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# PUBLICAÇÕES

IFUSP/P-940

**THE NUCLEAR TWO-NEUTRON REMOVAL CROSS-SECTION OF  $^{11}\text{Li}$ : A DYNAMIC POLARIZATION POTENTIAL APPROACH**

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Outubro/1991

THE NUCLEAR TWO-NEUTRON REMOVAL CROSS-SECTION OF  $^{11}\text{Li}$ :  
A DYNAMIC POLARIZATION POTENTIAL APPROACH

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Abstract

We determine the dynamic polarization potential associated to the two-neutron removal process from  $^{11}\text{Li}$ . This potential is then employed to calculate the nuclear break-up cross section of  $^{11}\text{Li}$  when it collides with different targets, as a function of its bombarding energy. An analytic expression for this cross section is obtained and shown to be in good agreement with the calculation based on the exact expression. We also discuss the relevance of this potential in the study of the fusion of  $^{11}\text{Li}$  with a heavy target

1. Introduction.

In the study of the structure of neutron-rich nuclei such as  $^{11}\text{Li}$  it is of paramount importance to have at hand a precise mean to determine the Coulomb dissociation cross section. Through this quantity it is possible to extract rather unambiguous information concerning the  $B(E1)$  (and  $B(E2)$ ) distribution. Recently, several models for the low energy response of  $^{11}\text{Li}$  have been developed<sup>1)</sup>. However, when confronted with data, none of these models seems to work well at low bombarding energy.<sup>2)</sup> We should clarify that by data we mean the *extracted* values of the Coulomb dissociation cross section  $\sigma_c$ , while what is actually available from experiment is the one associated to the total interaction. Therefore, in order to obtain  $\sigma_c$  one must subtract the calculated contribution from the nuclear field, making the usual assumption that the Coulomb-nuclear interference cross section is small. This naturally raises the question of how precise are the estimates of the nuclear contribution to the break-up process.

The purpose of the present paper is to develop a general theory of the  $^{11}\text{Li}$  nuclear two-neutron removal cross section when it collides with different targets. To attain this aim, first we derive the polarization potential for 2n-removal in section 2. In a recent paper<sup>3)</sup> this potential was evaluated using the Glauber method, valid at high energies. Here we extend the theory by incorporating Coulomb effects, which become increasingly important at lower collision energies. This potential is then used in section 3 in the derivation of an approximate closed expression for the nuclear 2n-removal cross section. In section 4 we apply the theory to  $^{11}\text{Li} + ^{12}\text{C}$  collisions for different collision energies. For this system  $\sigma_c$  is negligible, due to the small charges involved, so a comparison of our results with the available data is made easier. The extension of these calculations to the case of heavier targets is also discussed there. Finally, we present our main conclusions in section 5.

2. The 2n-removal polarization potential theory.

To calculate the 2n-removal polarization potential, we define the projection operators

$$P = |\phi_0\rangle\langle\phi_0|; \quad Q = 1 - P, \quad (1)$$

where  $\phi_0(\mathbf{x}) \equiv \phi_0(x)$  represents the bound state of the  $2n+^9\text{Li}$  system while  $Q$  is the projector onto states of the  $2n$  pair in the continuum. The polarization potential can then be written<sup>4)</sup>

$$V(\mathbf{r}, \mathbf{r}') = \langle \mathbf{r}; \phi_0 | v Q G_{QQ}^{(+)} Q v | \phi_0; \mathbf{r}' \rangle, \quad (2)$$

where  $v$  is the coupling interaction and  $G_{QQ}^{(+)}$  is the optical Green's function in the  $Q$ -subspace. In order to evaluate eq. (2), we write the projector  $Q$  in its spectral form

$$Q = \int |\phi_q \rangle \langle \phi_q| dq, \quad (3)$$

with  $q$  standing for the set of quantum numbers that characterize the continuum states.

If we now introduce representations in  $r$ -space and assume that the interaction  $v$  is local, we get

$$V(\mathbf{r}, \mathbf{r}') = \int \langle \phi_0 | v(\mathbf{r}) | \phi_q \rangle \langle \mathbf{r} | G^{(+)}(E - \varepsilon_q) | \mathbf{r}' \rangle \langle \phi_q | v(\mathbf{r}') | \phi_0 \rangle dq, \quad (4)$$

where  $\varepsilon_q$  is the energy associated to  $|\phi_q \rangle$ . Owing to the weak binding of state  $|\phi_0 \rangle$ , the matrix elements

$$\langle \phi_0 | v(\mathbf{r}) | \phi_q \rangle = \int \phi_0^*(\mathbf{x}) v(\mathbf{r}, \mathbf{x}) \phi_q(\mathbf{x}) d^3x \quad (5)$$

are negligible except in the case of states with low values \* of the energy  $\varepsilon_q$ , and thus we can safely approximate  $G^{(+)}(E - \varepsilon_q) \approx G^{(+)}(E)$  and consequently factor the Green's function out of the integrand in eq. (4).

Assuming  $\langle \phi_0 | v | \phi_0 \rangle = 0$  we can make use of the closure relations and write

$$V(\mathbf{r}, \mathbf{r}') = G^{(+)}(\mathbf{r}, \mathbf{r}') \int \phi_0^2(\mathbf{x}) v(\mathbf{r}, \mathbf{x}) v(\mathbf{r}', \mathbf{x}) d^3x. \quad (6)$$

\* On the basis of ref. 5) and using the fact that the radius of the neutron halo in  $^{11}\text{Li}$  is approximately 8 fm, one finds that only states with  $\varepsilon_q$  less than a couple of MeV make an appreciable contribution.

In order to evaluate  $V(\mathbf{r}, \mathbf{r}')$  we make use of the separable approximation for  $v(\mathbf{r}, \mathbf{x})$  introduced in ref. 5), namely

$$v(\mathbf{r}, \mathbf{x}) \approx U(r) u(x). \quad (7)$$

In this equation  $U(r)$  is the real part of the  $^{11}\text{Li}$ -target optical potential and  $u(x)$  is an internal excitation form factor. In this way we obtain

$$V(\mathbf{r}, \mathbf{r}') = \mathcal{F}(r) G^{(+)}(\mathbf{r}, \mathbf{r}') \mathcal{F}(r'), \quad (8)$$

where we have defined the form factor

$$\mathcal{F}(r) = U(r) \left[ \int \phi_0^2(x) u^2(x) dx \right]^{\frac{1}{2}}. \quad (9)$$

We now perform partial waves expansion for the Green's function,

$$\begin{aligned} G^{(+)}(\mathbf{r}, \mathbf{r}') &= \frac{1}{rr'} \sum Y_{\ell m}(\hat{r}) Y_{\ell m}^*(\hat{r}') \left[ -\frac{2\mu}{\hbar^2 k} f_{\ell}(kr_{<}) h_{\ell}^{(+)}(kr_{>}) \right] \\ &= \frac{1}{rr'} \sum Y_{\ell m}(\hat{r}) Y_{\ell m}^*(\hat{r}') G_{\ell}^{(+)}(r, r'). \end{aligned} \quad (10)$$

and for the potential,

$$V(\mathbf{r}, \mathbf{r}') = \frac{1}{rr'} \sum Y_{\ell m}(\hat{r}) Y_{\ell m}^*(\hat{r}') V_{\ell}(r, r'). \quad (11)$$

The  $\ell$ -components of the polarization potential are, therefore,

$$V_{\ell}(r, r') = \mathcal{F}(r) \left[ -\frac{2\mu}{\hbar^2 k} f_{\ell}(kr_{<}) h_{\ell}^{(+)}(kr_{>}) \right] \mathcal{F}(r'), \quad (12)$$

where  $f_{\ell}(kr_{<})$  and  $h_{\ell}^{(+)}(kr_{>})$  are respectively the regular and the outgoing solutions of the optical equation in the  $Q$ -space.

We follow Baltz *et al.*<sup>6)</sup> using the on-shell approximation for the Green's function ( $h_\ell^{(+)} \rightarrow i f_\ell$ ) and define the trivially equivalent local potential  $V_\ell^{pot}$ ,

$$V_\ell^{pot}(r) = -i \frac{2\mu}{\hbar^2 k} \mathcal{F}(r) \int_0^\infty \mathcal{F}(r') f_\ell^2(kr') dr'. \quad (13)$$

We now approximate  $f_\ell(kr) \simeq \sqrt{|S_\ell^{(1)}|} F_\ell(kr)$ , where  $S_\ell^{(1)}$  is the optical elastic S-matrix element in the Q-space and  $F_\ell(kr)$  is the regular Coulomb function. The polarization potential then becomes

$$V_\ell^{pot}(r) = -i \frac{2\mu}{\hbar^2 k} \mathcal{F}(r) |S_\ell^{(1)}| \int_0^\infty \mathcal{F}(r') F_\ell^2(kr') dr'. \quad (14)$$

In the r-region of interest for the break-up, only the tail of  $U(r)$  matters and accordingly the form factor can be written

$$\mathcal{F}(r) = \mathcal{F}_0 e^{-r/\alpha}, \quad (15)$$

where  $\mathcal{F}_0$  can be expressed as

$$\mathcal{F}_0 = C e^{R_0/\alpha}. \quad (16)$$

in which C is a constant related to the strength of the optical potential  $U(r)$  which can be trivially obtained from eq. (9),  $R_0 = R_{11Li} + R_{target}$ , and  $\alpha$  is the diffusivity associated to the optical potential  $U(r)$ . Replacing eq. (16) into eq. (13), the polarization potential takes the form

$$V_\ell^{pot}(r) = -i W_0(\ell, E) e^{-r/\alpha}. \quad (17)$$

The strength  $W_0(\ell, E)$  is given by

$$W_0(\ell, E) = \frac{|\mathcal{F}_0|^2}{E} |S_\ell^{(1)}| I_\ell(\eta, s), \quad (18)$$

in terms of the radial integral

$$I_\ell(\eta, s) = \int_0^\infty e^{-s\rho} F_\ell^2(\rho) d\rho. \quad (19)$$

Using the asymptotic WKB approximation for  $F_\ell(\rho)$ ,

$$F_\ell(\rho) \approx \left(1 - \frac{2\eta}{\rho} - \frac{\ell(\ell+1)}{\rho^2}\right)^{-1/4} \sin \left[ \frac{\pi}{4} + \int_{\rho_0}^{\rho} \sqrt{1 - \frac{2\eta}{\rho} - \frac{\ell(\ell+1)}{\rho^2}} d\rho \right], \quad (20)$$

where  $\eta$  is the Sommerfeld parameter,  $\rho \equiv kr$  and  $\rho_0$  is the value of  $\rho$  calculated at the turning point of the Rutherford trajectory, we obtain the approximate expression

$$I_\ell(\eta, s) = \frac{e^{-\eta s}}{2s} [\eta s K_0(X) + X K_1(X)]. \quad (21)$$

Above,  $K_0$  and  $K_1$  are modified Bessel functions and  $s$  and  $X$  are the variables

$$s = \frac{1}{k\alpha}; \quad X = \eta s \sqrt{1 + \frac{\ell(\ell+1)}{\eta^2}}. \quad (22)$$

The variable  $X$  measures the distance of closest approach in a Rutherford trajectory in units of  $\alpha$ . We note that at very high energies and for partial waves corresponding to distant collisions ( $\ell > kR_0$ ),  $\eta \rightarrow 0$  and  $S_\ell^{(1)} \simeq 1$ , and we obtain exactly the eikonal form of the polarization potential derived in ref. 3).

It is interesting to notice that the polarization potential can be put in a much simpler form if we use for  $I_\ell(\eta, s)$  the approximation

$$I_\ell(\eta, s) \simeq C_0 e^{-\gamma X}. \quad (23)$$

The constants  $C_0$  and  $\gamma$  are determined by fitting eq. (23) to the asymptotic forms of the Bessel functions  $K_0$  and  $K_1$  at some value  $\bar{X}$  larger than 1. Taking  $\bar{X} = 2$  we obtain

$$\gamma \approx 0.83; \quad C_0 \approx \frac{0.75}{s} \quad (24)$$

The accuracy of this approximation is illustrated in section 4, for the  $^{11}Li + ^{12}C$  system at several collision energies.

### 3. The nuclear two-neutron removal cross section.

We first remind the reader of the formal definition of the nuclear two-neutron removal cross section

$$\sigma^{bu} = \frac{k}{E} \langle \Psi_k^{(+)} | -Im\{V^{pol}\} | \Psi_k^{(+)} \rangle, \quad (25)$$

where  $|\Psi_k^{(+)}\rangle$  is the exact elastic scattering wave function and  $V^{pol}$  is the 2n-removal polarization potential, which accounts for the effect of the break-up process on the elastic scattering. If the total optical potential that generates  $|\Psi_k^{(+)}\rangle$  is denoted by  $U^{el}$  then the bare interaction that takes into account other channel-coupling effects is  $U^{opt} \equiv U^{el} - V^{pol}$ .

Performing partial waves expansion, we may write eq. (25) as

$$\sigma^{bu} = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell+1) T_{\ell}^{bu}, \quad (26)$$

where

$$T_{\ell}^{bu} = \frac{4k}{E} \int_0^{\infty} dr |f_{\ell}^{el}(kr)|^2 (-Im\{V_{\ell}^{pol}\}). \quad (27)$$

In the above equation  $f_{\ell}^{el}(kr)$  is the optical radial wave function which is the solution of the elastic-channel Schrödinger equation with  $U^{el}$ .

We now employ the same kind of approximation as that used in the derivation of eq. (14), namely  $f_{\ell}^{el}(kr) \approx |S_{\ell}^{(0)}|^{\frac{1}{2}} \hat{f}_{\ell}(kr)$ , where  $S_{\ell}^{(0)}$  is the nuclear elastic S-matrix calculated with  $U^{opt}$  and  $\hat{f}_{\ell}(kr)$  represents the scattering wave function generated by the potential  $V^{Coul} + V^{pol}$ . Approximating  $\hat{f}_{\ell}(kr)$ , by the properly normalized analytic extension of eq. (20), we obtain

$$\hat{f}_{\ell}(\rho) \approx N \left( 1 - \frac{2\eta}{\rho} - \frac{\ell(\ell+1)}{\rho^2} + i \frac{Im\{V_{\ell}^{pol}\}(\rho/k)}{E} \right)^{-1/4} \times \sin \left[ \frac{\pi}{4} + \int_{\rho_0}^{\rho} \sqrt{1 - \frac{2\eta}{\rho} - \frac{\ell(\ell+1)}{\rho^2} + i \frac{Im\{V_{\ell}^{pol}\}(\rho/k)}{E}} d\rho \right], \quad (28)$$

where the normalization factor N is given by

$$N^2 = \int_0^{\infty} dr |\hat{f}_{\ell}(kr)|^2 = \exp \left[ - \int_{\rho_0}^{\infty} d\rho \frac{Im\{V_{\ell}^{pol}\}/E}{\sqrt{1 - \frac{2\eta}{\rho} - \frac{\ell(\ell+1)}{\rho^2}}} \right] \equiv |\hat{S}_{\ell}|. \quad (29)$$

If we now expand  $|\hat{f}_{\ell}(\rho)|^2$  to lowest order in  $Im\{V_{\ell}^{pol}\}/E$ , substitute it in eq. (27) and perform the integration, we obtain

$$\begin{aligned} T_{\ell}^{bu} &= 2 |S_{\ell}^{(0)}| \cdot |\hat{S}_{\ell}| \cdot \sinh \left( \int_{\rho_0}^{\infty} d\rho \frac{Im\{V_{\ell}^{pol}\}/E}{\sqrt{1 - \frac{2\eta}{\rho} - \frac{\ell(\ell+1)}{\rho^2}}} \right) \\ &= \left[ 1 - \exp \left( -2 \int_{\rho_0}^{\infty} d\rho \frac{Im\{V_{\ell}^{pol}\}/E}{\sqrt{1 - \frac{2\eta}{\rho} - \frac{\ell(\ell+1)}{\rho^2}}} \right) \right] \cdot |S_{\ell}^{(0)}| \\ &= [1 - |\hat{S}_{\ell}|^2] \cdot |S_{\ell}^{(0)}|. \end{aligned} \quad (30)$$

Using the explicit form of  $V^{pol}$  (eqs.(17-21)) it is easy to show that  $|\hat{S}_{\ell}|$  can be written as

$$|\hat{S}_{\ell}| = \exp \left( -\frac{2F_0^2}{E^2} \cdot |S_{\ell}^{(1)}| \cdot I_{\ell}^2(\eta, s) \right). \quad (31)$$

At high energies it is safe to set  $\eta = 0$ , and for the distant collisions under consideration we may set  $|S_{\ell}^{(0)}| \simeq |S_{\ell}^{(1)}| \simeq 1$ . Under these conditions the latter approximation is valid because the nuclear break-up process almost completely dominates the reaction cross section. In this way the nuclear break-up cross section agrees with that found using the eikonal approximation (ref. 3)).

### 4. Applications.

In this section we apply our theory to the  $^{11}\text{Li} + ^{12}\text{C}$  case at several bombarding energies, and briefly discuss its extension to other systems. The reason for this choice is twofold: firstly this system has been extensively studied and there is reasonably reliable data available. Secondly the Coulomb 2n-removal cross section can be neglected owing to the small charges involved. Due to this latter fact it should be easier to relate our results to the measured total interaction cross section.

#### 4.1. Study of the $^{11}\text{Li} + ^{12}\text{C}$ system.

In order to calculate  $S_\ell^{(1)}$ , appearing in eq. (14), we used a typical strong absorption potential extended to the system under consideration. The strength of the form factor  $\mathcal{F}(r)$  was determined by fitting the experimental total reaction cross section<sup>7)</sup> determined at  $E_{lab} = 50$  A.MeV,  $\sigma_r^{exp} \approx 1.6$  b, to the theoretical expression

$$\sigma_r \approx \sigma^{bu} + \pi R_{abs}^2 (1 - V_0/E). \quad (32)$$

Above,  $R_{abs} = 5.37$  fm is the radius of the imaginary part of the strong absorption potential and  $\sigma^{bu}$  is taken from expressions (26-30). This procedure resulted in the value  $\mathcal{F}_0 = 2.7$  MeV, which is somewhat lower from the one obtained in ref. 3) at 800 A.MeV, namely  $\mathcal{F}_0 = 5.6$  MeV. This fact is hardly surprising considering the rather different energies in the two cases.

The polarization potential of eqs.(17-21) has been calculated for the case of a collision with a  $^{12}\text{C}$  target. Its strength,  $W_0(\ell, E)$ , is shown in fig. 1, for several bombarding energies. The dotted line indicates the values of the grazing collision angular momenta for each case. For values below this line the absorption is expected to be dominated by the fusion process. In fig. 2 we plot the radial integral  $I_\ell(\eta, s)$  vs. the variable  $X$  defined in eq. (22), as well as its exponential approximation (eq. (23)), for the same energies as in fig. 1. We note that the two calculations are in very good agreement for all energies and angular momentum values considered. It is important to remark that a similar conclusion holds for the scattering of  $^{11}\text{Li}$  from heavier targets. We should also emphasize that the use of the variable  $X$  has important advantages. Firstly, it makes possible the use of the simple parametrization of eq. (23). Furthermore, the region of values of this parameter where the polarization potential is important does not depend on the bombarding energy. It starts at  $X \approx R_{abs}/\alpha \approx 0.7$  and extends up to  $X \approx 3$ .

We are now in a position to calculate the break-up cross section for this system. To test the accuracy of the approximate formula for  $T_\ell$ , in fig. 3 we compare the results obtained from eq. (30) (dashed line) with those of eq. (27) (solid line). In the latter we used the exact radial

wave function  $f_\ell^{el}$  obtained by solving the Schrödinger equation with the full optical potential  $U^{el}$ . For all the energies considered the approximation of eq. (30) is quite reasonable, so we use it to calculate the break-up cross section, which is shown in fig. 4 as a function of the  $^{11}\text{Li}$  incident energy. We note that as the incident energy decreases, the break-up cross section reaches very large values. Since we know that the effect of Coulomb forces is to decrease the total reaction cross section in the barrier region, we expect that when heavier targets are considered the break-up cross section should peak at energies that increase with the target size. These effects are discussed in the next subsection.

#### 4.2. Dependence on the target size.

In order to study the  $^{11}\text{Li}$  break-up in collisions with different targets, we must scale the form factor parameter  $\mathcal{F}_0$  according to eq. (16). Namely,

$$\mathcal{F}_0(A_T) = \mathcal{F}_0(^{12}\text{C}) \exp\left(\frac{R_T - R_C}{\alpha}\right), \quad (33)$$

where  $R_T$  and  $R_C$  stand for the radii of the nuclei of mass  $A_T$  and the  $^{12}\text{C}$ , respectively.

In fig. 5 we show the target mass dependence of the nuclear 2n-removal cross section at laboratory energies 20, 50 and 100 A.MeV. A slight increase of this cross section with  $A$  has already been discussed at the higher energy of 800 A.MeV by several authors<sup>8)</sup>, and our results show that this trend is enhanced at lower energies.

### 5. Conclusions.

In this paper the nuclear  $^{11}\text{Li}$  2n-removal cross section has been calculated for collisions with several targets at intermediate energies. For this purpose we derived the polarization potential associated to this process and used it to calculate the break-up cross section. A closed approximate expression for the transmission coefficients was found. Through a comparison with values obtained through numerical integration with the exact solutions of the radial Schrödinger equation, this approximation was shown to be quite accurate. It was then

used to study the target mass dependence of the nuclear  $2n$ -removal cross section. The results indicate that the slightly increasing trend with target mass found at 800 A·MeV is more pronounced at lower energies.

Our estimate of the nuclear contribution to the break-up cross section should be useful to extract from the data the Coulomb contribution to this process, which carries important information on the nuclear response function of  $^{11}\text{Li}$ . Extensions of our theory to other neutron-rich nuclei is simple, and will be reported in a future publication. Furthermore, we think that the closed expression for the transmission coefficients should be of practical use in assessing the importance of the break-up process in  $^{11}\text{Li}$ -induced fusion reactions. These, and other radioactive projectile-induced fusion reactions have been recently discussed in the literature.<sup>9,10</sup> In these references, however, the low-lying soft giant dipole mode is taken to be infinitely lived. The coupling to the break-up channel, ignored in these fusion studies, should be quite relevant and is currently being investigated.<sup>11</sup>

This work was supported in part by the Financiadora de Estudos e Pesquisas, the Conselho Nacional de Pesquisas e Desenvolvimento Científico and the Fundação de Amparo à Pesquisa do Estado de São Paulo - Brazil.

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Figure Captions.

- Fig. 1:** Strength of the imaginary part of the polarization potential as a function of  $\ell$  for different collision energies.
- Fig. 2:** The radial integral (eq. (19)) (solid lines) and its approximate form (eq. (23)) (dashed lines), as a function of the variable  $X$  for different collision energies.
- Fig. 3:** The transmission coefficient as a function of  $\ell$  for three different energies. The solid lines represent the results of the calculation using eq. (27) and the dashed lines those using the approximation of eq. (30).
- Fig. 4:** Cross section for the break-up of  $^{11}\text{Li}$  incident on a  $^{12}\text{C}$  target, as a function of its collision energy.
- Fig. 5:** Target mass dependence of the  $^{11}\text{Li}$  nuclear 2n-removal cross section shown at three different collision energies. For simplicity a sharp cutoff model for  $|S_\ell^{(0)}|$  (eq. (30)) is used. See text for details.

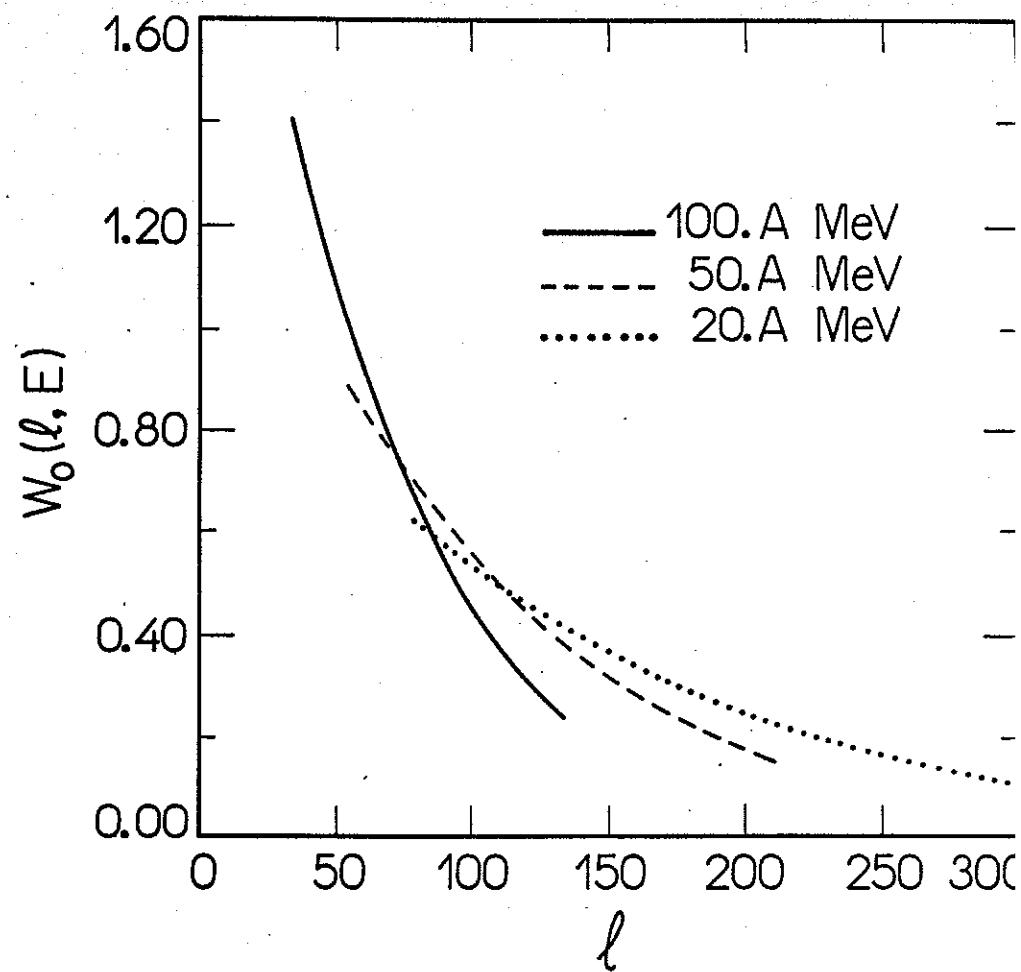


FIG. 1



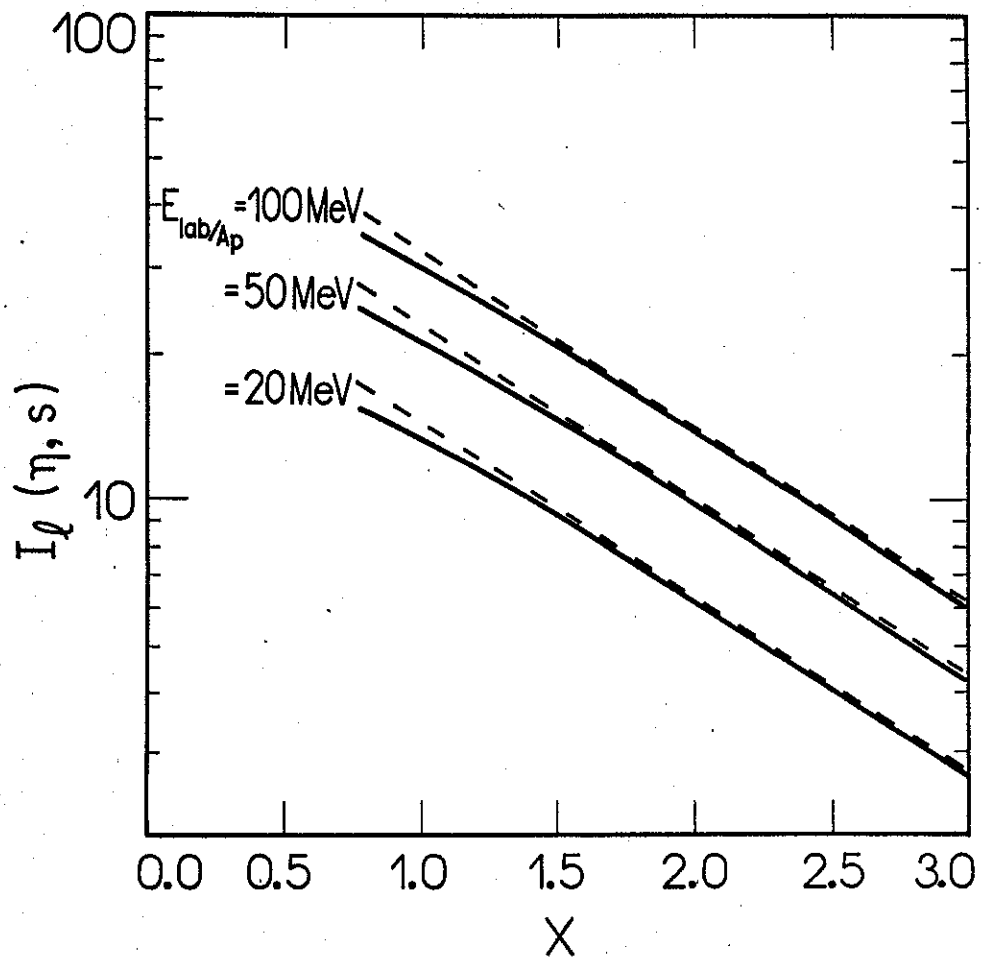


FIG. 2

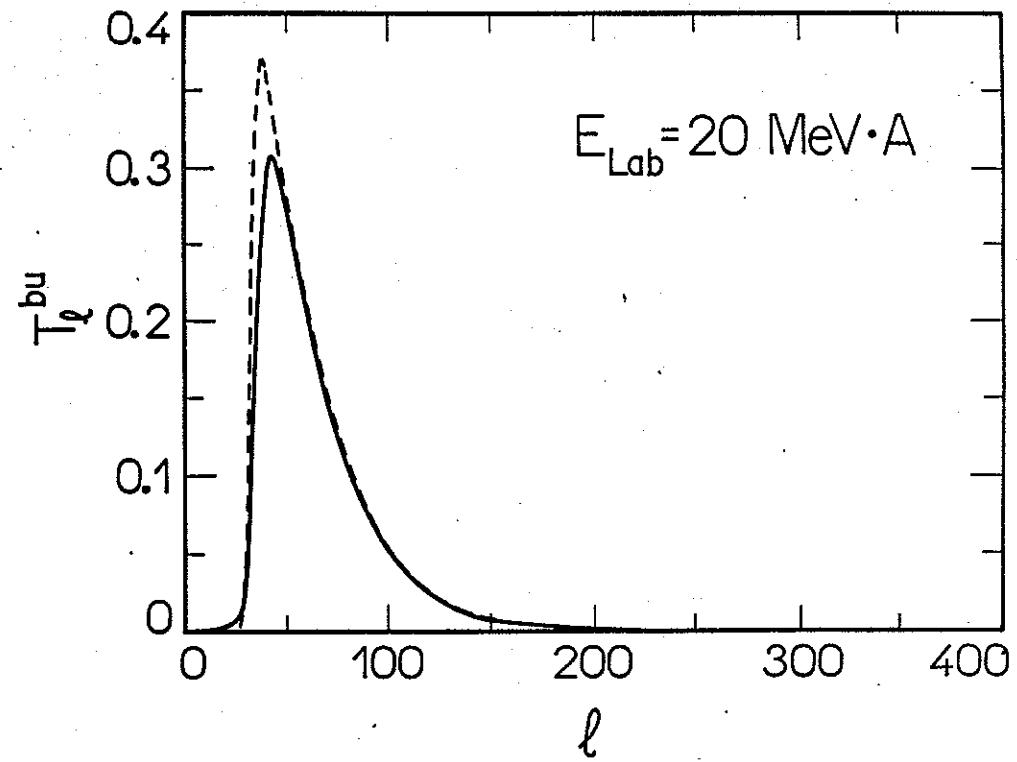


FIG. 3a

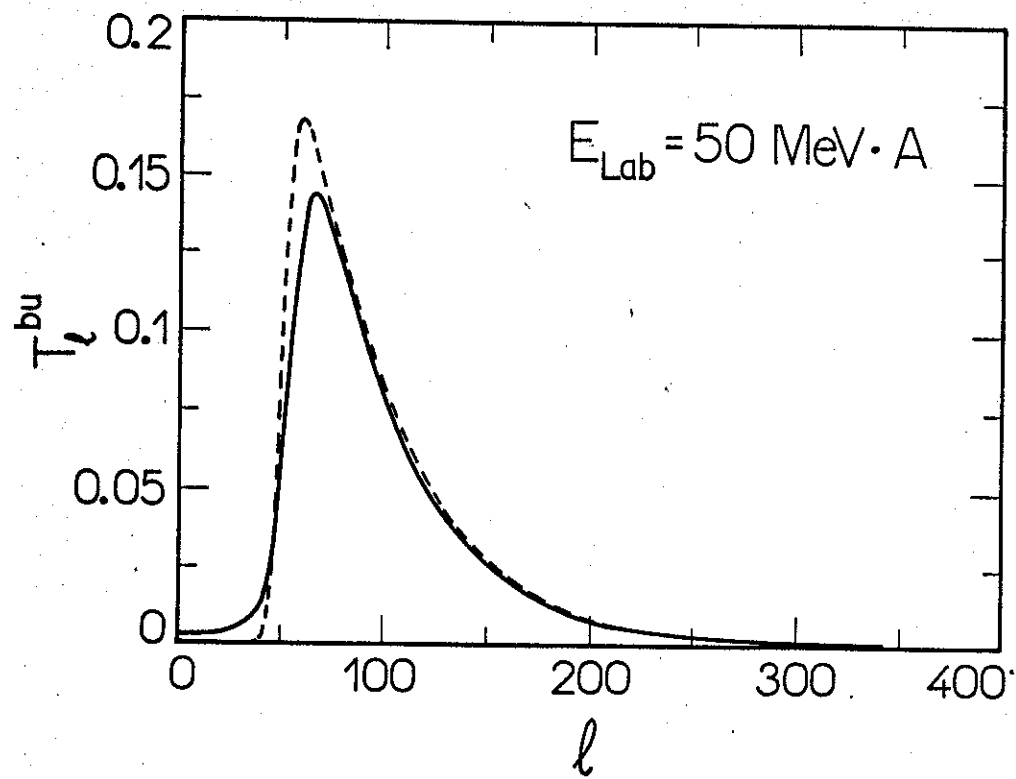


FIG. 3b

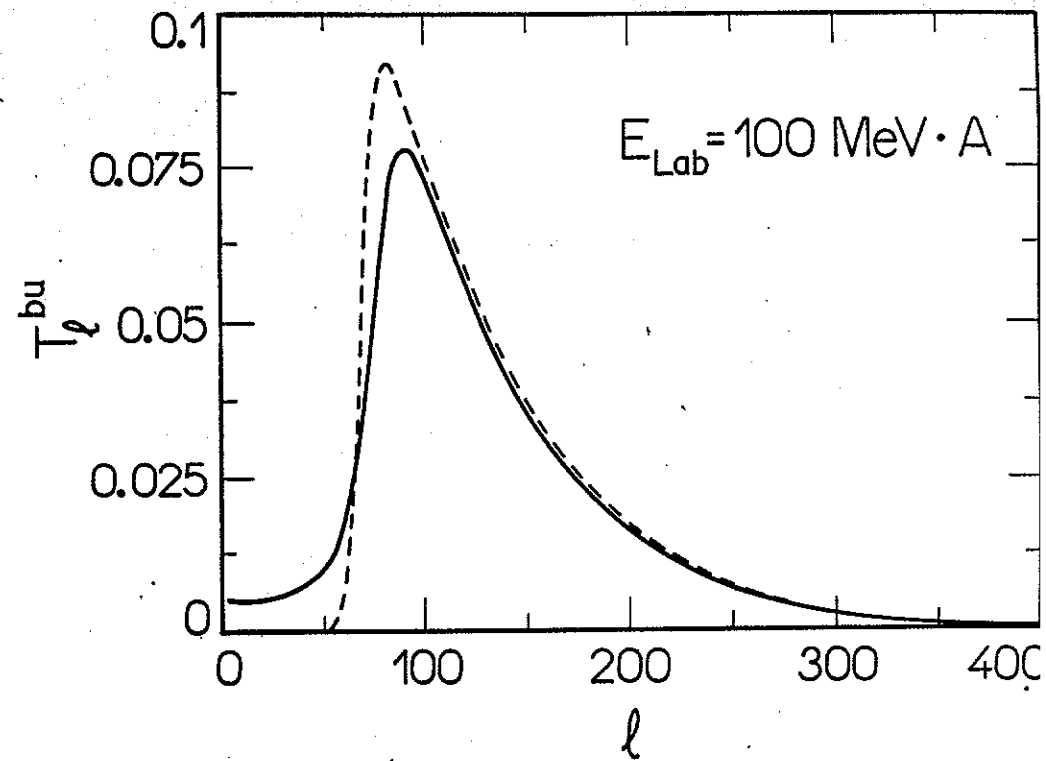


FIG. 3c

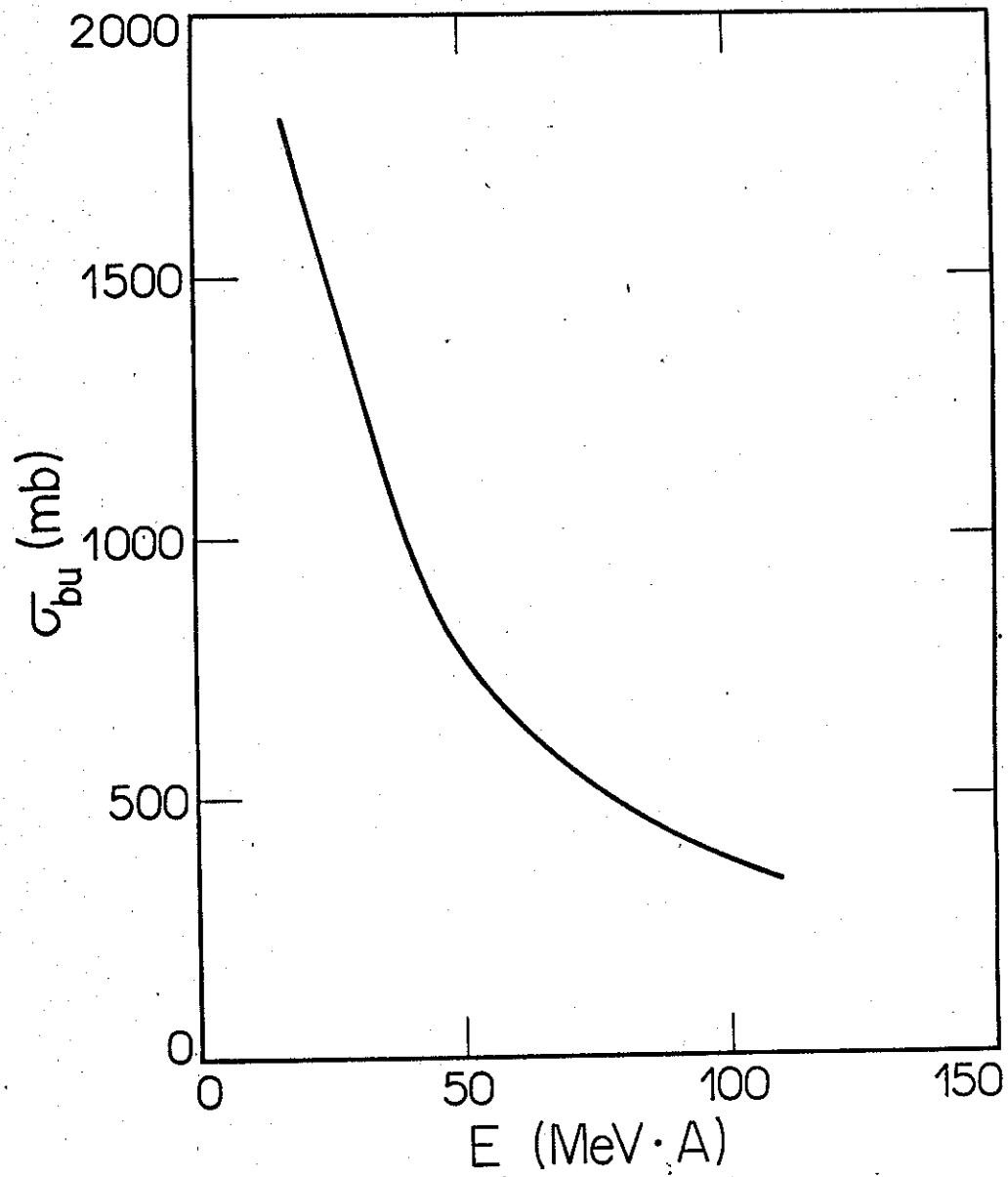


FIG. 4

