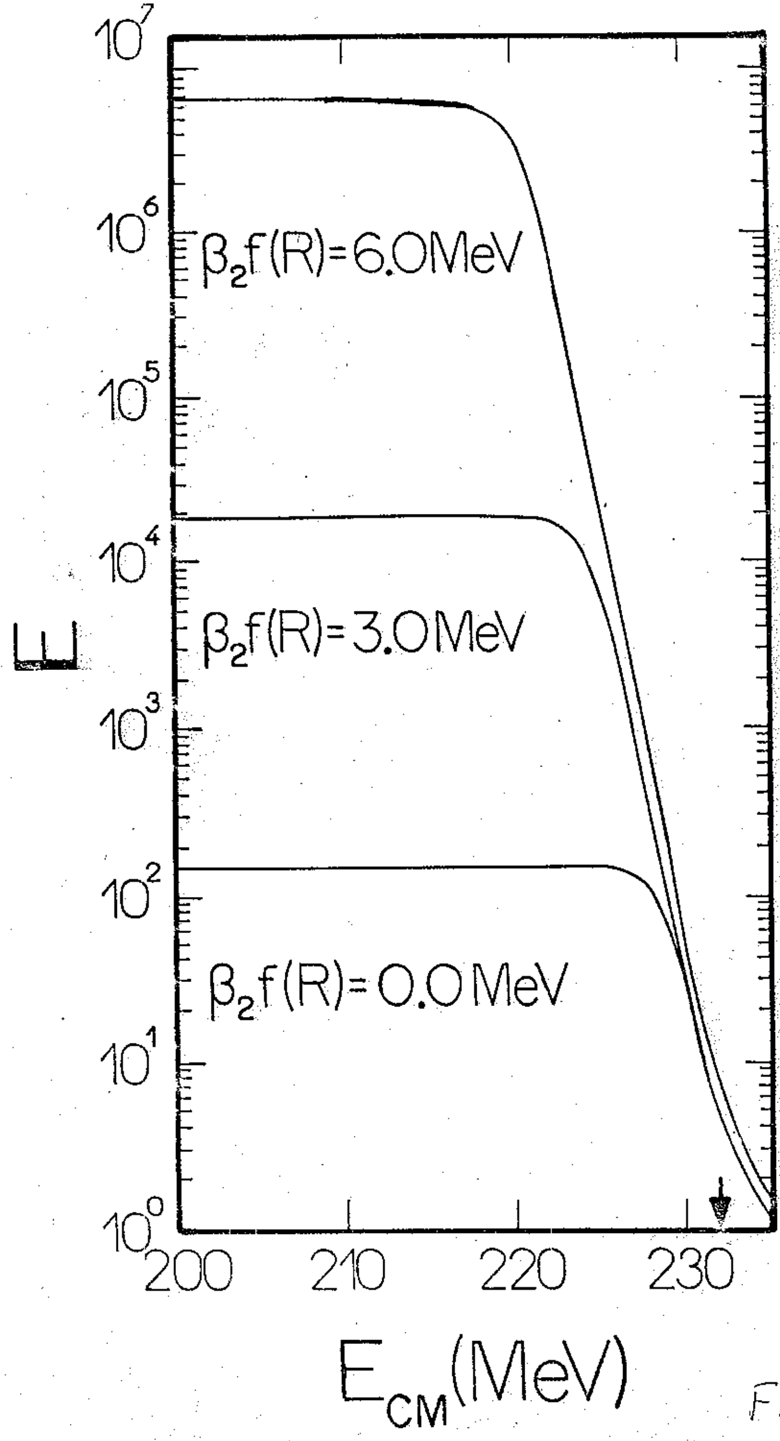
$\beta_2 f(R)$ = 6.0 MeV
$\beta_2 f(R)$ = 3.0 MeV
$\beta_2 f(R)$ = 0.0 MeV
E
E_{CM}(MeV)

Fig. 4

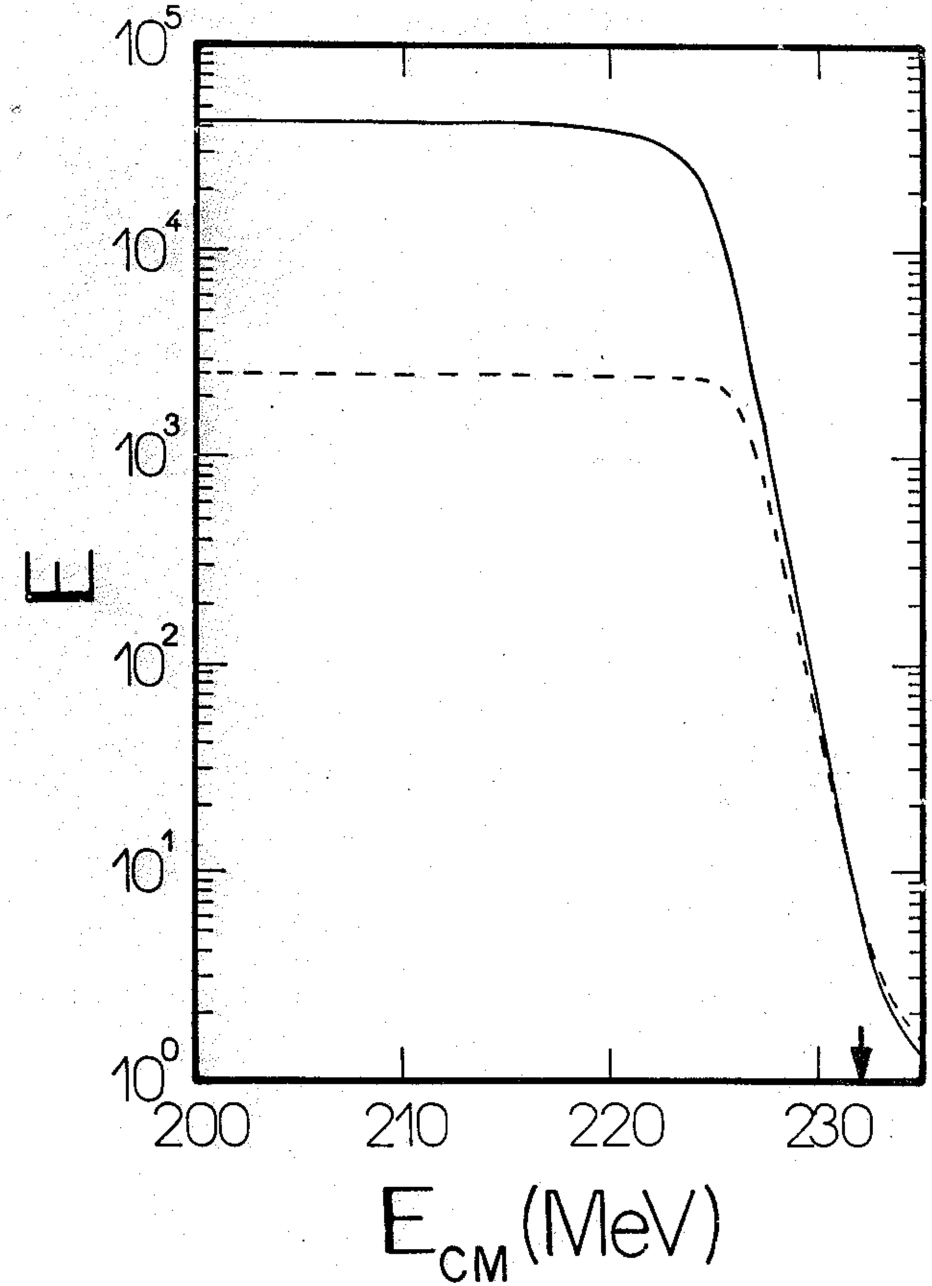

Fig. 5