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ABSTRACT

The Coulomb dissociation cross–section for ^{11}Li on ^{197}Au is calculated for different models of the distribution of dipole response strength in ^{11}Li. All the available models fail in accounting for the low–energy behaviour of the cross–section.

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Recently, Sustich[1)], presented a detailed calculatioon of the Coulomb dissociation cross–section for three models of dipole strength distribution as a function of ^{11}Li bombarding energy for reactions on a ^{197}Au target. This type of calculation is quite important as it clearly shows the sensitivity of the cross section at low energies model used for the ^{11}Li dipole response. Surtich found that the data point at 30 MeV/nucleon measured by Anne et. al.[2)] can be accounted for best by the single particle model of Bertsch and Foxwell[3)]. The more recent correlated state model of Bertsch and Esbensen[4)] underestimates the cross section by factor of 2, wherever the cluster model[5)] overestimates the cross section by a factor of 2. Both the single particle and correlated state model account well for the data at 790 MeV/nucleon[6)] whereas the cluster model again overestimates the cross section at this energy by about 15% .

The formula used by Sustich for the Coulomb dissociation cross section, however, is only valid at high energies, as pointed out by Bertulani and Baur[7)]. At lower energies, a more complicated expression for the cross section must be used. The purpose of this note is to present the result of the calculation with the correct formula for the Coulomb cross section. As shown in ref. 7, this formula reproduces both the low and the high energy regimes; and incorporates relativistic and recoil effects properly. We also include the RPA–Cluster model of Teruya et. al.[8)], not considered in ref. 1.

The Coulomb dissociation cross section is given by 7) (we observe that the original formula for the dipole case appearing in ref. 7 has a misprinted sign in one of its terms)

$$\sigma_c = \frac{15\pi^3}{9}\,\alpha \int dE\; n_{E1}(E)\, \frac{dB(E1)}{e^2\, dE} \qquad (1.a)$$

where E is the excitation energy, α is the fine structure constant, and

$$n_{E1}(E) = \frac{2}{\pi} Z_1^2 \alpha e^{-\pi\eta} \left[\frac{c}{v}\right]^2 \Bigg\{ -\xi K_{i\eta} K'_{i\eta} - \frac{1}{2}\left[\frac{c}{v}\right]^2 \xi^2 \Bigg[K_{i\eta+1} K_{i\eta-1} + - K_{i\eta}^2 - \frac{i}{\varepsilon_0}\left[K_{i\eta}\left[\frac{\partial K'_\mu}{\partial \mu}\right]_{i\eta} - K_{i\eta}\left[\frac{\partial K_\mu}{\partial\mu}\right]_{\mu=i\eta}\right]\Bigg]\Bigg\} , \tag{1.b}$$

where Z_1 and v are the target charge and the projectile (^{11}Li) velocity, respectively. α is the fine structure constant, ε_0 is the eccentricity factor of the lowest allowed Coulomb trajectory, that is

$$\varepsilon_0 = \begin{cases} 1 & \text{for } E_{c.m.} > V_B \\ \sqrt{1+4\left[\frac{E_{c.m.}}{V_B}\right]^2\left[1-\frac{V_B}{E_{c.m.}}\right]} & \text{for } E_{c.m.} < V_B \end{cases} \tag{1.c}$$

where V_B is the Coulomb barrier potential. The quantities η and ξ are defined by

$$\eta = \frac{\omega a}{\gamma v} \quad \text{and} \quad \xi = \varepsilon_0 \eta \tag{1.d}$$

where ω is the excitation frequency, $a = \frac{Z_1 Z_2 e^2}{2E_{c.m.}}$ is half the distance of closest approach for a head–on collision, $\gamma = \left[1 - \frac{v^2}{c^2}\right]^{-1/2}$.

The function $K_{i\eta}$ is the modified Bessel function with imaginary order. $K'_{i\eta}$ means the derivative of $K_{i\eta}$ with respect to the argument. At high energies the above expression for n_{E1} reduces to

$$n_{E1}(E) = \frac{2}{\pi} Z_1^2 \alpha \left[\frac{c}{v}\right]^2 \left[\xi K_0 K_1 - \frac{v^2\xi^2}{2c^2}\left(K_1^2 - K_0^2\right)\right] , \tag{2}$$

which is the form used by Sustich, even at the rather low energy of 30 MeV/nucleon. We should point out, however, that Sustich[1)] included recoil corrections to (2) which should render his calculation accurate to within 20%.

Before presenting our calculation of σ_c based on Eq.(1) for the different models of $\frac{dB(E1)}{dE}$, we first discuss the behaviour of the function $K_{i\eta}$ given by the integral[9)]

$$K_{i\eta}(\xi) = \int_0^\infty e^{-\xi \cosh x} \cos\eta x \, dx \tag{3}$$

these functions are not tabulated, and have to be obtained by means of the numerical evaluation of the integral at the r.h.s. of (3). The functions $K_{i\eta+1}$ and $K_{i\eta-1}$ are not needed since

$$K_{i\eta+1}(\xi) K_{i\eta-1}(\xi) = \frac{\eta^2}{\xi^2} K_{i\eta}^2(\xi) + K'^2_{i\eta} . \tag{4}$$

In figure 1 we show the functions $K_i(\xi)$ and $K_{5i}(\xi)$ vs. ξ. It is easy to understand the oscillatory behaviour of $K_{i\eta}(\xi)$ vs. ξ for small values of ξ, by using the stationary phase method. By writing $\cos\eta x = \frac{1}{2}(e^{i\eta x} + e^{-i\eta x})$ and since the integral of Eq.(3) is even in x, one may take only the $e^{i\eta x}$ branch of the cosine and extend the lower limit of integration to $-\infty$. Changing x to $x + i\pi$ and using the stationary phase method, we find

$$K_{i\eta}(\xi) \cong \frac{\pi e^{-\pi\eta/2}}{2} \left[\frac{4Y}{\xi^2 - \eta^2}\right]^{1/4} \mathrm{Ai}(Y) \tag{5}$$

$$Y = -\left[\eta \cosh^{-1}\frac{\eta}{\xi} - \sqrt{\eta^2 - \xi^2}\right]^{2/3} \left[\frac{3}{2}\right]^{2/3} , \qquad \eta > \xi$$

$$Y = \left[\xi \sqrt{1 - \frac{\eta^2}{\xi^2}} - \eta \sin^{-1}\sqrt{1 - \frac{\eta^2}{\xi^2}}\right]^{2/3} \left[\frac{3}{2}\right]^{2/3} , \qquad \eta < \xi \ .$$

In Eq.(5) Ai(Y) is the Airy function. This function oscillates for negative values of its argument ($\xi < \eta$) and dies out as $e^{-\xi}$ for large positive values of Y, just as figure 1b shows. Further, the local period of the oscillations goes as $\Delta\xi \simeq 2\pi\,\xi/\eta$. Thus, even for small values of η, the function $K_{i\eta}(\xi)$ oscillates at very small values of ξ. In figure 1.a, these oscillation are not shown.

We further verified that the representation (5) is also valid for $K_0(\xi)$. Finally we remark that for our purpose here, the argument of the modified Bessel function is related to its order through $\xi = \varepsilon_0\,\eta$ and since $\varepsilon_0 \geq 1$, ξ is equal or larger than η and thus the low–ξ oscillations are not relevant.

We turn now to the results obtained for σ_c, Eq.(1), using for the dipole strength distribution $\frac{dB(E1)}{dE}$ different models discussed recently in the literature. In figure 2 we show a comparison among the cross sections obtained with the modified independent particle model[3], the correlated state model[4], the hybrid RPA–Cluster model[8], and the cluster model[5]. Our result diverge in an important way from those of Sustich in that not any of the models account for the low energy data point (E_{lab} = 30 MeV/nucleon). Whereas the cluster model overestimates the cross–section the other models fall short in value. The recent calculation of Lenske and Wambach, using the quasiparticle RPA method,

also fall short in value (the cross section for this case is not shown in figure 2 as it almost coincides with the independent particle result).

In conclusion, we have calculated in this paper the Coulomb dissociation cross section for ^{11}Li on ^{197}Au using different models for the dipole strength distribution of ^{11}Li. All available models fail in accounting for the low–energy cross–section. Further theoretical studies and experiments are clearly needed to settle the matter.

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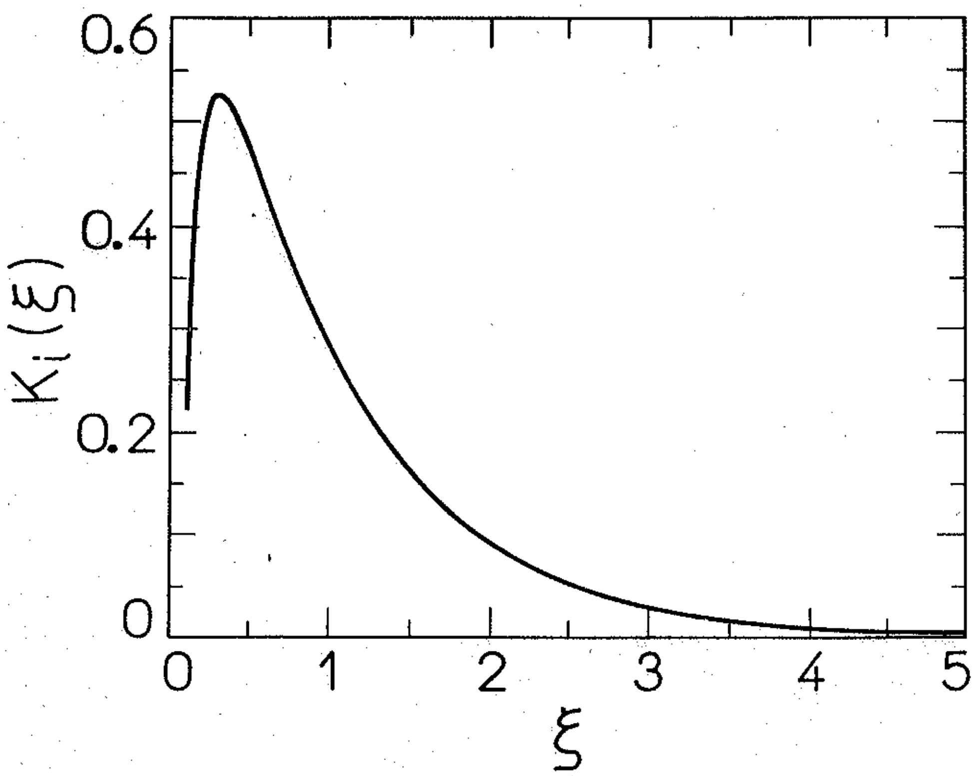
FIGURE CAPTIONS

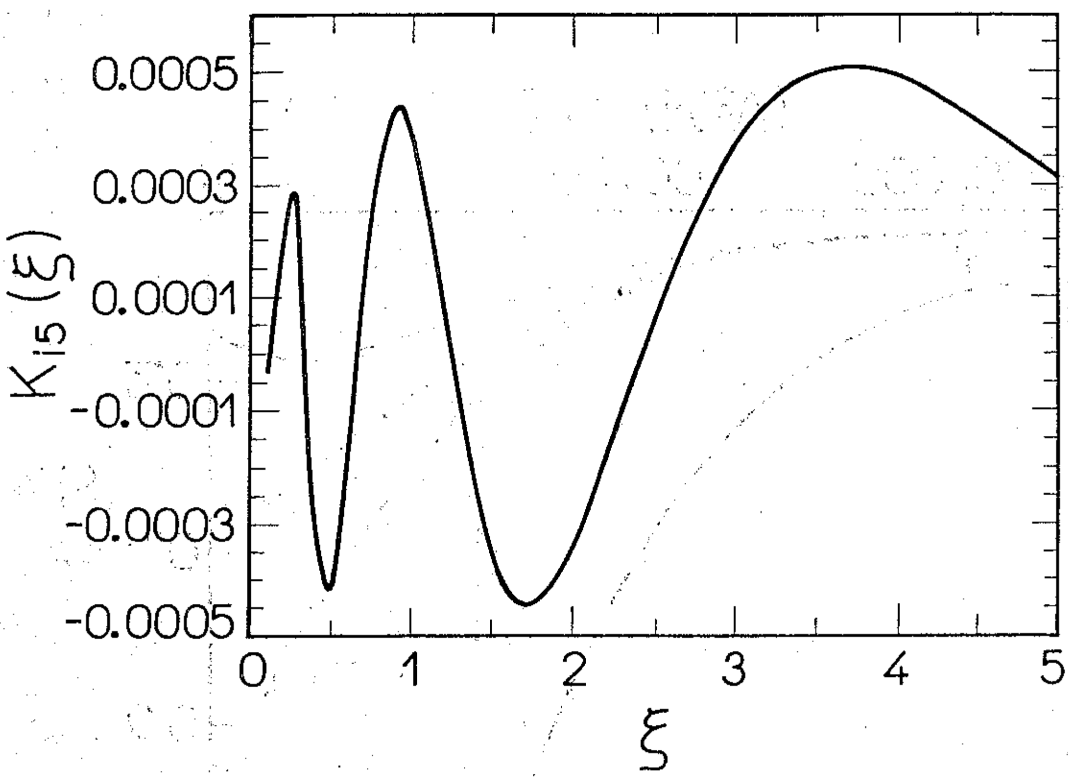
Figure 1. The function $K_{i\eta}(\xi)$ vs. ξ . a) $\eta = 1$ b) $\eta = 5$.

Figure 2. The Coulomb dissociation cross section for different models of $\frac{dB}{dE}$. The two data point are from ref.(6) (E = 790 MeV/n) and ref.(2) (E = 30 MeV/n). Full curve cluster model. Dashed curve: independent particle model. Dotted curve: correlated state model. Dashed–dotted curve: hybrid RPA–Cluster model. See text for details.



0.6
0.4
0.2
0
$K_1(\xi)$
0
1
2
3
4
5
ξ

Fig. 1a



0.0005
0.0003
0.0001
-0.0001
-0.0003
-0.0005
$K_{15}(\xi)$
0
1
2
3
4
5
ξ

Fig. 1b

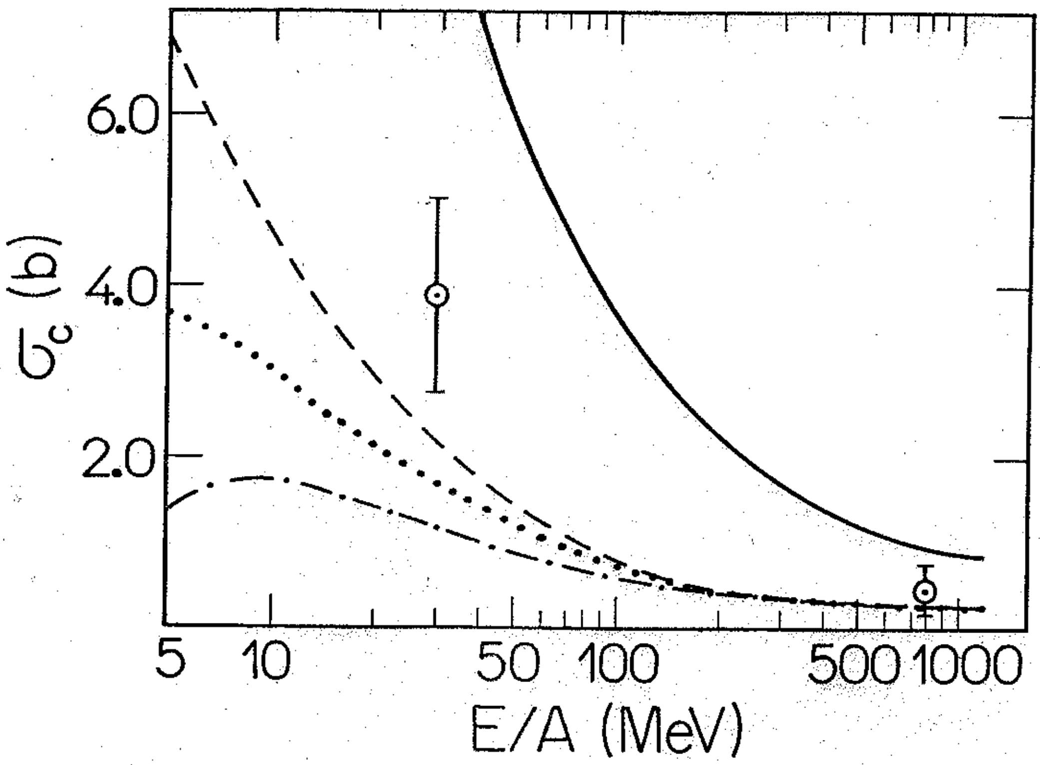


Fig. 2