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SPIN FORMALISM IN THE ALGEBRAIC SCATTERING THEORY FOR HEAVY IONS

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Abstract

We develop a simple method of treating spin effects in the frame of algebraic scattering theory and apply it to the system $^{16}O + {}^{65}Cu$ at $E_{C.M.} = 44.77$ MeV.

The algebraic scattering theory (AST) has been developed by Alhassid and Iachello [1] and has proved to be a very useful technique for the analysis of heavy-ion scattering data when a large number of channels are open [2,3]. The version of AST used in the present work is based on the assumption that the Hamiltonian for heavy-ion scattering exhibits an approximate SO(3,1) symmetry. In this case, the S matrix can be written in closed form as a ratio of two Euler gamma functions:

$$S_l = \frac{\Gamma(l+1+iV)}{\Gamma(l+1-iV)}. \qquad (1)$$

V is called algebraic potential and is equal to the Sommerfeld parameter , $\eta(k) = \frac{\mu Z_1 Z_2 e^2}{\hbar^2 k}$, for pure Coulomb scattering, when the SO(3,1) symmetry is exact. If there is strong interaction , the algebraic potential can be generalized to a function of wave number, k, and angular momentum, l

$$V(k, l) = \eta(k) + V^N(k, l), \qquad (2)$$

where $V^N(k, l)$ is the nuclear potential. The adoption of eqs. (1-2) is justfied by the dominance of Coulomb interaction in heavy-ion processes.

When reaction channels are explicitly taken into account in the interaction of two spinless particles, eq. (1) assumes a matrix form

$$\mathbf{S} = \frac{\Gamma(\mathbf{L}+1+i\mathbf{V})}{\Gamma(\mathbf{L}+1-i\mathbf{V})}, \qquad (3)$$

where V is the algebraic potential matrix, which contains the dynamics of the scattering in the different channels and L is a diagonal matrix, whose non-zero elements are all equal to the orbital angular momentum, L. In the spinless case, L is thus a multiple of the identity matrix I and commutes with V , so that the

problem can be solved by means of a simple algorithm: (i) diagonalize V; (ii) write the S matrix in the representation where both S and V are diagonal,

$$S_{\alpha\alpha} = \frac{\Gamma(L+1+iV_{\alpha\alpha})}{\Gamma(L+1-iV_{\alpha\alpha})},$$

where $V_{\alpha\alpha}$ is the αth eigenvalue of V; (iii) go back to the original representation by means of the transformation $\mathbf{S} = \mathbf{Z}\mathbf{S}_{diag}\mathbf{Z}^{-1}$,where Z is the matrix whose columns are the eigenvectors of V .

Introduction of spin makes the algebraic approach more difficult to treat, because, in general, L is not multiple of the identity matrix and does not commute with the algebraic potential matrix. In this paper we propose a simple method of solving the coupled- channels problem in AST , based on the assumption that we can replace L in the argument of the gamma function in formula (2) by the total angular momentum , J . This approximation , equivalent to replacing $L(L+1)$ by $J(J+1)$ in the centrifugal potential of the coupled Schrödinger equations, was first proposed by Tanimura [4] and analyzed in detail by Esbensen, Landowne and Price [5]. The S matrix can now be written as

$$\mathbf{S} = \frac{\Gamma(\mathbf{J}+\mathbf{1}+i\mathbf{V})}{\Gamma(\mathbf{J}+\mathbf{1}-i\mathbf{V})}, \qquad (4)$$

where J and V commute, so that we can use the same algorithm as in the spinless case. We do not make approximations in the potential matrix , which remains the same as in the exact treatment.

From now on , for the sake of simplicity, we suppose that only the target nucleus has spin different from zero and that the projectile cannot be excited during the collision: the matrix elements are thus labeled as $V^J_{\alpha',L';\alpha,L}$ and

$S^J_{\alpha',L';\alpha,L}$, where L (L') are the possible values of the initial (final) orbital angular momentum and α (α') the other quantitites characterizing the initial (final) state, such as masses, charges, and quantum numbers of the intrinsic states. It is important to note that, in the spinless case, V is a $n \times n$ matrix, where n is the number of states of the target taken explicitly into account.; when we have spin different from zero, the dimension of the algebraic potential matrix for a given J and parity is determined by summing the numbers of possible angular momentum substates for each state of the target nucleus explicitly taken into account.

The scattering amplitudes can now be written in the form [6]

$$f_{\alpha',M_{\alpha'};\alpha,M_\alpha}(\theta) = \frac{\sqrt{4\pi}}{2ik_\alpha}\sum_{LL'J}\hat{L}(LS_\alpha 0M_\alpha \mid JM_\alpha)(L'S_{\alpha'}m'M_{\alpha'} \mid JM_\alpha) \\ (\hat{S}^J_{\alpha'L',\alpha L} - \delta_{\alpha\alpha'}\delta_{LL'}\delta_{S_\alpha S_{\alpha'}})e^{i(\sigma_{\alpha L}+\sigma_{\alpha'L'})}Y^{m'}_{L'}(\theta) \qquad (5)$$

Here, the z-axis has been chosen along the direction of the incident beam ; $\hat{L} = \sqrt{2L+1}$, $m' = M_\alpha - M_{\alpha'}$, S_α is the spin of the target in state α , and

$$\hat{S}^J_{\alpha'L',\alpha L} = \frac{S^J_{\alpha'L',\alpha L}}{e^{i(\sigma_{\alpha L}+\sigma_{\alpha'L'})}} \qquad (6)$$

The unpolarized cross section for the $\alpha \to \alpha'$ reaction is given by

$$\frac{d\sigma_{\alpha\alpha'}(\theta)}{d\Omega} = \frac{k_{\alpha'}}{k_\alpha}\frac{1}{2S_\alpha+1}\sum_{M_\alpha M_{\alpha'}}\sum |f_{\alpha',M_{\alpha'};\alpha,M_\alpha}(\theta) + f_C(\theta)\delta_{M_{\alpha'}M_\alpha}|^2, \qquad (7)$$

where $f_C(\theta)$ is the usual Coulomb amplitude [6].

One comment is necessary at this point : when we have spin in the entrance channel ($L \neq J$) , the elastic S-matrix element $S^J_{L,\alpha L}$ of formula (4) does not match exactly the Coulomb S-matrix element, $S^C_{L,\alpha L} = \exp(2i\sigma_{\alpha L})$ at high J, when

$V^{N} = 0$, and formula (6) gives spurious terms in the partial wave summation. These spurious terms can be avoided if we rewrite expression (6) as

$$\hat{S}^{J}_{\alpha L,\alpha L} = \frac{S^{J}_{\alpha L,\alpha L}}{S^{C}_{\alpha J,\alpha J}},$$

where

$$S^{C}_{\alpha J,\alpha J} = \frac{\Gamma(J+1+i\eta_{\alpha})}{\Gamma(J+1-i\eta_{\alpha})}. \tag{8}$$

We have tested formulae (5), (7) and (8) for elastic scattering of ^{16}O on ^{63}Cu by assigning to the target spin either a fictitious value 0, or the true value 3/2 : the computed ratios , $\frac{\sigma}{\sigma_R}(\theta)$, are practically the same in the complete angular range, $0° - 180°$, with differences only on the fourth significant digit.

As a preliminary calculation, we have applied our formalism to the analysis of cross sections for the system $^{16}O + ^{63}Cu$ at $E_{c.m.} = 44.77$ MeV, measured by Chamon and Pereira (7). The experimental data consist of the differential elastic cross section and the cumulative excitation of the five lowest-lying levels of ^{63}Cu

The elements, $V_{\alpha L,\alpha' L'}$, of the potential matrix, V, have been calculated according to the general expression (6)

$$V_{\alpha L,\alpha' L'} = \frac{1}{\sqrt{4\pi}} \sum_{\lambda} (-1)^{J-S_{\alpha}} \hat{L}\hat{L}'\hat{S}_{\alpha'}(LL'00\,|\,\lambda 0) W(L,L',S_{\alpha},S_{\alpha'}\,|\,\lambda,J) \; V^{\lambda}_{\alpha\alpha'}(\bar{L})(S_{\alpha'}\,||\,V\,||\,S_{\alpha}) \tag{9}$$

Here, $\hat{L} = \sqrt{2L+1}$, λ is the multipolarity of the transition, S_{α} the spin of the target in state α, W is a Racah coefficient and the reduced matrix element is defined as in ref. (6); the expressions adopted for the transition form factors, $V^{\lambda}_{\alpha\alpha'}(\bar{L})$, are the following: nuclear elastic (8) :

$$V^{0}_{\alpha\alpha}(\bar{L}) = (1+2\frac{\bar{L}}{L_0})^{-\pi/2} \frac{V_R + iV_I}{1+e^{\frac{\bar{L}-L_0}{\Delta}}}, \tag{10}$$

nuclear inelastic:

$$V^{\lambda}_{\alpha\beta}(\bar{L}) = -\eta_{nucl.} \frac{dV^{\lambda}_{\alpha\alpha}(\bar{L})}{d\bar{L}}, \tag{11}$$

where $V^{\lambda}_{\alpha\alpha}$ has the same structure as formula (9), but, in principle, different depths , U_R and U_I, and different critical momentum and diffuseness;

Coulomb excitation (1) :

$$V^{C\lambda}_{\alpha\beta}(\bar{L}) = \eta_{Coul.} \begin{cases} (\bar{L}/L_C)^{\lambda+1}, & \bar{L} < L_C \\ (L_C/\bar{L})^{\lambda}, & \bar{L} > L_C. \end{cases} \tag{12}$$

Here, V_R, V_I, U_R, U_I, $\eta_{nucl.}$ and $\eta_{Coul.}$ are to be adjusted on the experimental data. The diagonal form factors are calculated at the proper value of the orbital angular momentum, $\bar{L} = L$, and the off-diagonal factors at $\bar{L} = \frac{1}{2}(L+L')$, the arithmetic mean of the initial and final angular momenta. In the same way , the critical angular momentum and the diffuseness in form factor (10) are the arithmetic mean of the corresponding values in formula (9) for the entrance and exit channels. Moreover, the algebraic potential does not contain spin-spin , or spin-orbit interactions.

In these preliminary calculations, we have coupled to the ground state of ^{63}Cu the five lowest excited states through $\lambda = 2$ transitions. The parameters that define the diagonal matrix elements of the potential have been determined by least-squares fit of the experimental angular distribution for elastic scattering (7) The inelastic coupling strengths ($\eta_{Coul.}$ and $\eta_{nucl.}$) have been derived from ex-

perimental B(E2) values (9), with a normalization obtained by fitting the experimental cumulative excitation to the five lowest-lying states of ^{65}Cu (7)

The elastic and inelastic scattering cross sections for $^{16}O + {}^{65}Cu$ at $E_{c.m.} = 44.77$ MeV , calculated with the parameters listed in Table I, are compared in Fig. 1 with the experimental data (7). The spin effects are emphasized in Fig. 2, where we compare two calculations of pure nuclear excitation with the true spins and with spin zero in all channels. We observe that the effect of the spin is to wash out the oscillations in the angular distribution, as expected from general considerations. The differences in the magnitude of the cross sections are due to different values of the geometric part of formula (9) in the two cases.

The quality of these preliminary results encourages us to extend the AST analysis to other incident energies and reaction channels, by including also existing data of one-proton transfer (7), with the main goal of extracting coupled-channels effects on the fusion cross section. In the more extended analysis, the transition matrix elements for the various channels will be obtained by means of calculations based on the interacting-boson and interacting boson-fermion models, so as to give a simultaneous algebraic description of excitation of intrinsic states and scattering.

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Final State	V_R	V_I	L_0	Δ	L_C	U_R	U_I	$\eta_{nucl.}$	$\eta_{Coul.}$
$^{65}Cu\,(\frac{3}{2})^-_1$	2.0	3.4	26.0	1.3	10.0	–	–	–	–
$^{65}Cu\,(\frac{1}{2})^-_1$	2.0	3.4	24.7	2.2	10.0	7.0	5.0	0.14	-.038
$^{65}Cu\,(\frac{5}{2})^-_1$	2.0	3.4	24.7	2.2	10.0	7.0	5.0	0.15	-.040
$^{65}Cu\,(\frac{7}{2})^-_1$	2.0	3.4	24.7	2.2	10.0	7.0	5.0	0.13	-.036
$^{65}Cu\,(\frac{5}{2})^-_2$	2.0	3.4	24.7	2.2	10.0	7.0	5.0	.021	-.006
$^{65}Cu\,(\frac{3}{2})^-_2$	2.0	3.4	24.7	2.2	10.0	7.0	5.0	.035	-.010

Table 1. AST parameters for elastic and inelastic scattering of $^{16}O + ^{65}Cu$.

Figure Captions

Fig.1 (a) Elastic scattering cross section divided by Rutherford cross section for $^{16}O + ^{65}Cu$ at $E_{c.m.}$ = 44.77 MeV. (b) Inelastic scattering to the five lowest lying levels of ^{65}Cu. 300 partial waves have been included in the scattering amplitudes. The experimental data are taken from ref. (7).

Fig.2 Nuclear contribution to inelastic scattering to the five lowest-lying levels of ^{65}Cu Solid curve: experimental spin value assigned to each level; dashed curve: spin 0 assigned to each level. The experimental data are the same as in Fig. 1(b).

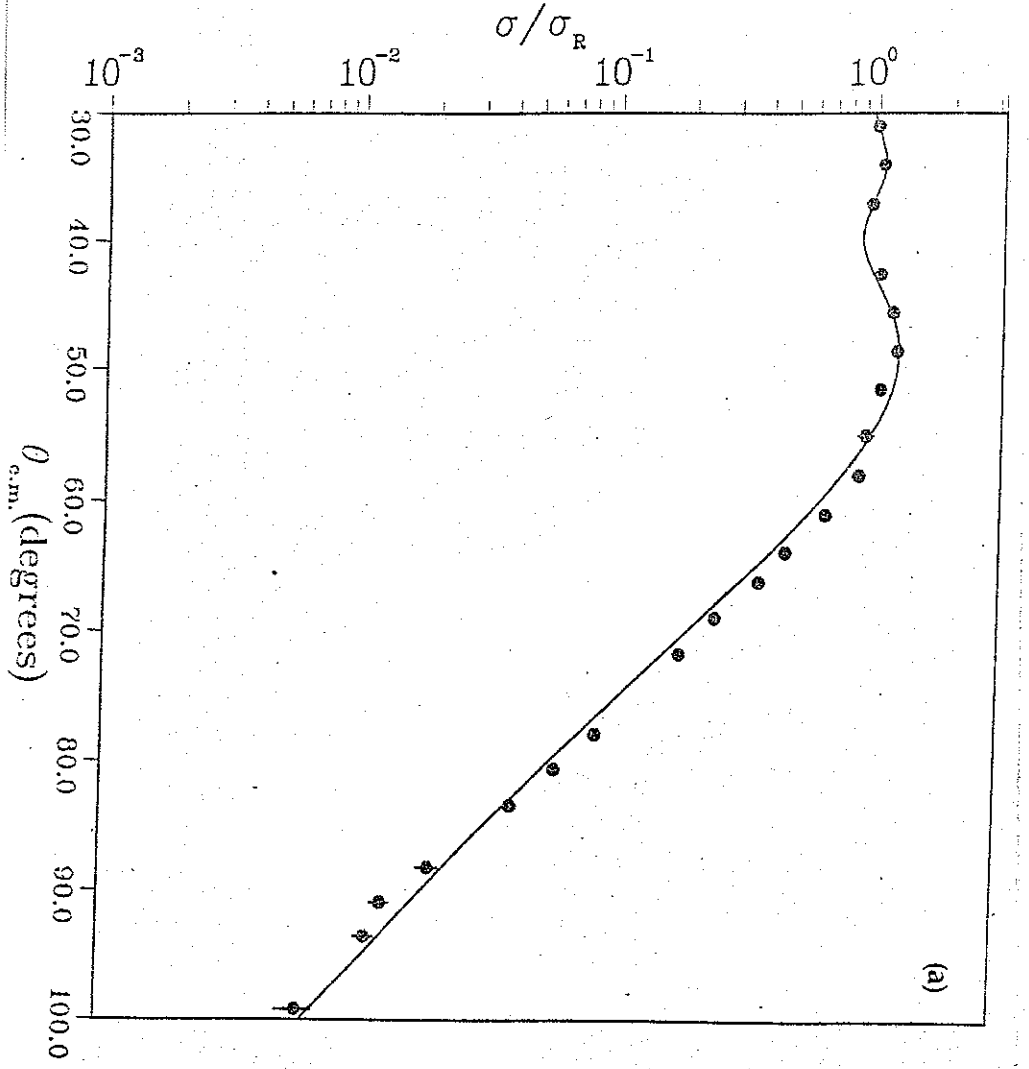
σ/σ_R
10^{-3}
10^{-2}
10^{-1}
10^{0}
30.0
40.0
50.0
60.0
70.0
80.0
90.0
100.0
$\theta_{c.m.}$(degrees)
(a)

Fig. 1(a)

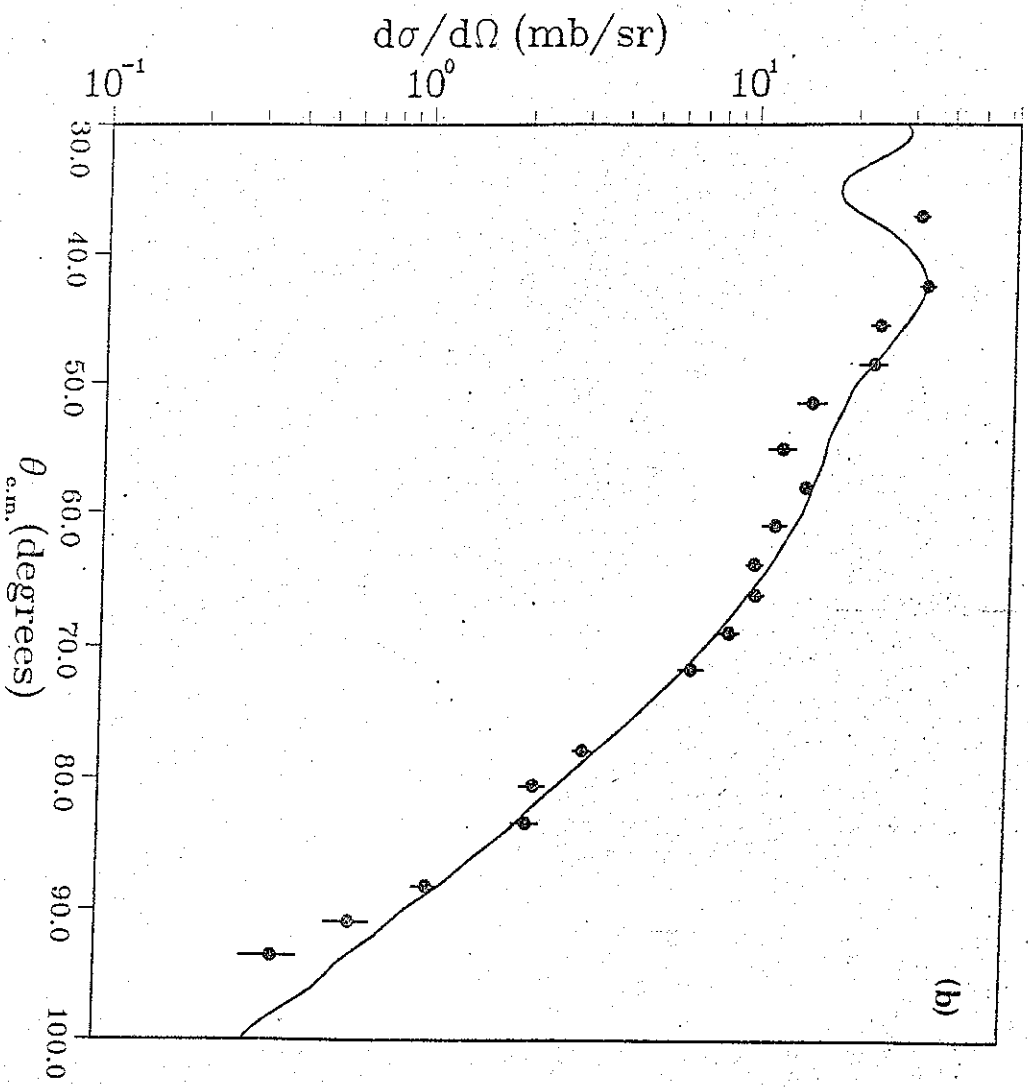
$d\sigma/d\Omega$ (mb/sr)
10^{-1}
10^{0}
10^{1}
30.0
40.0
50.0
60.0
70.0
80.0
90.0
100.0
$\theta_{c.m.}$(degrees)
(b)

Fig. 1(b)

dσ/dΩ (mb/sr)
10^{-1}
10^{0}
10^{1}
0.0
10.0
20.0
30.0
40.0
50.0
60.0
70.0
80.0
90.0
100.0
$\theta_{c.m.}$ (degrees)

Fig. 2