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INSTITUTO DE FÍSICA
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PHASE TRANSITIONS AND FORMATION OF BUBBLES IN THE EARLY UNIVERSE

G.C. Marques and R.O. Ramos
Instituto de Física, Universidade de São Paulo

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PHASE TRANSITIONS AND FORMATION OF BUBBLES IN THE EARLY UNIVERSE *

G.C. Marques and R.O. Ramos
Instituto de Física, Universidade de São Paulo
C.P. 20516, 01498 São Paulo, S.P., Brazil

Abstract

We analyse the bubble formation process that takes place as a result of phase coexistence in the early Universe. We have been concerned with the determination of quantities relevant to cosmology such as the number density of bubbles, the contrast density and the most probable sizes of bubbles (critical radius). We show that all these quantities can be expressed as a function of the surface tension. The surface tension is shown to acquire a very simple dependence in the high temperature limit and easily predicted up to the one–loop approximation. In this limit it is possible to get simple expressions to all cosmologically relevant quantities. In the case of the SU(5) GUT model we get the Zel'dovich spectrum with the proper order of magnitude as well as other interesting consequences to cosmology.

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I. INTRODUCTION

As one probes the large scale structure of the Universe some puzzling features, in the distribution of galaxies on large scales, emerges. In particular, there are strong evidences that the distribution of galaxies is made up of thin sheets (or surfaces) in which the galaxies lie. These sheets surround surround vast voids (regions in space containing few bright galaxies). Furthermore the sheets seems to be surfaces of several adjacent "*bubbles*".[1]

The nowadays accepted picture for the development of structures in the Universe is based upon the growing, due to gravitational instability after the recombination era, of small disturbances of the density. Furthermore, there is an widespread belief that these initial perturbations should result from processes operating in the very early Universe, that is, processes that took place very close to the singularity. In this context, cosmological phase transition might play an important role, since the appearance of inhomogeneities (defects) in the system is a common feature of theories, whose symmetries are spontaneously broken. In fact, there are suggestions that topological defects such as strings and domain walls generates the required contrast density for giving rise to the observed structures in the Universe.

The formation of bubbles (or droplets) is a feature of systems that exhibits phase coexistence along the phase transition. The approach that we have used (the droplet picture of phase transition[2,3]) has been developed for dealing with bubble formation in phase transitions that are very familiar to physicists. One of our motivations for dealing with this problem, and its role in cosmology, is the similarity of the observed geometrical structures and the ones formed along some phase transitions. The idea is then to see whether this familiar physical phenomena might give rise to the geometrical structure observed, yielding, as the Universe envolves, a pattern very much like the one defined by the galaxies.

In this paper we will explore the possibility that the observed large scale structure of the Universe emerged from the existence of interfaces separating regions of different phases in the Universe. We imagine that at some stage of the Universe there was a phase coexistence. There was an era in the Universe in which two bulk thermodynamic phases coexisted in such a way that regions of the space (bubbles) were separated by a relatively narrow region, the interfacial region, over which the properties of the system must change from those of one phase to those of the second phase. In the case of a magnetic material the interfacial region is planar and is refered as the bloch wall. In the case of theories with spontaneous symmetry breakdown the planar interfacial region will be refered as domain walls.

We shall infer some relevant parameters for cosmology and achieve a description of phase transitions from the the knowledge of the interfacial free energy per unit area (that will be refered from now on as surface tension). The idea is that one can define first the thermodynamics of a single interface and afterwards to extend it to a description of the system as a whole.

In field theory there are two circumstances under which the Universe might developed bubbles or domains. We will distinguish these two situations and will refer to them as degenerate and nondegenerate case.

The nondegenerate case occurs when the order parameter has more than one component and the effective Hamiltonian is different for each value of the order parameter. In cosmology we would say that the two phases would have different cosmological constants. Under these circumstances below a certain temperature the phase with the order parameter $\rho_0 = 0$ becomes metastable. The change from a metastable to a stable phase occurs as the result of fluctuations in a homogeneous medium. Within the homogeneous medium there is formation of small quantities–droplets–of the new phase.

The degenerate case occurs when the order parameter has, say, n components ρ_i

but the effective Hamiltonian depends on only in the sum of the square of these components. The effective Hamiltonian is independent of the direction of the n–dimensional vector ρ. In field theory we would say that the vacuum of the theory is degenerate. A typical and familiar example of a degenerate system is a purely exchange ferromagnet, whose energy is independent of the direction of the magnetization vector.

The plan of the paper is the following: In section II we review the general aspects of surfaces and their thermodynamics and the bubble formation. In section III we establish the general framework and give formal expressions, in field theory at finite temperature, for the free energy per unit area (surface tension) of a domain wall: As an example we found the surface tension and the critical temperature for the minimal SU(5) model. The expressions obtained are fairly simple in the high temperature limit. In section IV we consider the case of bubbles. We analysed the case for a nondegenerate theory and for a degenerate one we determined relevant results for cosmological applications such as the number density of bubbles and the contrast density in the dilute gas approximation. In section V we gave the results of the last section for the minimal SU(5) model and we analyse the possibility that bubbles have a bearing on the formation of structures in the Universe. Conclusions are presented in section VI.

II. SURFACES – CLASSICAL RESULTS

2.1. Surface Tension

The thermodynamical properties of an interface can be entirely characterized by the surface tension σ. This thermodynamical variable is defined in terms of the work (dW) needed to vary the surface by an amount dA by:

$$dW = \sigma \, dA \quad . \tag{2.1}$$

The surface tension depends on the temperature as well as other variables, that we call "*external variables*" such as magnetic fields,

$$\sigma = \sigma(T, x_1, ..., x_n) \tag{2.2}$$

where x_i is the ith external variable: one can say that x_i accounts for the bulk enviromment action over the surface.

In order to take into account surface effects, by taking the volume fixed, one writes

$$dF = dE - T\, dS = \mu \, dN + \sigma \, dA \tag{2.3}$$

where the differential stands for these elements in the two phase system. For T and μ constant one gets from the equation for the thermodynamic potential, $d\Omega = - S\, dT - - N\, d\mu + \sigma\, dA$, that

$$d\Omega = \sigma \, dA \tag{2.4}$$

whereas within the canonical ensemble (taking N fixed in (2.3)) one gets

$$dF = \sigma \, dA \quad . \tag{2.5}$$

From (2.5) it follows that f (the free energy per unit area) is equal to σ From (2.4) it follows then that the entropy (per unit area) is given by

$$s = - \frac{d\sigma}{dT} \tag{2.6}$$

and the surface energy is

$$\varepsilon = f + Ts = \sigma - T \frac{d\sigma}{dT} \quad . \tag{2.7}$$

If one represents by E^o, S^o and F^o the internal energy, entropy and free energy of the two phase system without the surfaces (that is, excluding the interfacial region) then the same quantities when a single surface of area dA is present in the system are given by

$$S = S^o + s(T,x)\, dA \tag{2.8a}$$

$$E = E^o + \varepsilon(T,x)\, dA \tag{2.8b}$$

$$F = F^o + \sigma(T,x)\, dA \tag{2.8c}$$

with $s(T,x)$ and $\varepsilon(T,x)$ defined by (2.6) and (2.7) respectively.

The main conclusion is that, as pointed out earlier, the surface tension, defined in (2.1), is the essential thermodynamic variable of the interface. From it one gets the free energy, entropy and energy of an interface of area A , as

$$F_S = \sigma A \tag{2.9a}$$

$$S_S = -A \frac{d\sigma}{dT} \tag{2.9b}$$

$$E_S = \left[\sigma - T\left[\frac{d\sigma}{dT}\right]\right] A \ . \tag{2.9c}$$

Furthermore, from the definition (2.1) it follows that σ represents the cost in energy, per unit area, for introducing an interface in the system. This cost in energy can be expressed as a difference in the free energy of the two phase system.

2.2. Phase Transition

The bubbles on which we will be concerned in this paper are associated to phase coexistence in some stage of the Universe. We imagine two bulk thermodynamic phase separated by relatively narrow region, the interfacial region, over which the properties of the system changes from one phase to the other. Phase coexistence occurs in simple fluids, binary fluids and in anisotropic magnets. The latest case is a prototype of models in which there is spontaneous symmetry breakdown and the interfaces are refered as domain walls.

There is, then, a strong correlation between the existence of interfaces and the occurence of phase transitions.

At the critical temperature the surface tension vanishes

$$\sigma\,(T_c, x_i, ..., x_n) = 0 \ . \tag{2.10}$$

The condition (2.11) implies, for theories in which the vacuum is degenerate, that the cost for introducing a surface of arbitrary size in the system is zero and consequently the system is "*insensible*" to boundary conditions.

In this paper we will show that in field theory at finite temperature it is possible to account for the vanishing of the surface tension since one can provide a definite scheme for computing this relevant parameter. That allows us to determine many relevant parameters in cosmology. The approach, in order to achieve this, is then from the thermodynamical variable $\sigma(T)$, which is associated to a single bubble, to extend it to the description of the system as a whole.

The dependence of the surface tension in the temperature in some cases is a universal function of T/T_c. From the law of corresponding states it follows that the surface tension can be written as

$$\frac{\sigma(T)}{\sigma(T=0)} = f\left[\frac{T}{T_c}\right] \ . \tag{2.11}$$

A dependence of the form (2.11) we shall call a corresponding state dependence. The surface tension in the high temperature limit and up to the one–loop approximation, for any renormalizable theory, can be written in the form (2.11), with $f(T/T_c) = 1 - T^2/T_c^2$, so that the corresponding state dependence is valid in field theory.

2.3. Bubble Formation and Critical Sizes

According to the thermodynamic theory of fluctuations the probability for producing a bubble of radius R is given by

$$\omega \sim \exp\left[-\frac{\Delta F(R)}{T}\right] \tag{2.12}$$

where $\Delta F(R)$ is the cost in energy for introducing such an object into the system[4]. Usually, and as well be done in this paper, the cost in energy can be expressed as a difference of thermodynamical potentials.

In order to determine the most probable bubbles we just look for the value of R that minimizes $\Delta F(R)$,

$$\left.\frac{d\Delta F(R,T,...)}{dR}\right|_{R=R_c} = 0 \quad . \tag{2.13}$$

The sizes of bubbles as well as their densities is one of our concerns in this paper.

As a simple example we consider the formation of bubbles in the case of liquid–vapour phase transition. The bubbles will be considered as spheric ones of radius R. Under these circumstance all one has to do is to consider the variation in the thermodynamic potential Ω.

Before the appearence of the bubble the potential is given by $\left(V = V_0 + \frac{4\pi}{3} R^3\right)$

$$\Omega^0 = -P_0 \left(V_0 + \frac{4\pi}{3} R^3\right) \tag{2.14}$$

after the appearence of the bubble in the system of which the pressure is P

$$\Omega = -P_0 V_0 - P \frac{4\pi}{3} R^3 + \sigma 4\pi R^2 \quad . \tag{2.15}$$

From (2.14) and (2.15) it follows that

$$\Delta F(R) = \Omega - \Omega^0 = -(P - P_0) \frac{4\pi}{3} R^3 + \sigma 4\pi R^2 \quad . \tag{2.16}$$

The probability for producing a bubble of radius R is then, from (2.12),

$$\omega \sim \exp\left[\frac{4\pi}{3} R^3 (P-P_0) - 4\pi R^2 \sigma\right] \quad . \tag{2.17}$$

A dependence of the form (2.17) is known as the capillarity approximation[3]. We shall see that for nondegenerate vacua it is possible to get a dependence of the form (2.17) within the one–loop approximation in the high T limit. The critical size is then (from (2.13))

$$R_{cr} = \frac{2\sigma}{P-P_0} \tag{2.18}$$

and the probability for the most favorable bubble will be given, following (2.12), by

$$\omega \sim \exp\left[\frac{-16\,\pi\sigma^3}{3(P-P_0)^2\ T}\right] \quad . \tag{2.19}$$

As can be seen from (2.18) and (2.19) that one can get relevant informations on the size of most probable bubbles (critical bubbles) and their distribution from the knowledge of the surface tension.

Formally, the critical sizes of bubbles tends to zero at the critical temperature,

$$\lim_{T \to T_c} R_{cr}(T) = 0 \quad . \tag{2.20}$$

This is a consequence of the capillarity approximation which, in field theory, follows from the fact that the two phases exhibits different cosmological constants.

III. SURFACES IN FIELD THEORY

3.1. Surface Tension — Definitions

We have shown in the last section that the relevant quantity, whenever there is phase coexistence, is the surface tension. We will show in the following that in field theory at finite temperature one has a well defined approach for computing this thermodynamical variable.

Let $\phi_D(x)$ represents a field configuration describing a defect (for example a bubble) in the system. We shall be interested in the thermodynamical properties of the system in the presence of such a background field. This, in the other hand, should be infered from the partition function $Z(\phi_D)$ defined as

$$Z(\phi_D) = \int [D\phi]\, e^{-S[\phi+\phi_D]} \quad . \tag{3.1}$$

The free energy of the system in the presence of the background field $\phi_D(x)$ is

$$F(\phi_D) = -\beta \ln Z(\phi_D) \quad . \tag{3.2}$$

One might be interested also in analysing the free energy associated to a uniform background field that we represent by ϕ_0. The free energy of the system in the presence of this uniform background field ϕ_0 is

$$F^0(\phi_0) = -\beta \ln Z^0(\phi_0) \tag{3.3}$$

where $Z^0(\phi_0)$ is obtained from (3.1) by substituting ϕ_D in (3.1) by ϕ_0.

The vacuum of the theory is associated to the field configuration that minimizes $F^0(\phi_0)$:

$$\left.\frac{\delta F^0(\phi_0)}{\delta \phi_0}\right|_{\phi_0 = \varphi_v} = 0 \quad . \tag{3.4}$$

$F(\phi_D)$ defined in (3.2) can be though as a thermodynamical potential associated to a spatially innhomogeneous system. The correponding equilibrium condition is found by solving the following variational problem:

$$\left.\frac{\delta F(\phi)}{\delta\phi}\right|_{\phi = \phi_c} = 0 \quad . \tag{3.5}$$

One is then led to a variational problem which, more generally, can be stated as follows: let ϕ_i be a solution of the following variational problem:

$$\left.\frac{\delta \Gamma(\phi)}{\delta\phi}\right|_{\phi = \phi_i} = 0 \tag{3.6}$$

where Γ is a Group–invariant functional.

It is possible to show, by using the background field method, that under condition (3.6) one can write the cost in energy for introducing an interface in the system as

$$\Delta F = F(\phi_c) - F(\phi_v) = \Gamma(\phi_c) - \Gamma(\phi_v) \tag{3.7}$$

where Γ is the effective action defined by

$$\Gamma = \sum_n \frac{1}{n!} \int \dots \int dx_1 \dots dx_n \, \Gamma^{(n)}(x_1, \dots, x_n) \, \phi(x_1) \dots \phi(x_n) \tag{3.8}$$

where $\Gamma^{(n)}$ is the one–particle irreducible Green's functions of the theory. Then, one can see that the free energy of the system in the presence of the background ϕ_c satisfying the classical equation

$$\left. \frac{\delta \Gamma}{\delta \phi} \right|_{\phi = \phi_c} = 0 \tag{3.9}$$

is given by $\Gamma(\phi_c) = F(\phi_c)$, the effective action computed at this configuration.

If one uses the Fourier transform of $\Gamma^{(n)}$, given by

$$\Gamma^{(n)}(\tau_1 \vec{x}_1, \dots, \tau_n \vec{x}_n) = \beta^{-n} \prod_{j=1}^{n} \sum_{n_j=-\infty}^{+\infty} \int \frac{d^3 \vec{k}_j}{(2\pi)^3} \, \tilde{\Gamma}^{(n)}(\omega_1 \vec{k}_1, \dots, \omega_n \vec{k}_n) \times$$

$$\times \exp\left[-i \sum_{\ell=1}^{n} \left(\omega_\ell \tau_\ell + \vec{k}_\ell \cdot \vec{x}_\ell \right) \right] \tag{3.10}$$

where $\omega_\ell = 2\pi\ell/\beta$, and remembering that translational symmetry allows us to set

$$\tilde{\Gamma}^{(n)}(\{\omega_i \vec{k}_i\}) = \beta(2\pi)^3 \, \delta\big(\sum_i \omega_i\big) \, \delta^3\big(\sum_i \vec{k}_i\big) \, \tilde{\Gamma}^{(n)}(\{\omega_i \vec{k}_i\}) \tag{3.11}$$

then, for static field configurations (those with which we will be concerned in this paper), the general structure of $\Gamma(\phi_c)$ is

$$\Gamma(\phi_c) = \beta \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{j=1}^{n} \int d^3 \vec{k}_j \, \tilde{\phi}(-\vec{k}_j) \, \tilde{\Gamma}^{(n)}(\{\vec{k}_j, \omega_j = 0\}) \, \delta^3\big(\sum_j \vec{k}_j\big) \; . \tag{3.12}$$

The graphs that contribute to $\tilde{\Gamma}^{(n)}$ will envolve sums over the discrete ω_j which, once performed, yield a term independent of temperature plus one which has the full T dependence. This separation can always be implemented[5]. One can then split $\tilde{\Gamma}^{(n)}$ into two parts

$$\tilde{\Gamma}^{(n)}(\{\vec{k}_i, \omega_i = 0\}) = \tilde{\Gamma}_0^{(n)}(\{\vec{k}_i\}) + \tilde{\Gamma}_T^{(n)}(\{\vec{k}_i, \omega_i = 0\}) \tag{3.13}$$

where the second term contains all the T–dependence. The general structure of this dependence can be inferred by making a change in all internal momenta integration variables. This change is just a replacement $\vec{p} \to \vec{p}\,' = \vec{p}\,\beta$. After this scaling in the internal momenta one can predict, from pure dimensional analysis, that $\tilde{\Gamma}_T^{(n)}(\{\vec{k}_i, \omega_i = 0\})$ have following structures[6]

$$\tilde{\Gamma}_T^{(n)}(\{\vec{k}_i, \omega_i = 0\}) = \sum_{\gamma_n} T^{d(\gamma_n)} \, G_{\gamma_n}\left[\frac{\vec{k}_i}{T}, \frac{n}{T} \right] \tag{3.14}$$

where $d(\gamma_n)$ is the superficial degree of divergence of a graph γ_n contributing to $\tilde{\Gamma}$ and G_{γ_n} is dimensionless. Putting (3.11), (3.13) and (3.14) together, we have

$$\Gamma(\phi_c) = \Gamma_0(\phi_c) + \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{j=1}^{n} \int d^3 \vec{k}_j \, \tilde{\phi}_c(-\vec{k}_j) \sum_{\gamma_n} T^{d(\gamma_n)} \, G_{\gamma_n}\left[\frac{\vec{k}_i}{T}, \frac{n}{T} \right] \delta^3\big(\sum_j \vec{k}_j\big) \tag{3.15}$$

where $\Gamma_0(\phi_c)$ is the effective action computed at the background field ϕ_c at zero temperature.

Using (3.7) and remembering that the surface tension σ can be defined as $\Delta F/L^2$, then we can write the following general expression for the surface tension involving a background field ϕ_c

$$\sigma(T) = \frac{1}{L^2}\left[\Gamma_0(\phi_c) - \Gamma_0(\phi_v)\right] + \frac{1}{L^2}\left\{ \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{j=1}^{n} \int d^3 \vec{k}_j \, \tilde{\phi}_c(-\vec{k}_j) \times \right.$$

$$\left. \times\, \delta^3\Big(\sum_j \vec{k}_j\Big) \sum_{\gamma_n} T^{d(\gamma_n)}\, G_{\gamma_n}\left[\frac{\vec{k}_i}{T}, \frac{n}{T}\right] - L^3 \sum_{n=1}^{\infty} \frac{1}{n!} \phi_v^n \sum_{\gamma_n} T^{d(\gamma_n)}\, G_{\gamma_n}\left[0, \frac{n}{T}\right] \right\} \quad (3.16)$$

From (3.16) it follows that the general structure of $\sigma(T)$ is

$$\sigma(T) = \sigma(0) - T^2 g^{(2)}(T,m) + T\, g^{(1)}(T,m) + \ldots$$

where $\sigma(0) = \frac{1}{L^2}\left[\Gamma_0(\phi_c) - \Gamma_0(\phi_v)\right]$ and where we have separated the contribution whose graphs have superficial degree of divergence 2 and 1 leading, from (3.16), to the powers T^2 and T.

From expression (3.16) one can see that, in the high temperature limit, the leading contributions comes from graphs that have higher superficial degree of divergence. As we will show in the next section, these graphs up to a given order in the semiclassical expansion, are easy to isolate.

3.2. Domain Wall Free Energy

Field theories whose gauge symmetry is spontaneously broken might exhibit topologically stable defects. The prediction of the type of defect relies upon topological arguments. Under certain circumstances one can predict the existence of domain walls. These defects corresponds to infinite interfaces separating vacua configurations (planar interfaces). At the classical level these objects are associated to solutions of (3.5) when one takes Γ computed at zero loop level.

In this section we will review the approach for getting the surface tension in Field theory[7]. In this case we will be concerned with the computation of free energies associated to domain walls. Let σ_w represet the free energy associated to a domain wall. In the field theory σ_w is given as

$$\sigma_w = -\frac{\beta^{-1}}{L^2} \ln\left[\frac{Z_w}{Z_v}\right] \quad (3.17)$$

where Z_w stands for the partition function of the system evaluated when one imposes boundary conditions that force the existence of a domain wall defect in the system, while Z_v is the partition function obtained using topological trivial boundary conditions (vacuum sector). L is the size of the system.

The various thermodynamical functions can be written, in the one loop approximation, as shown in the last section, as differences of the effective action of the theory evaluated at certain field configurations. Let $\Gamma(\phi)$ be the effective action of the theory and ϕ_v be the constant field configuration associated to the vacuum of the theory. Then, in terms of the effective action one write σ_w by (3.7), with ϕ_c changed by ϕ_w, the field configuration associated to the wall,

$$\sigma_w = \frac{1}{L^2}\left[\Gamma(\phi_w) - \Gamma(\phi_v)\right] \quad . \tag{3.18}$$

The special field theoretical configurations φ_c (ϕ_w for the wall), within the semiclassical scheme, are the defects associated to the classical solutions of the Euler–Lagrange equations of the model.

The dependence of σ_w on T is given in (3.16). The critical temperature T_c is given from the condition (2.10), i.e., $\sigma_w(T_c) = 0$. The interpretation in this case is that above T_c there is symmetry restauration as a result of condensation of domain walls[7].

As an example let us consider the minimal SU(5) GUT at finite temperature. Its Euclidean Lagrangean density is

$$\mathcal{L} = -\tfrac{1}{4}\,T_r\left[G_{\mu\nu}\,G^{\mu\nu}\right] + \tfrac{1}{2}\,T_r\left[|D_\mu\,\phi|^2\right] + V(\phi) \tag{3.19}$$

where ϕ is the Higgs multiplet belonging to the adjoint representation and

$$V(\phi) = -\frac{\mu^2}{2}\,T_r(\phi^2) + \frac{a}{4}\left[T_r(\phi^2)\right]^2 + \frac{b}{2}\,T_r[\phi^4] \tag{3.20}$$

$$G_{\mu\nu} = \sum_{i=1}^{24} G^i_{\mu\nu}\,\frac{\lambda^i}{\sqrt{2}} \tag{3.21a}$$

$$W_\mu = \sum_{i=1}^{24} W^i_\mu\,\frac{\lambda^i}{\sqrt{2}} \tag{3.21b}$$

$$\phi = \sum_{i=1}^{24} \phi^i\,\frac{\lambda^i}{\sqrt{2}} \tag{3.21c}$$

$$D_\mu\phi = \partial_\mu\phi - \frac{ig}{\sqrt{2}}\,T_r\left[W_\mu\,,\,\phi\right] \tag{3.21d}$$

and λ^i(i=1, ..., 24) are the generators of SU(5) in the fundamental representation (normalized so that $T_r[\lambda^i\,\lambda^j] = 2\delta^{ij}$). We also impose that $b > 0$ and $a > -7/15\,b$.

This model exhibits two different topological defects: domain walls and magnetic monopoles. The background fiels describing a domain wall is the type of solution which one is interested and is given by

$$\bar{\phi}_w = \frac{\mu}{\sqrt{\lambda}}\tanh\left[\frac{\mu}{\sqrt{2}}\,x\right]\frac{\lambda_{24}}{\sqrt{2}} \tag{3.22a}$$

$$\bar{W}^a_\mu = 0 \tag{3.22b}$$

with $\lambda = a + 7/15\,b$. Note that this solution depends only on one spatial coordinate, which we choose to be the x one.

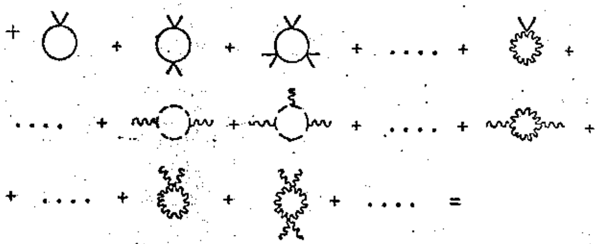
Let us exhibit the structure of the free energy of the system under this background field in the one–loop approximation. In the zero–loop approximation one has, from (3.16),

$$\sigma_w(0) \equiv \Delta\varepsilon^0_w = \frac{1}{L^2}\left[\Gamma_0(\bar{\phi}_w) - \Gamma_0(\phi_v)\right] = \frac{T}{L^2}\left[S_{cl}(\bar{\phi}_w) - S_{cl}(\varphi_v)\right] \tag{3.23}$$

where ϕ_v is the vacuum value of the classical potential $V(\varphi_v)$, equation (3.20), given by $\varphi_v = \mu/\sqrt{\lambda}$, with $\lambda = a + 7/15$ b. This from (3.23), in the zero–loop approximation, the free energy of the topological defect is just the difference between the classical action associated to the wall and the energy of the vacuum. $\Delta\varepsilon^0_w$ is the mass per unit area of the wall at $T = 0$.

Within the one–loop approximation $\Gamma(\bar{\phi},\bar{W}_u)$ will have the structure predicted from (3.8) which, for the example that we are considering, has the structure

$$\Gamma(\bar{\phi},\bar{W}_\mu) = S_{cl}(\bar{\phi},\bar{W}_\mu) +$$



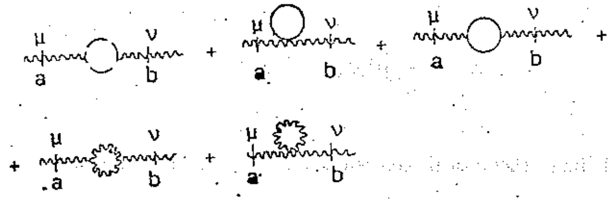
$$= S_{cl}(\bar{\phi},\bar{W}_\mu) - \frac{1}{2!}\Sigma^{ab}(T)\int_0^\beta d\tau \int d^3\bar{x}\ \bar{\phi}^a\,\bar{\phi}^b = \frac{1}{2!}\Pi^{ab}_{\mu\nu}(T)\int_0^\beta d\tau\int d^3\bar{x}\ \bar{W}^a_\mu\,\bar{W}^b_\nu + \ldots \tag{3.24}$$

where S_{cl} is the classical action associated to the background field, $\Sigma^{ab}(T)$ can be represented graphycally as

$$\Sigma^{ab}(T) = \tag{3.25}$$

whereas $\Pi^{ab}(T)$ can be represented as

$$\Pi^{ab}_{\mu\nu}(T) = \tag{3.26}$$



The wavy, solid and dotted lines stand, respectively, for the gauge bosons, Higgs and ghost fields (for the fluctuations we are working in Landau gauge). $\Pi^{ab}(T)$ can be identified as the polarization tensor for zero external momenta[8]. Following our earlier prescription (3.13), we also split (3.25) and (3.26) into the zero temperature and temperature dependent parts:

$$\Sigma^{ab}(T) = \Sigma^{ab}_0 + \Sigma^{ab}_T(\{\vec{k}_i,\omega_i=0\}) \tag{3.27}$$

and

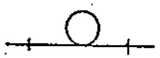
$$\Pi^{ab}_{\mu\nu}(T) = \Pi^{ab}_{\mu\nu_0} + \bar{\Pi}^{ab}_{\mu\nu}(T) \tag{3.27b}$$

First of all one notes, looking at (3.24), the appearence of ultraviolet divergences. These, however, can be treated, as usual, by adding appropriate renormalization counterterms. Which are just the usual ones at zero temperature. This means that the zero temperature renormalization scheme suffices for getting finite expressions to free energies of topological defects. Substituting (3.27) into (3.24), one can obtain the topological defect free energy of the SU(5) model, which for a wall with $\bar{\phi}$ and $\bar{W}_\mu$ given by (3.22), one has

$$\sigma_w(T) = \Delta\varepsilon_w - \frac{1}{2!}\frac{\Sigma^{24,24}(T)}{L^2}\int_0^\beta d\tau\int d^3\bar{x}\left[\bar{\phi}^w_{24}(x)\,\bar{\phi}^w_{24}(x) - \bar{\phi}^2_V\right] + \ldots \tag{3.28}$$

where $\Delta\varepsilon_w$ stands for the classical energy density of the wall, $\bar{\phi}^{w}_{24}(x)$ is given by (3.22), $\bar{\phi}_v = \mu/\sqrt{2\lambda}\,\lambda_{24}$, $\Sigma^{24,24}(T)$ is given by (3.27a) and the dots represents one loop contributions not included explicity in (3.28). One could go further and write down similar expression for all the one–loop graphs for the topological wall structure of the SU(5) model. However, instead of doing this explicitly, we will just analyse the high temperature limit of the free energy. In this limit, the form (3.16) is particularly useful, since the leading power in T of series (3.15) is easily obtained. Property (3.14) permits us to identify these contributions, which are the ones with higher superficial degree of divergence. These contributions are precisely the ones we have written explicitly.

In the high temperature limit, the graphs appearing in (3.25) are the ones we need and yield



$$= -\left[26\,a + \frac{282}{15}\,b\right]\frac{T^2}{12}\,\delta^{mn} \tag{3.29a}$$

and

$$= -5/4\; g^2\, T^2\, \delta^{mn}\ . \tag{3.29b}$$

From (3.29) we have the asymptotic expression for $\Sigma^{mn}(T)$

$$\Sigma^{mn}(T) = -\frac{T^2}{4}\left[5g^2 + \frac{1}{3}\left[26\,a + \frac{282}{15}\,b\right]\right]\delta^{mn} \tag{3.30}$$

and from (3.28) one obtains the high temperature behavior for $\sigma_w(T)$

$$\sigma_w(T) = \Delta\varepsilon_w + \frac{T^2}{4}\left[5g^2 + \frac{1}{3}\left[26\,a + \frac{282}{15}\,b\right]\right]\int dx\left[\bar{\phi}^2_w(x) - \phi^2_v\right] \tag{3.31}$$

The substitution of (3.22) into (3.31) and from (3.23) leads to

$$\sigma_w(T) = \frac{(2\mu^2)^{3/2}}{3\lambda} - \frac{T^2}{12}\frac{\mu\sqrt{2}}{\lambda}\left[26\,a + \frac{282}{15}\,b + 5g^2\right] \tag{3.32}$$

with $\lambda = a + \frac{7}{15}\,b$.

From the expression for $\sigma_w(T)$ and from the condition (2.10) we obtain the critical temperature T_c :

$$(T_c)^2 = \frac{60\mu^2}{225/2\; g^2 + 13(15a + 7b) + 50b}\ . \tag{3.33}$$

From (3.33) and (3.32) one can also write $\sigma_w(T)$ as

$$\sigma_w(T) = \sigma(0)\left[1 - \frac{T^2}{T_c^2}\right] \tag{3.34}$$

where $\sigma(0) = \frac{(2\mu^2)^{3/2}}{3\lambda}$. This is the result predicted in (2.11), where $\sigma(0)$ and T_c depends on the parameters (masses and coupling constants) of the SU(5) model.

3.3. Semiclassical Approach

Within the zero loop level or classical level one has

$$\Gamma_0(\phi) = \int d^3\vec{x}\; S_{cl}(\phi,\partial_\mu\phi) \tag{3.35}$$

where $S_{cl}(\varphi,\partial_\mu\phi)$ is the classical actions.

The condition (3.9)

$$\left.\frac{\delta\,\Gamma_0(\phi)}{\delta\phi}\right|_{\phi=\phi_c} = 0 \qquad (3.36)$$

leads to the classical Euler–Lagrange equation

$$\Box\phi_c - V'(\phi_c) = 0 \ . \qquad (3.37)$$

At the zero loop level the cost in energy is given by

$$F = -\beta^{-2} \ln Z \ . \qquad (3.38)$$

Supposing that the system under study is described by a scalar field ϕ one can write a path integral representation for Z

$$Z = \int [D\phi] \exp[-S(\phi)] \qquad (3.39)$$

where S is the effective action of the field ϕ.

$$S(\phi) = \int dx \left[1/2\, (\partial_\mu\varphi)^2 + V(\phi) \right] \qquad (3.40)$$

that is, $S(\phi)$ is replaced by the first term of its low momentum expansion[9].

The approximation (3.40) is very good in the high temperatures limt since the leading terms (in T) of S are exactly the ones that come from the zero momentum terms of $S(\phi)$ [10].

In the semiclassical limit, the leading contributions to Z, given by (3.39) and (3.40), come from the field configurations which minimize the effective action and therefore obey the Euler–Lagrange equation (3.37).

Now one makes a functional Taylor expansion of $S(\phi)$ around ϕ_c and keeps only the quadratic terms in $\eta = \varphi - \phi_c$

$$Z^{(1)} = e^{-S(\phi_c)} \int [D\eta] \exp\left[-\int_0^\beta d\tau \int d^3x \left[\frac{1}{2}(\partial_\mu\eta)^2 + \frac{1}{2}\,\eta\, V''(\phi_c)\,\eta \right] \right] . \qquad (3.41)$$

The gaussian integral in (3.41) is easy to perform and one gets formally

$$Z^{(1)} = e^{-S(\phi_c)} \det{}^{-1/2} \left[-\Box_{Eucl} + V''(\phi_c) \right] \ . \qquad (3.42)$$

This expression gives the contribution of just one bounce solution.

Using the dilute gas approximation one obtains

$$Z = Z^{(0)} \exp\left[\frac{Z^{(1)}}{Z^{(0)}} \right] \qquad (3.43)$$

where

$$Z^{(0)} = e^{-S(\phi_c)} \det{}^{-1/2} \left[-\Box_{Eucl} + V''(\phi_v) \right] \qquad (3.44)$$

and ϕ_v is the vacuum of the theory.

By (3.37) one obtains, by treating separately the zero eigenvalues[11]:

$$F = -T\left[\frac{S(\phi_c)}{2\pi}\right]^{\gamma/2}\left[\frac{\det'(-\Box_{Eucl}+V''(\phi_c))}{\det(-\Box_{Eucl}+V''(\phi_v))}\right]^{-1/2} e^{-S(\phi_c)} \qquad (3.45)$$

where the prime indicates that the zero eigenvalues of $-\Box_{Eucl}+V''(\phi_c)$ must be omitted from the determinant and γ is the number of these eigenvalues, which in tridimensional theories is three.

Label Λ the ratio of determinants which appears in (3.45). We shall develop a formal expansion for Λ that will be useful in order to extract it dependence on T at high temperatures. Λ can be written as

$$\Lambda = \exp\left\{-\tfrac{1}{2}\left[\mathrm{Tr}'\ln\left(-\Box_{Eucl}+V''(\phi_c)\right)-\mathrm{Tr}\ln\left(-\Box_{Eucl}+V''(\phi_v)\right)\right]\right\} \qquad (3.46)$$

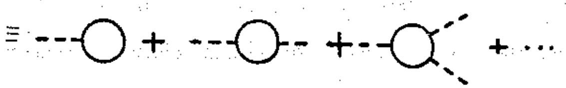
that can be put in the form

$$\Lambda = \exp\left\{-\tfrac{1}{2}\left[\mathrm{Tr}\ln\left[1+G_\beta\left(V''(\phi_c)-V''(\phi_v)\right)\right]\right]\right\} \qquad (3.47)$$

where $G_\beta = 1/\left[-\Box_{Eucl}+V''(\phi_v)\right]$ is just the free propagator at finite temperature, with mass $\sqrt{V''(\phi_v)}$.

If we expand the ln above in powers of $G_\beta\left[V''(\phi_c)-V''(\phi_v)\right]$, we get formally

$$\mathrm{tr}\ln\left\{1+G_\beta\left[V''(\phi_c)-V''(\phi_v)\right]\right\} \equiv$$



(3.48)

where the dashed lines correspond to the "*background field*" $\left(V''(\phi_c)-V''(\phi_v)\right)$, and the internal lines denote propagators G_β.

As in the section (3.2), one can isolate (in the high T limit, $\beta\to 0$) the terms which have higher superficial degree of divergence and then the contribution with leading power in T. These contribution is just the first term of the series (3.48). Then, we have

$$\Lambda = \exp\left\{-\tfrac{1}{2}\mathrm{Tr}\left[\frac{1}{-\Box_{Eucl}+V''(\phi_v)}\left(V''(\phi_c)-V''(\phi_v)\right)\right]\right\} \qquad (3.49)$$

for $\beta\to 0$ $(T\to\infty)$.

We can develop more the exponent in (3.49) so that, in (3+1) dimensional space one can write

$$\Lambda = \exp\left\{-\frac{1}{2}\frac{1}{\beta}\sum_{n=-\infty}^{+\infty}\int\frac{d^3K}{(2\pi)^3}\,\frac{1}{(2\pi\ n/\beta)^2+K^2+V''(\phi_v)}\int_0^\beta d\tau\int d^3\vec{x}\left[V''(\phi_c)-V''(\phi_v)\right]\right\} \qquad (3.50)$$

where we have used the usual representation for the trace and the Feynmann rules at finite temperature to the first graph in (3.48). Performing the n summation and taking into account a static classical field ϕ_c one obtains, for a renormalizable theory (and making the argument of the exponent in (3.50) free of divergences), that Λ can be written as

$$\Lambda = \exp\left\{-\frac{1}{2}\beta\int d^3\vec{x}\left[V''(\phi_c)-V''(\phi_v)\right]\int\frac{d^3K}{(2\pi)^3}\times\right.$$

$$\left.\times\frac{1}{\sqrt{K^2+V''(\phi_v)}\left(e^{\beta\sqrt{K^2+V''(\phi_v)}}-1\right)}\right\}. \qquad (3.51)$$

In the high temperature limit, Λ behaves as

$$\Lambda \sim \exp\{-AT\} \tag{3.52}$$

where

$$A = \frac{1}{2}\int d^3\vec{x}\left[V''(\phi_c) - V''(\phi_v)\right]\int \frac{d^3K}{(2\pi)^3}\frac{1}{K(e^K-1)} \tag{3.53}$$

Therefore (3.45) in the high temperature limit becomes

$$F \simeq -T\left[\frac{S(\phi_c)}{2\pi}\right]^{\gamma/2}\left[\frac{1}{\beta}\right]^{\gamma}\exp\left[-S(\phi_c,T)\right] \tag{3.54}$$

where $S(\phi_c,T) = S(\phi_c) + AT$, with AT being the result (3.52) for Λ, which is the quantum correction to $S(\phi_c)$, given to it a temperature dependence term. The factor $(1/\beta)^\gamma$ comes from a careful manipulation of (3.46) when on excludes the zero eigenvalues of the determinant.

In the nexts sections we will use this expression to obtain the total free energy for a specific field configuration ϕ_c, describing a spherical bubble.

IV. FIELD THEORETICAL DESCRIPTION OF CONDENSATION

In this section we will show how the evaluation of the partition function for a collection of noninteracting droplets may lead to the thermodynamic properties of a condensing system and the derivation of macroscopic features of a two phase system. This is the so called condensation problem.

For a nondegenerate system one can treated the condensation problem by using the droplet picture of phase transitions. The basic idea of the droplet picture, suggested as early as 1939[12] is that a transition from a phase A to a phase B might be preceeded by the formation of small nuclei of the phase B within A. The droplet model in field theory at zero temperature has been developed earlier by J.S. Langer[2] as a statistical theory of the condensation phenomenon.

Under the hypothesis of noninteracting bubbles, the thermodynamics of the system can be derived from the knowledge of the partition function. For a system of n particles, one can write the partition function, $Z_\ell(T)$, associated to an isolated cluster of ℓ particles moving in the volume V.

Within the droplet picture, and this is the basic assumption of the model, $Z_\ell(T)$ is written phenomenologically as[13]

$$\frac{Z_\ell(T)}{V} = q_0\,\ell^{-\tau}\,\mathrm{exo}\,a_0\,\ell^{\gamma}[(W-\omega t)/T]\,\exp\,\ell\left[\frac{F_V}{K_BT}\right] \tag{4.1}$$

where $q_0\,\ell^{-\tau}$ is a geometric term, whereas the other terms represents the surface and bulk contribution to the free energy. The surface term has a contribution associated to the surface energy W and a surface entropy ω associated to the wiggles of the surface. $a_0\,\ell^\gamma$ is the effective surface. q_0, τ and γ are phenomenological parameters. F_V/K_BT is the bulk contribution to the free energy.

Expression analogous to (4.1) has been already obtained, within the context of field theory, for phase transition in which the system goes to a metastable phase (the vacuum is metastable). That is, phase transitions for which there is a difference in energy density of the vacua of the theory (the true one and the false, the one in which the system is trapped). In this context it is possible, in semiclassical approximations, to identily all the elements present in (4.1). In fact, the classical action $S_{cl}(R)$ associated to a bounce solution, describing a bubble of radius R, can be cast (in three dimension) under the form

$$S_{cl}(R) = -\frac{4\pi}{3} R^3 \Delta\Gamma + 4\pi R^2 \sigma \tag{4.2}$$

where $\Delta\Gamma$ is just the difference in energy density between the vaccum states and σ is the surface tension. The first term thus represents the bulk contribution (volume energy) and the second one represents the surface energy.

An analogous term to the geometric one can be obtained only within the one–loop approximation[11]. Taking into account just the zero modes we have a pre–exponential term that goes like $S^{\gamma/2}$, where γ is the number of zero modes[11].

The droplet model pictures the system as a "*dilute gas*" of small droplets of radius R. The number of bubbles of size R might be approximated by a simple Boltzman factor, that is

$$N(R) \sim \exp\left\{-\beta\Delta F(R)\right\} \tag{4.3}$$

where $\Delta F(R)$ is the energy cost for introducing a bubble in the system.

As shown in previous sections, the cost in energy for introducing an interface in the system can be defined by (3.7)

$$\Delta F = F(\phi_B) - F(\phi_v) = -\beta^{-1} \ln\left[\frac{Z(\phi_B)}{Z(\phi_v)}\right] . \tag{4.4}$$

For spherical bubbles of radius $R, \Delta F$ is a function for R, and one can write $\Delta F \equiv \Delta F(R)$.

Only bubbles whose size R is above a critical value R_{cr} are stable and they survive in the system. This critical value is given by the condition

$$\left.\frac{d\Delta F(R)}{dR}\right|_{R=R_{cr}} = 0 . \tag{4.5}$$

Bubbles with radius smaller than R_{cr} are unstable and disappear again. These bubbles are assumed to be macroscopic objects.

The value $R = R_{cr}$ determined by (4.5) corresponds to the limit beyond which large quantities of the new phase begin to be formed. Bubbles beyond the critical range (with $R > R_{cr}$) will inevitably develop into a new phase.

For nondegenerate system one can picture the condensation process as a two stage process.

In the first stage (metastable phase) the system is metastable. In this stage there is formation of bubbles with radius below the critical one.

In the second stage (condensed phase) there is the growing of the critical droplets. Bubbles of size larger than the critical one developed and become stable.

Within the one–loop approximation and for temperature below, but close to the critical one, one can write

$$\Delta F(R) = -\frac{4\pi}{3} R^3 \left[\Gamma(\bar{\varphi}_{out},T) - \Gamma(\bar{\varphi}_{in},T)\right] + 4\pi R^2 \sigma(T) \tag{4.6}$$

where $\bar{\phi}_{out}$ is the local minimum, of the potential $V(\phi)$, which dominates the region outside the bubble and $\bar{\phi}_{in}$ is the global minimum which dominates the inside. In this case the solution which interpolates between these two minima is the kink–like solution $\phi_k(\vec{x})$ and it describes the surface of the bubble. $\sigma(T)$ in (4.6) is the surface tension.

The result (4.6) above can be seen when one uses (3.54) for a nondegenerate system and replacing ϕ_c by ϕ_B, representing a spherical bubble of radius R. ϕ_B consistes of: $\bar{\phi}_{in}$, for $R < R_{cr} - \Delta R$; ϕ_k, for $R \in (R_{cr} - \Delta R, R_{cr} + \Delta R)$ and $\bar{\phi}_{out}$, for $R > R_{cr} + \Delta R$. Then one can divide the integral of the classical action[11] in three regions: the inside of the bubble, the skin of the bubble, and the outside of the bubble. In the thin wall approximation, that is, $\Delta R \ll R_{cr}$ and using the result (3.52) (in the case of a nondegenerate system) as the temperature corrections to the classical action, (4.6) gives a gut approximate description to the bubble action[14]. From (4.6) one obtain that R_{cr} is given by

$$R_{cr}(T) = \frac{2\,\sigma(T)}{\Delta\Gamma(T)} . \tag{4.7}$$

As an example (of a nondegenerate system) one can consider the Hamiltonian density for a scalar field theory given by

$$\mathcal{H} = \frac{1}{2}\,\pi^2(\vec{x},t) + \frac{1}{2}\,[\vec{\nabla}\phi(\vec{x},t)]^2 + \frac{1}{2}\,m^2\,\phi^2(\vec{x},t) + \left(\frac{\lambda}{4!}\right)\phi^4(\vec{x},t) + j\phi(\vec{x},t) + \frac{3}{2}\,m^4/\lambda \tag{4.8}$$

where j is an external current assumed to be time and position independent.

Following the ideas of Langer[2], metastability arises if we consider what happens as we vary the value of the external current j, for suitable how temperatures.

A simple analysis of the classical potential shows that at the vicinity of $j = 0$, one can have two minima, one local and the other global. The semiclassical correction around each minimum brings the temperature into the problem and leads to a two–phase picture of the system: large regions dominated by the global minimum configuration where, due to thermal fluctuations, there occur bubbles (or droplets) dominated by the local minimum[14].

From (4.7) one can make contact with the phase transition (second–order one) that takes place as $j \to 0$ and $T \to T_c$ by remarking that, at the transition temperature, the critical radius $R_{cr}(T_c)$ should vanish (by (4.7) one can see that at $T = T_c$, with $R_{cr}(T_c) = 0$, imply that $\sigma(T_c) = 0$, signing the phase transition).

4.4. The Total Free Energy of Bubbles – Dilute Gas Approximation

In this section we will be concerned with situations in which the difference between the energy of the degenerate vacua is zero. That is there is no external source to drive one of them, preferred, energetically, with regard to the other. Under this circumstances $\Delta\Gamma$ in (4.2) is zero. In this way, the deformation of the radius of critical bubble, for instance, cannot be done only by using (4.2). So that one has to adopt the droplet picture in field theory. This is precisely what we intend to do here.

Within the droplet model picture symmetry restoration occurs as a result of formation of droplets along the volume in which inside the droplet there is a region in which there lives a vacuum of the other phase. Close to the critical temperature these bubbles are more numerous and larger. In fact, at the critical temperature bubbles with infinite radius are favored to appear in the system. That is why we will be concerned in this paper with infinite domain wall ("magic carpets" in Fisher words)[13].

Besides implementing, under these circumstances, the droplet picture of phase transition in field theory, we will derive expressions for physical quantities which are relevant for phenomenological applications in Cosmology. In this context we have concentrated our attention on the question of contrast density and the size of bubbles.

We will assume that the distribution of bubbles is a dilute one. Under these circumstances one can write, for the partition function Z, equation (3.43), as

$$\frac{Z}{Z^{(0)}} = \exp\left[\frac{Z^{(1)}}{Z^{(0)}}\right] \tag{4.9}$$

where $Z^{(0)}$ now stands for the partition function in the vacuum field configuration, ϕ_v, and $Z^{(1)}$ in the bubble field configuration ϕ_B.

From the results of section (3.3), one can write the free energy of the bubble, $F = -\beta^{-1} \ln Z$, by equation (3.54) with ϕ_c replaced by ϕ_B. In the high temperature limit and considering spherical bubbles one can find a general form to F given by

$$F = -T\left[\frac{-4\pi/3\ R^3\ \Delta\Gamma + 4\pi\ R^2\ \sigma(0)}{2\pi\ T}\right]^{3/2}\left[\frac{1}{\beta}\right]^3 \exp\left[\frac{4\pi/3\ R^3\ \Delta\Gamma(T) - 4\pi\ R^2\sigma(T)}{T}\right] \tag{4.10}$$

where we have used (4.2) to $S_{cl}(\phi_B)$ and (4.6) to $S(\phi_B,T)$, the classical action associated to the bubble and $\sigma(T)$ is the surface tension.

In (4.10) we have considered the factor γ appearing in (3.50) as being three (for bubbles in three spatial dimensions there are three translational zero modes and therefore three zero eigenvalues).

Let one takes the vacua as a degenerate one, i.e., $\Delta\Gamma$ in (4.10) is equal zero. In these situation (4.10) becomes

$$F = -T^4\left[\frac{4\pi\ R^2\ \sigma(0)}{2\pi\ T}\right]^{3/2} \exp\left[\frac{-4\pi\ R^2\ \sigma(T)}{T}\right] \tag{4.11}$$

The critical radius of the bubble can be obtained by minimizing the free energy (4.11) and one obtains

$$R_{cr}^2(T) = \frac{3T}{8\pi\ \sigma(T)} \tag{4.12}$$

From this expression for $R_{cr}(T)$, one can see that for $T = T_c$ the bubble radius becomes infinite. This is the principal difference between our point of view from the usual one considering nondegenerate vacua where the critical radius is given by (4.7).

If one expands the expression (4.11) for $R \sim R_{cr}$ and writing $\sigma(T)$ in terms of R_{cr} using (4.12), one obtains that free energy F can be written as

$$F \underset{R\sim R_{cr}}{\simeq} -T^{5/2}\left[\frac{2\sigma(0)}{e}\right]^{3/2} R_{cr}^3 \exp\left[-3\,\frac{(R - R_{cr})^2}{R_{cr}^2}\right] \tag{4.13}$$

for $R = R_{cr}$ given by (4.12), one obtains the contribution to the free energy as

$$F = -T^4\left[\frac{3}{4\pi e}\ \frac{\sigma(0)}{\sigma(T)}\right]^{3/2} . \tag{4.14}$$

Within the dilute gas approximation the average number of bubbles is

$$N(T) = \frac{Z^{(1)}}{Z^{(0)}} \tag{4.15}$$

and then from (4.13) we have that for $R \sim R_{cr}$

$$N(T,R) \underset{R\sim R_{cr}}{\simeq} V \left[\frac{2T\sigma(0)}{e}\right]^{3/2} R_{cr}^3 \exp\left[-3\frac{(R-R_{cr})^2}{R_{cr}^2}\right] . \qquad (4.16)$$

The energy density associated to the bubbles whose average number is $N(T,R)$ and with radius between R and $R+dR$ is

$$d\rho_{bubble} = \left[\frac{2T\sigma(0)}{e}\right]^{3/2} R_{cr}^3 \exp\left[-3\frac{(R-R_{cr})^2}{R_{cr}^2}\right] 4\pi R\, dR\, \sigma(0) . \qquad (4.17)$$

If one excludes those bubbles with radius less than R_{cr}, which are energetically unfavourables, one can integrate the expression above from R_{cr} to infinity and one obtains

$$\rho_{bubble} = \frac{(1+\sqrt{3\pi})}{4}\left[\frac{3}{4\pi e}\right]^{3/2}\left[\frac{\sigma(0)}{\sigma(T)}\right]^{5/2} T^4 \qquad (4.18)$$

where one uses the expression (4.12) for R_{cr} in (4.18).

The contrast density associated to bubbles is defined as

$$\frac{\delta\rho}{\rho} = \frac{\rho_{bubbles}}{\rho_{elem.part.} + \rho_{bubbles}} \qquad (4.19)$$

where $\rho_{elem.part.}$ is the energy density associated to the elementary particles and it can be written in terms of the number of degrees of freedom fermionic (N_F) and bosonic (N_B) as

$$\rho_{elem.part.} = \frac{\pi^2}{30}\left(N_B + \frac{7}{8}N_F\right) T^4 . \qquad (4.20)$$

Then from the expressions above one can see that all one need to know is the form of $\sigma(T)$, the surface tension, to determine all the quantities of interest.

From the results of the last sections, one can write $\sigma(T)$, in the one loop order and in the high temperature approximation, in the general form given by (3.34)

$$\sigma(T) = \sigma(0)\left[1 - \frac{T^2}{T_c^2}\right] \qquad (4.21)$$

where $\sigma(0)$ and T_c depends on the parameters (masses and coupling constants) of the model that we want to consider.

Then by (4.18) and (4.20) one can write the contrast density (4.21), by taking $\sigma(T)$ given by (4.21), as

$$\frac{\delta\rho}{\rho} = \frac{1}{1 + \pi^2/30\,\left(N_B + 7/8\,N_F\right)\frac{4}{(1+\sqrt{3\pi})}\left[\frac{4\pi e}{3}\right]^{3/2}\left[1 - \frac{T^2}{T_c^2}\right]^{5/2}} . \qquad (4.22)$$

Furthermore, taking $T < T_c$, but not much more less than T_c, one obtains the simple result

$$\frac{\delta\rho}{\rho} \simeq \frac{1}{1 + 25/2\,\left(N_B + 7/8\,N_F\right)} \qquad (4.23)$$

which is completely general.

V. APPLICATIONS TO COSMOLOGY

5.1. Formation of Structures in the Early Universe

In this section we will analyse the possibility that the defects that we have studied throughout this paper (domain walls and bubbles) have a bearing on the question of formation of structures in the Universe. That is, we propose that these defects work as structure seeds. The following conditions must be fullfiled through in order that this propositions be consistent:

1. The structures that act as seeds should not dissipate until recombination.
2. The magnitude of the primordial density fluctuations should satisfy Zel'dovich's condition[15], that is

$$\frac{\delta\rho}{\rho} \lesssim 10^{-4} \ . \tag{5.1}$$

3. The length of fluctuations must be larger than Jeans length, so as to enable it to trigger the gravitation modes when recombination occur.

As far as walls are concerned they do not obey condition (1) since they are not supposed to exist for temperatures below the temperature for which $\sigma(T_c) = 0$. The thermodynamical argument for preventing the appearance of such topological structures is that they will require, for low temperatures, an infinite amount of energy.

Since domain walls exists however for temperatures above the critical one and, in fact, their presence is intimately connected with the picture sketched in this paper one obviously need to check if their presence in the system would lead to unnacceptable, from the cosmological point of view, contrast densities. We will check below that this is not so within the context of SU(5) model.

In the minimum SU(5) model, with Lagrangean density described by (3.19), with (3.20), the vacuum field configuration Φ_v and the bubble field configuration Φ_B can be

written as

$$\Phi_v = \frac{\mu}{\sqrt{\lambda}}\sqrt{\frac{2}{15}}\begin{bmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & -3/2 & \\ 0 & & & & -3/2 \end{bmatrix} = \frac{\Phi_{24}}{\sqrt{2}}\lambda_{24} \tag{5.2}$$

and

$$\Phi_B = \frac{\mu}{\sqrt{\lambda}}\sqrt{\frac{2}{15}}\tanh\frac{\mu}{\sqrt{2}}\left[\frac{|\vec{x} - \vec{x}_0(t)| - R}{\sqrt{1-v^2}}\right]\begin{bmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & -3/2 & \\ 0 & & & & -3/2 \end{bmatrix} \tag{5.3}$$

with $\lambda = a + 7/15b$. Φ_B describe a spherical bubbles with radius R, and Φ_B can be think as a generalization of the domain wall field configuration given by (3.22).

From the results of section (4.4), one can find the free energy of the bubble equation (4.14), by considering the limit of large R (or in other words, when $T \sim T_c$) and then one can approximate the bubble configuration by a domain wall configuration, and using the results of section (3.2) for the domain wall free energy for the minimum SU(5) model, one can write the expression for F in the high temperature limit (and for spherical bubbles at rest) as

$$F = -T^4\left[\frac{3}{4\pi e}\right]^{3/2}\left[1 - \frac{T^2}{T_c^2}\right]^{-3/2} \tag{5.4}$$

where we have used the expression (4.21) for $\sigma(T)$

$$\sigma(T) = \frac{2\sqrt{2}\,\mu^3}{3\lambda} - \frac{T^2}{12}\frac{\mu\sqrt{2}}{\lambda}\left[26a + \frac{282b}{15} + 15g^2\right] = \frac{2\sqrt{2}\,\mu^3}{3\lambda}\left[1 - \frac{T^2}{T_c^2}\right] \quad (5.5)$$

By (4.21) one can see here that $\sigma(0) = \frac{2\sqrt{2}\,\mu^3}{3\lambda}$. T_c^2 is given by (3.33)

$$T_c^2 = \frac{60\mu^2}{\frac{225}{2}\,g^2 + 13(15a + 7b) + 50b} \quad (5.6)$$

By (4.12), one can write the critical radius R_{cr} as

$$R_{cr}^2(T) = \frac{9\lambda T}{16\pi\sqrt{2}\,\mu^3}\left[1 - \frac{T^2}{T_c^2}\right]^{-1} \quad (5.7)$$

From (5.7), one can see that for $T = T_c$ the critical radius becomes infinite, which means that at this temperature the bubbles wall in fact becomes a plane domain wall. Within this context it seems appropriate to identify T_c in (5.7) as the critical temperature of the theory. However, although this consistency holds in our scheme, it can be easily argued that a dilute gas approximation will no longer be valid for temperatures close to T_c. One expects then, that the critical temperature in indeed lower than T_c given by (5.6).

Let us turn now to the computation of the contrast density to bubbles, defined by (4.22).

$$\frac{\delta\rho}{\rho} = \frac{1}{1 + \frac{\pi^2}{30}\left[N_B + \frac{7}{8}N_F\right]\left[1 - \frac{T^2}{T_c^2}\right]^{5/2}\frac{4}{1+\sqrt{3\pi}}\left[\frac{4\pi e}{3}\right]^{3/2}} \quad (5.6)$$

For T below T_c and for $N_B + 7/8\,N_F \gg 1$ the contrast density is small and can be approximated by

$$\frac{\delta\rho}{\rho} \simeq \frac{1}{\frac{\pi^2}{30}\left[N_B + \frac{7}{8}N_F\right]\left[1 - \frac{T^2}{T_c^2}\right]^{5/2}\frac{4}{1+\sqrt{3\pi}}\left[\frac{4\pi e}{3}\right]^{3/2}} \quad (5.9)$$

In the minimal SU(5) model, $N_B + 7/8\,N_F = 160{,}75$, so that for $T \sim T_c/3$ one gets

$$\frac{\delta\rho}{\rho} \sim 6.10^{-4} \quad (5.10)$$

This result is compatible with the bounds imposed by the anisotropy of the background radiation ($\delta\rho/\rho$ satisfy Zel'dovich's condition).

Let us analyze if condition (3) can be met in this picture. The lenght of fluctuations that we propose here is essentially the distance between two bubbles. Unfortunately we are not able to compute this distance, by using thermodynamical arguments, for the range of temperatures covering the critical temperture (10^{15} GeV) untill recombination (1 eV). We can do this however, for temperature close to the critical one. For this range of temperatures, one has that if the average number of bubbles is given by (4.16) with $R = R_{cr}$ and using (5.5), its density will be given by

$$\bar{n} = \left[\frac{3T^2}{4\pi e}\right]^{3/2}\frac{1}{\left[1 - \frac{T^2}{T_c^2}\right]^{3/2}} \quad (5.11)$$

If one assume further that the bubbles are uniformly distributed over the space the (average) distance between two bubbles (theirs centers) will be given by

$$d = \frac{1}{\sqrt[3]{\bar{n}}} \quad . \tag{5.12}$$

From (5.11) and (5.12) one gets

$$d(T) = \frac{\left[1 - \frac{T^2}{T_c^2}\right]^{1/2}}{T\left[\frac{3}{4\pi e}\right]^{1/2}} \quad . \tag{5.13}$$

For $T \simeq T_c/3$ ($T_c \sim 10^{15}$ GeV) one than has

$$d^{GUT} \sim 8.2 \times 10^{-5}\ \text{GeV}^{-1} \simeq 10^{-28}\ \text{cm} \quad . \tag{5.14}$$

In order to estimate the lenght of fluctuation in the recombination era, one just makes the hypothesis that the distances between bubbles (λ^B) expands conformally, that is, the ratio between this distance and the horizon distance is constant. Consequently at any time one has

$$\lambda^B(T) = \frac{d^{GUT}}{d_H(0, t_{GUT})}\, d_H(0,t) \quad . \tag{5.15}$$

So that during recombination ($t = t_R$) one has, by using (5.14)

$$\lambda^B(T \simeq 1\ \text{eV}) = \frac{d_0^{GUT}}{d_H(0, 2 \times 10^{-37}\ \text{s})}\, d_H(0, t_R)$$

$$\lambda^B \sim 1.2 \times 10^{21}\ \text{cm} \quad . \tag{5.16}$$

Since the Jeans length at recombination is

$$\lambda_J(t_R) \simeq 2.9 \times 10^{19}\ \text{cm} \tag{5.17}$$

it follows from (5.16) that $\lambda^B > \lambda_J$.

The mass associated to the distance is

$$M^{bubb.} = \frac{4\pi}{3}\rho_{rec}(\lambda^B)^3 \sim 10^{10}\ M_\odot \tag{5.18}$$

which fits very well in the galactical mass spectrum and is probably consistent with all of them if the dynamics of the bubbles below T_c is considered.

A legitimate conclusion would be that the number of aglutination centers is roughly the number of great structures observed in the Universe today. In fact, one can estimate the number of aglutination centers. This number is roughly given by

$$n_{agl.cent.} \simeq \left[\frac{d_H(0, t^{GUT})}{d_0^{GUT}}\right]^3 \simeq 1{,}9 \times 10^6 \quad . \tag{5.19}$$

The greatest known structures are the superclusters of galaxies that consist of groups with an average of 10^5 galaxies, that have densities close to critical $\rho_c \sim 10^{-29}\text{g cm}^{-3}$ and spread over dimensions from 50 to 100 Mpcs (from 1.5 to 3.0 × 10^{26} cm). The number of these structures (sub–clusters) may be estimated by the ratio

$$n_{sc} \simeq \left[\frac{d_H(0,t_p)}{d_{sc}}\right]^3 \simeq 7\cdot 10^5 - 6\cdot 10^6 \qquad (5.20)$$

because $t_p \sim 10^{10}$ years and $d_H(0,t_p) = 3t_p \simeq 2.7 \times 10^{18}$ cm .

We see that the results from (5.19) and (5.20) are quite close to each other.

VI. CONCLUSIONS

Bubbles might appear in cosmological phase transitions for theories with nondegenerate or degenerate vacua. In both cases one can predict phase coexistence in the Universe and the appearence of bubbles as a result of thermal fluctuations. The basic ingredient for making relevant predictions to cosmology is the cost in energy to introduce such an object in the system.

Within the droplet picture of phase transitions, and admitting a dilute gas of droplets, the free energy of a collection of bubbles with radius R can be written as[2,3]

$$\mathcal{F} \sim \int dR\, F^{(1)}(R)$$

where $F^{(1)}(R)$ is the free energy associated to a single bubble of radius R and this free energy is the cost in energy for introducing a single bubble in the system.

In this paper we have proposed an extension of the droplet picture of phase transitions in field theory that allows us to get estimates of the critical radius, their dependence with temperature and the contrast density due to bubbles. The droplet picture has been applied, in field theory, to the description of phase transition in which the system goes through a metastable phase[2]. These situations are characterized by the existence, at least for a certain range of temperatures, of nondegenerate vacua[17]. In the case of theories with nondegenerate vacua expression (4.10) permit us to make better estimates of critical radius than the usual "classical theory"[3] since this expression takes into account the translational modes as well as the temperature dependence of the surface tension.

When the theory exhibits degenerate vacua as a result of a discrete symmetry, and as has been suggested in the literature[7,16,18] the phase transition is supposed to be Ising–like. In this paper we have shown how the droplet picture can be applied in these circumstances.

For temperatures no too different from the critical one we have been able to compute the radius and the average number of bubbles within the dilute gas approximation. The expressions obtained depends on the knowledge of $\sigma(T)$, the surface tension. The surface tension is for us, therefore, the fundamental quantity to be determined in field theory at finite temperature.

Whereas in the nondegenerate case the radius of critical bubbles tends, formally, to zero at the critical temperature, in the degenerate case the critical bubbles tends to infinite. This, on the other hand, implies that only for temperatures above the critical one condensation of domain walls takes place and consequently above this temperature there will be formation of domain walls[19], below this temperature domain walls are not favourable.

As an application to cosmology we have analysed the GUT phase transition in the minimal SU(5) model. In this application we have assumed that these bubbles survive until the recombination era. This is a dynamical problem that one has to solve in order to be sure that these objects acts as seeds for structure formation. Our simple estimates based only upon the interbubble distance indicates that one might get a surprisingly good picture for the formation of structures in the Universe.

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