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CORRELATION FUNCTIONS IN NON CRITICAL (SUPER) STRING THEORY

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Correlation Functions in Non Critical (Super) String Theory

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Abstract

We consider the correlation functions of the tachyon vertex operator of the super Liouville theory coupled to matter fields in the super Coulomb gas formulation, on world sheets with spherical topology. After integrating over the zero mode and assuming that the s parameter takes an integer value, we subsequently continue it to an arbitrary real number and compute the correlators in a closed form. We also included an arbitrary number of screening charges and, as a result, after renormalizing them, as well as the external legs and the cosmological constant, the form of the final amplitudes do not modify. The result is remarkably parallel to the bosonic case. For completeness, we discussed the calculation of bosonic correlators including arbitrary screening charges.

1- Introduction

Two dimensional gravity is not only a toy model for the theory of gravitation, but also describes phenomena such as random surfaces and string theory away from criticality[1]. The discretized counterpart, namely matrix models, proved to be an efficient means to obtain information, especially about non critical string theory, while computing correlation functions[2].

In the continuum approach[4], in the conformal gauge, we have to face Liouville theory[5]. However, although several important developments have been achieved[6], we still lack some important points, in spite of much effort which has been spent.

In particular, it is difficult to calculate correlation functions in a reliable way because perturbation theory does not apply. Recently, however, several authors[7−14] succeeded in taming the difficulties of the Liouville theory and computed exactly correlation functions in the continuum approach to conformal fields coupled to two dimensional gravity. The technique is based on the integration over the zero mode of the Liouville field. The resulting amplitude is a function of a parameter s which depends on the central charge and on the external momenta. The amplitudes can be computed when the above parameter is a non-negative integer. Later on, one analytically continues that parameter to real (or complex) values. The results for the correlation functions of the tachyon operator thus obtained agree with the matrix model approach.[2] General correlation functions including arbitrary screening charges are also computed, and may be useful for the purpose of studying fusion rules of minimal models coupled to two dimensional gravity.[25]

Opposite to the bosonic case supersymmetric matrix models are up to now scarcely known. There are few papers in the literature concerning this approach[3,25], and it is desirable to have results from an alternative approach for comparison. For this reason, several groups are studying the continuum approach to two dimensional supergravity[17−21]. Our aim here is to investigate the supersymmetric Liouville theory. We shall compute supersymmetric correlation functions on world sheets with spherical topology in the Neveu-Schwarz sector, where the super-Liouville is coupled to superconformal matter with central charge $\hat{c} \leq 1$, represented as a super Coulomb gas[23,17]. The results are remarkable, and very parallel to the bosonic case; after a redefinition of the cosmological constant, and of the primary superfields, the resulting amplitudes have the same form as those of the bosonic theory obtained by Di Francesco and Kutasov[9]. Our present results generalize those presented in a recent paper[19], as well as others recently obtained in the literature[20,21,26].

The paper is organized as follows. In section 2 we review some computations of the bosonic correlators and generalize them to include an arbitrary number of s.c. . In section 3 we calculate the N-point correlator of the Neveu-Schwarz vertex operator in the $n = 1$ two dimensional supergravity coupled to $N = 1$ supermatter including an arbitrary number s.c..

Our results include the limiting cases $c = 1$ ($\hat{c} = 1$) and in those situations some physical conclusions can be drawn in the section 4: the amplitude factorizes, and the expected intermediate poles have zero residue, due to strong kinematic constraints. All results are at least consistent with the matrix model approach, with possible exception about the inclusion of screening charges, in the 3-point correlation function. In the latter case, fusion rules deserve careful study.

2- Bosonic Correlation Functions

Some N-point tachyon correlation function in Liouville theory coupled to $c \leq 1$ conformal matter were recently calculated by Di Francesco and Kutasov[9]. They worked in the DDK's framework[22] where the total action is given by:

$$S = \frac{1}{2\pi} \int d^2w \sqrt{\hat{g}} \left[\hat{g}^{ab} \partial_a \phi \partial_b \phi - \frac{Q}{4} \hat{R} \phi + 2\mu e^{\alpha\phi} + \hat{g}^{ab} \partial_a X \partial_b X + \frac{i\alpha_0}{2} \hat{R} X \right] \quad , \tag{2.1}$$

here ϕ represents the Liouville mode and X is the matter field with the central charge given by $c = 1 - 12\alpha_0^2$. From the literature[17] we know that the constant Q is determined by imposing a vanishing total central charge, and is given by

$$Q = 2\sqrt{2 + \alpha_0^2}.$$

The value of α is determined by requiring $e^{\alpha\phi}$ to be a (1,1) conformal operator, yielding the equation $\frac{-\alpha}{2}(\alpha + Q) = 1$, whose solutions are labeled by $\alpha_\pm$

$$\alpha_\pm = -\frac{Q}{2} \pm |\alpha_0| \quad , \quad \alpha_+ \alpha_- = 2 \tag{2.2}$$

the semiclassical limit ($c \to -\infty$) fixes $\alpha = \alpha_+$.

The gravitationally-dressed tachyon amplitudes are the objects we are interested in:

$$\langle T_{k_1} \cdots T_{k_N} \rangle = \left\langle \prod_{j=1}^{N} \int d^2 z_j e^{ik_j X(z_j) + \beta(k_j)\phi(z_j)} \right\rangle \tag{2.3}$$

where we fix the dressing parameter β imposing $e^{ik_j X + \beta_j \phi}$ to be a (1,1) conformal operator and supposing the space-time energy to be positive:

$$E = \beta(k) + \frac{Q}{2} = |k_j - \alpha_0| \quad . \tag{2.4}$$

In the calculation of the amplitudes $\langle T_{k_1} \cdots T_{k_N} \rangle$ the main ingredient is the integration over the matter (X_0) and the Liouville (ϕ_0) zero modes. This is the so called zero mode technique: one splits[4,5] both, the matter and the Liouville fields as a sum of the zero mode $(X_0), (\phi_0)$ plus fluctuations $(\tilde{X}), (\tilde{\phi})$, where the fluctuations are orthogonal to the zero mode. After such splitting we are left with the following integrals:

$$\begin{aligned} \int_{-\infty}^{\infty} \mathcal{D}X_0 e^{iX_0\left(\sum_{i=1}^N k_i - 2\alpha_0\right)} &= 2\pi\delta\left(\sum_{i=1}^{N} k_i - 2\alpha_0\right) \quad , \\ \int_{-\infty}^{\infty} \mathcal{D}\phi_0 e^{i\phi_0\left(\sum_{j=1}^N \beta_j + Q\right) - e^{\alpha_+\phi_0}\left(\frac{\mu}{\pi}\int d^2 w e^{\alpha_+\tilde{\phi}}\right)} &= \frac{\Gamma(-s)}{-\alpha_+}\left(\frac{\mu}{\pi}\int d^2 w e^{\alpha_+\tilde{\phi}}\right)^s \quad , \end{aligned} \tag{2.5}$$

where we have used that on the sphere $\frac{1}{8\pi}\int d^2w\sqrt{\hat{g}}\hat{R} = 1$ and

$$s = -\frac{1}{\alpha_+}\left(\sum_{j=1}^{N} \beta_j + Q\right) \quad . \tag{2.6}$$

We thus obtain for the amplitude

$$\begin{aligned} \langle T_{k_1} \cdots T_{k_N} \rangle = 2\pi\delta\left(\sum_{j=1}^{N} k_j - 2\alpha_0\right) \frac{\Gamma(-s)}{-\alpha_+}\left(\frac{\mu}{\pi}\right)^s \\ \times \left\langle \prod_{j=1}^{N} \int d^2 z_j e^{ik_j + \beta_j\phi(z_j)} \left(\int d^2 w e^{\alpha_+\phi}\right)^s \right\rangle_0 \end{aligned} \tag{2.7}$$

where $\langle \cdots \rangle_0$ means that now the correlation functions are calculated as in the free theory ($\mu = 0$). The strategy to obtain $\mathcal{A}_N$ is to assume first that s is a non-negative integer and to continue the result to any real s at the end. Thus, using free propagators

$$\langle X(w)X(z)\rangle_0 = \langle \phi(w)\phi(z)\rangle_0 = \ln|w - z|^{-2} \tag{2.8}$$

and fixing the residual $SL(2,\mathbb{C})$ invariance of the conformal gauge on the sphere by choosing ($z_1 = 0\,,\, z_2 = 1\,,\, z_3 = \infty$), the 3-point function is written as:

$$\mathcal{A}_3(k_1, k_2, k_3) = \frac{\Gamma(-s)}{-\alpha_+}\left(\frac{\mu}{\pi}\right)^s \int \prod_{j=1}^{s} d^2 w_j |w_j|^{2\alpha} |1 - w_j|^{2\beta} \prod_{i<j}^{s} |w_i - w_j|^{4\rho} \quad , \tag{2.9}$$

where we have defined $\alpha = -\alpha_+\beta_1\,,\, \beta = -\alpha_+\beta_2\,,\, \rho = -\alpha_+^2/2$. Choosing the kinematics $k_1, k_3 \geq \alpha_0\,,\, k_2 < \alpha_0 \leq 0$ (notice that our notation differs from Ref.[9] by the exchange of k_2 and k_3.) we can eliminate β using (2.4), (2.6) and the momentum conservation, one can write the 3-point amplitude in a rather compact form

$$\mathcal{A}_3 = [\mu\Delta(-\rho)]^s \prod_{j=1}^{3} \Delta\left(\frac{1}{2}(\beta_j^2 - k_j^2)\right) \quad , \tag{2.10}$$

where $\Delta(x) = \Gamma(x)/\Gamma(1 - x)$. After redefinitions of the cosmological constant and of the external fields as

$$\mu \to \frac{\mu}{\Delta(-\rho)} \quad , \quad T_{k_j} \to \frac{T_{k_j}}{\Delta\left(\frac{1}{2}(\beta_j^2 - k_j^2)\right)} \quad , \tag{2.11}$$

Di Francesco and Kutasov[9] obtained for the three-point function

$$\mathcal{A}_3 = \mu^s \quad , \tag{2.12}$$

which is also obtained in the matrix model approach. In the following we shall see that a similar expression holds for general N-point tachyon amplitudes with an arbitrary number of screening charges.

The screening charges are introduced in the form of n operators e^{id_+X} and m operators e^{id_-X}, with $d_\pm$ solutions of: $\frac{1}{2}d(d-2\alpha_0)=1$, $(d_+d_- = -\alpha_+\alpha_- = -2)$. Integrating over the zero-modes again we get:

$$\begin{aligned}&\left\langle T_{k_1}T_{k_2}T_{k_3}\left(\frac{1}{n!}\prod_{i=1}^{n}\int d^2t_i e^{id_+X(t_i)}\right)\left(\frac{1}{m!}\prod_{i=1}^{m}\int d^2r_i e^{id_-X(r_i)}\right)\right\rangle \\ &= 2\pi\delta(\sum_{i=1}^{3}k_i + nd_+ + md_- - 2\alpha_0)\mathcal{A}_3^{nm}(k_1,k_2,k_3)\end{aligned} \tag{2.13}$$

where the amplitude $\mathcal{A}_3^{nm}(k_1,k_2,k_3)$ is given by the expression

$$\begin{aligned}\mathcal{A}_3^{nm}(k_1,k_2,k_3) = &\frac{\Gamma(-s)}{-\alpha_+}\left(\frac{\mu}{\pi}\right)^s\prod_{i=1}^{n}\int d^2t_i|t_i|^{2\tilde{\alpha}}|1-t_i|^{2\tilde{\beta}}\prod_{i<j}^{n}|t_i-t_j|^{4\tilde{\rho}} \\ &\times\prod_{i=1}^{m}\int d^2r_i|r_i|^{2\tilde{\alpha}'}|1-r_i|^{2\tilde{\beta}'}\prod_{i<j}^{m}|r_i-r_j|^{4\tilde{\rho}'} \\ &\times\prod_{i=1}^{n}\prod_{j=1}^{m}|t_i-r_j|^{-4}\prod_{i=1}^{s}\int d^2z_i|z_i|^{2\alpha}|1-z_i|^{2\beta}\prod_{i<j}^{s}|z_i-z_j|^{4\rho} \quad .\end{aligned} \tag{2.14}$$

The parameters α,β and ρ are defined as before, and the remaining parameters are

$$\begin{aligned}\tilde{\alpha} &= d_+k_1 \quad , \quad \tilde{\beta} = d_+k_2 \quad , \quad \tilde{\rho} = \frac{1}{2}d_+^2 \\ \tilde{\alpha}' &= d_-k_1 \quad , \quad \tilde{\beta}' = d_-k_2 \quad , \quad \tilde{\rho}' = \frac{1}{2}d_-^2 \quad .\end{aligned} \tag{2.15}$$

Notice that the gravitational part of the amplitude (integrals over z_i) is the same as in the case without screening charges. The integrals over t_i and r_j (matter contributions) have been calculated by Dotsenko and Fatteev[16] (See their formula (B.10) in the second paper of [16]); the result turns out to be

$$\begin{aligned}\mathcal{A}_3^{nm} = &\left(\frac{\mu}{\pi}\right)^s\Gamma(-s)\Gamma(s+1)\pi^{s+n+m}\tilde{\rho}^{-4nm}[\Delta(1-\tilde{\rho})]^n[\Delta(1-\tilde{\rho}']^m\prod_{i=1}^{m}\Delta(i\tilde{\rho}'-n)\prod_{i=1}^{n}\Delta(i\tilde{\rho}) \\ &\times\prod_{i=0}^{m-1}\Delta(1-n+\tilde{\alpha}'+i\tilde{\rho}')\Delta(1-n+\tilde{\beta}'+i\tilde{\rho}')\Delta(-1+n-\tilde{\alpha}'-\tilde{\beta}'-(n-1+i)\tilde{\rho}') \\ &\times\prod_{i=0}^{n-1}\Delta(1+\tilde{\alpha}+i\tilde{\rho})\Delta(1+\tilde{\beta}+i\tilde{\rho})\Delta(-1+2m-\tilde{\alpha}-\tilde{\beta}-(n-1+i)\tilde{\rho}) \\ &\times[\Delta(1-\rho)]^s\prod_{i=1}^{s}\Delta(i\rho)\prod_{i=0}^{s-1}\Delta(1+\alpha+i\rho)\Delta(1+\beta+i\rho)\Delta(-1-\alpha-\beta-(s-1+i)\rho)\end{aligned} \tag{2.16}$$

Assuming $\alpha_0 < 0$, and the same kinematics (k_1, $k_3 \geq \alpha_0$, $k_2 < \alpha_0$) we can eliminate β again, using (2.4), (2.6) and momentum conservation. In this way, we have

$$\begin{aligned}\tilde{\alpha} &= \alpha - 2\rho \quad , \quad \tilde{\alpha}' = -2 + \tilde{\rho}\alpha \\ \beta &= -1 - m - (s+n)\rho \quad , \\ \tilde{\beta} &= m - 1 + (s+n)\rho \quad , \quad \tilde{\beta}' = s + n + \rho^{-1}(m-1) \\ \tilde{\rho} &= -\rho \quad , \quad \tilde{\rho}' = -\rho^{-1} \quad .\end{aligned} \tag{2.17}$$

Substituting in (2.16) we obtain the three point function with arbitrary screenings

$$\begin{aligned}\mathcal{A}_3^{nm} = &\left(\frac{\mu}{\pi}\right)^s\Gamma(-s)\Gamma(s+1)\pi^{s+n+m}(\tilde{\rho})^{-4nm}\left[\Delta(1+\rho^{-1})\right]^m[\Delta(1+\rho)]^n \\ &\times\prod_{i=1}^{m}\Delta(i\rho^{-1}-n)\prod_{i=1}^{n}\Delta(-i\rho)\,[\Delta(1-\rho)]^s\prod_{i=1}^{s}\Delta(i\rho) \\ &\times\prod_{i=0}^{n-1}\Delta(m+(s+n-i)\rho)\prod_{i=0}^{s-1}\Delta(-m-(s+n-i)\rho) \\ &\times\prod_{i=0}^{m-1}\Delta(1+s+(m-1-i)\rho^{-1}) \\ &\times\prod_{i=0}^{m-1}\Delta(-1-n+\rho^{-1}\alpha-i\rho^{-1})\Delta(1-s-\rho^{-1}\alpha+i\rho^{-1}) \\ &\times\prod_{i=0}^{n-1}\Delta(1+\alpha-(i+2)\rho)\Delta(m-\alpha-(s-1-i)\rho) \\ &\times\prod_{i=0}^{s-1}\Delta(1+\alpha+i\rho)\Delta(m-\alpha+(n+1-i)\rho) \quad .\end{aligned} \tag{2.18}$$

To get a simpler expression for $\mathcal{A}_3^{nm}$ we look for the term $\Delta(\rho-\alpha)\Delta(\rho(s-n+1)+\alpha-m+1)\times\Delta(-m\rho^{-1}-(s+n))$ which corresponds to $\prod_{i=1}^{3}\Delta(\frac{1}{2}(\beta_i^2-k_i^2))$. After algebraic manipulations we get

$$\mathcal{A}_3^{nm} = [\mu\Delta(-\rho)]^s\left[-\pi\Delta(\rho^{-1})\right]^m[-\pi\Delta(\rho)]^n\prod_{i=1}^{3}(-\pi)\Delta\left(\frac{1}{2}(\beta_i^2-k_i^2)\right) \quad . \tag{2.19}$$

This result has been also obtained by Di Francesco and Kutasov[9,20], as well as Aoki and D'Hoker[14]. Note that the factors $\Delta(\rho)$ and $\Delta(\rho^{-1})$ can be easily understood; the screening operators are renormalized like the tachyon vertex operators T_k with vanishing dressing $\beta(k)$.

Remembering the momentum conservation law:

$$\sum_{i=1}^{3}k_i = (1-n)d_+ + (1-m)d_- \quad , \tag{2.20}$$

we can use (2.19) with $n = m = 1$ and the formula below to obtain the partition function $\mathcal{Z}$:

$$\frac{\partial^3 \mathcal{Z}}{\partial \mu^3} = \mathcal{A}_3^{11}(k_i \to 0) \tag{2.21}$$

thus,

$$\mathcal{Z} = \Delta(\rho)\Delta(\rho^{-1})\frac{\rho^3[\mu\Delta(-\rho)]^{1-\rho^{-1}}}{(\rho-1)(\rho+1)} \quad . \tag{2.22}$$

We can also get the two-point function by taking, e.g., $k_3 \to 0$ in (2.19) and (2.21):

$$\mathcal{A}_3^{\tilde{n}\tilde{m}}(k_1, k_2, k_3 \to 0) = \frac{\partial}{\partial \mu}\mathcal{A}_2^{\tilde{n}\tilde{m}}(k_1, k_2) \quad , \tag{2.23}$$

where $(\tilde{n}, \tilde{m})$ are fixed by (2.20) with $k_3 = 0$. Thus we finally arrive at

$$\mathcal{A}_2^{\tilde{n}\tilde{m}}(k_1, k_2) = \frac{[\mu\Delta(-\rho)]^{\rho^{-1}\left(\frac{\alpha_+}{2}\sum_{i=1}^2 \beta_i + \rho - 1\right)}}{\rho^{-1}\left(\frac{\alpha_+}{2}\sum_{i=1}^2 \beta_i + \rho - 1\right)}\left[\Delta(\rho^{-1})\right]^{\tilde{m}}\left[\Delta(\rho)\right]^{\tilde{n}}\prod_{i=1}^{2}\Delta\left(\frac{(\beta_i^2 - k_i^2)}{2}\right) \tag{2.24}$$

We are able now to calculate ratios of correlation functions to compare with other results in the literature. We have (in a generic kinematic region):

$$R = \frac{\mathcal{A}_3^{nm}(k_1, k_2 k_3)\mathcal{Z}}{\mathcal{A}_2^{\tilde{n}_1,\tilde{m}_1}(k_1, k_1)\mathcal{A}_2^{\tilde{n}_2,\tilde{m}_2}(k_2, k_2)\mathcal{A}_2^{\tilde{n}_3,\tilde{m}_3}(k_3, k_3)} = \frac{\prod_{i=1}^3 \alpha_+|k_i - \alpha_0|}{(\rho-1)(1+\rho)} \tag{2.25}$$

In the case of minimal models the momenta assume the following form:

$$k_{r_i r_i'} = \frac{(1-r_i)}{2}d_+ + \frac{(1-r_i')}{2}d_- \quad . \tag{2.26}$$

Plugging back in (2.25) we find for the ratio R the result

$$R = \frac{\langle T_{r_1 r_1'} T_{r_2 r_2'} T_{r_3 r_3'}\rangle^2 \mathcal{Z}}{\langle T_{r_1 r_1'} T_{r_1 r_1'}\rangle\langle T_{r_2 r_2'} T_{r_2 r_2'}\rangle\langle T_{r_3 r_3'} T_{r_3 r_3'}\rangle} = \left(\frac{\alpha_+}{2}\right)^3 \frac{\prod_{i=1}^3 |r_i d_+ + r_i' d_-|}{(\rho-1)(\rho+1)} \tag{2.27}$$

That is exactly the same result obtained by Dotsenko[10] (in a given kinematic region) which reduces to the result of Goulian and Li for $r_i = r_i'$; all these results agree with those obtained by the matrix model approach.

At this point the following remark is in order. After renormalizing the screening charges:

$$e^{id_+X} \to \frac{e^{id_-X}}{\Delta(\rho)} \quad , \quad e^{id_-X} \to \frac{e^{id_-X}}{\Delta(\rho^{-1})} \quad , \tag{2.28}$$

and the tachyon operators T_{k_i} as well as the cosmological constant μ as before (see (2.11)) we get the renormalized amplitude.

$$\mathcal{A}_3^{nm} = \mu^s \quad . \tag{2.29}$$

Using the above result we would be able to exactly reproduce the ratio in (2.27). Thus, the comparison of those ratios with matrix models is not a precise test and only measures the scalling of the amplitudes w.r.t. the cosmological constant. All singularities contained in the Δ functions cancel out in such ratios. If on the other hand compare, the 3-point function (2.19) directly with the matrix model result we would find a precise agreement only for $c = 1$ $(\alpha_0 = 0)$ where the amplitude can be written as a function of the renormalized cosmological constant as:

$$\mathcal{A}_3^{nm} \sim (\tilde{\mu})^s \prod_{i=1}^{3}\Gamma(1 - \sqrt{2}|k_i|) \quad . \tag{2.30}$$

For $c < 1$ the amplitude obtained via matrix model are finite and the singularities contained in (2.19) are not observed. Neither do the fusion rules for minimal models (see discussion in [27,28]) appear in (2.19), although they can be seen via matrix models.

We now generalize the cases known in the literature for $N > 3$ including screening charges. Repeating the zero-mode technique in the most general case of an N-point function with arbitrary screening charges we have

$$\begin{aligned}\mathcal{A}_N^{nm} = (-\pi)^3\left(\frac{\mu}{\pi}\right)^s \Gamma(-s)\prod_{i=1}^{N}\int d^2 z_i \prod_{j=1}^{n}\int \frac{d^2 t_j}{n!}\prod_{k=1}^{m}\int\frac{d^2 r_k}{m!} \\ \times\prod_{l=1}^{s}\int d^2 w_l \left\langle e^{ik_i X(z_i)} e^{id_+X(t_j)} e^{id_-X(r_n)}\right\rangle_0 \left\langle e^{\beta_i\phi(z_i)} e^{\alpha_+\phi(w_l)}\right\rangle_0 \quad ,\end{aligned} \tag{2.31}$$

where $s = -\frac{1}{\alpha_+}(\sum_{i=1}^N \beta_i + Q)$ and a factor π^3/α_+ has been absorbed in the measure of the path integral. Fixing the $SL(2,\mathbb{C})$ symmetry we get:

$$\begin{aligned}\mathcal{A}_N^{nm} &= (-\pi)^3\left(\frac{\mu}{\pi}\right)^s \Gamma(-s) I_N^{nm} \quad , \\ I_N^{nm} &= \int\prod_{j=4}^{N} d^2 z_j |z_j|^{2\alpha_j}|1 - z_j|^{2\beta_j}\prod_{i<j=4}^{N}|z_i - z_j|^{4\rho_{ij}} \\ &\times\int\prod_{i=1}^{s} d^2 w_i |w_i|^{2\alpha}|1 - w_i|^{2\beta}\prod_{i<j=1}^{s}|w_i - w_j|^{4\rho}\prod_{i=1}^{s}\prod_{j=4}^{N}|w_i - z_j|^{2p_j} \\ &\times\prod_{i=1}^{n} d^2 t_i |t_i|^{2\tilde{\alpha}}|1 - t_i|^{2\tilde{\beta}}\prod_{i<j}^{n}|t_i - t_j|^{4\tilde{\rho}}\prod_{i=1}^{n}\prod_{j=4}^{N}|z_j - t_i|^{2\tilde{\alpha}_j} \\ &\times\int\prod_{i=1}^{m} d^2 r_i |r_i|^{2\tilde{\alpha}'}|1 - r_i|^{2\tilde{\beta}'}\prod_{i<j=1}^{m}|r_i - r_j|^{4\tilde{\rho}'}\prod_{i=1}^{m}\prod_{j=4}^{N}|z_j - r_i|^{2\tilde{\alpha}_j'} \\ &\times\prod_{i=1}^{n}\prod_{j=1}^{m}|t_i - r_j|^{-4} \quad ,\end{aligned} \tag{2.32}$$

where

$$\begin{aligned}\alpha_j &= k_1 k_j - \beta_1 \beta_j \quad , \quad \tilde{\alpha}_j = d_+ k_j \\ \beta_j &= k_2 k_j - \beta_2 \beta_j \quad , \quad \tilde{\alpha}_j = d_- k_j \\ \rho_{lj} &= \frac{1}{2}(k_l k_j - \beta_l \beta_j) \quad , p_j = -\alpha_+ \beta_j \quad , \quad 4 \le j,l \le N \quad .\end{aligned} \tag{2.33}$$

The integral above has been calculated by Di Francesco and Kutasov[9], for the case $n = m = 0$. For arbitrary n, m we shall use the same technique. Notice that translation invariance

$$w_i \to 1 - w_i \, , \, z_i \to 1 - z_i \, , \, t_i \to 1 - t_i \, , \, r_i \to 1 - r_i$$

implies the symmetry relations

$$\alpha \leftrightarrow \beta \, , \, \alpha_j \leftrightarrow \beta_j \, , \, \tilde{\alpha} \leftrightarrow \tilde{\beta} \, , \, \tilde{\alpha}' \leftrightarrow \tilde{\beta}'$$

so that after the elimination of the remaining parameters as a function of α, β, p_j and ρ $(j = 4, 5, \cdots, N-1)$, I_N^{nm} exhibits an α-β symmetry

$$I_N^{nm}(\alpha, \beta, p_j, \rho) = I_N^{nm}(\beta, \alpha, p_j, \rho) \quad . \tag{2.34}$$

Similarly by the inversion of all variables w_i, z_i, t_i, r_i we have:

$$I_N^{nm}(\alpha, \beta, p_j, \rho) = I_N^{nm}(-2 - \alpha - \beta - 2\rho(s-1) - p_N - P, \beta, p_j, \rho) \tag{2.35}$$

where $P = \sum_{j=4}^{N-1} p_j$. Further information about I_N^{nm} can be obtained in the limit $\alpha \to \infty$ (or $\beta \to \infty$), by using a technique applied by Dotsenko and Fatteev[16] in the case of contour integrals, we found

$$I_N^{nm} \approx \alpha^{2\beta + 2\rho(s-N-n+3)+2P-2m} \tag{2.36}$$

where we have used the kinematics: $k_1, k_2, \cdots, k_{N-1} \ge \alpha_0$, $k_N < \alpha_0$ and assumed $\alpha_0 < 0$ To eliminate most of the parameters as a function of α, β, p_j and ρ we use (2.4), (2.6) and momentum conservation. After such elimination the symmetry (2.25) becomes:

$$I_N^{nm}(\alpha, \beta, p_j, \rho) = I_N^{nm}(m - 1 - P - \alpha - \beta + \rho(N + n - 1 - s), \beta, p_j, \rho) \tag{2.37}$$

Using Stirling's formula, it is not difficult to check that the following Ansatz is consistent with (2.34), (2.36) and (2.37):

$$\begin{aligned}\mathcal{A}_N^{nm} &= f_N^{nm}(\rho, p_j)\Delta(\rho - \alpha)\Delta(\rho - \beta)\Delta(1 - m + P + \alpha + \beta + \rho(s + 2 - N - n)) \\ \mathcal{A}_N^{nm} &= f_N^{nm}(\rho, p_j)\prod_{j=1}^{3}\Delta\left(\frac{1}{2}(\beta_j^2 - k_j^2)\right) \quad .\end{aligned} \tag{2.38}$$

Now we can fix $f_N^{nm}(\rho, p_j)$ by using the 3-point function $\mathcal{A}_3^{nm}$.

$$\mathcal{A}_N^{nm}(k_1, k_2, k_j \to 0, k_N) = (-\pi)^{N-3}\frac{\partial}{\partial \mu}^{N-3}\mathcal{A}_3^{nm}(k_1, k_2, k_N) \quad , \quad 3 \le j \le N-1 \quad . \tag{2.39}$$

Now using the result for $\mathcal{A}_3^{nm}$ we get:

$$f_N^{nm}(\rho, p_j) = [-\pi\Delta(\rho^{-1})]^m[-\pi\Delta(\rho)]^n\left(\frac{\partial^{N-3}}{\partial_\mu}\mu^{s+N-3}\right)[\Delta(-\rho)]^s\prod_{j=4}^{N}(-\pi)\Delta(\frac{1}{2}(\beta_j^2 - k_j^2)) \quad . \tag{2.40}$$

We finally return to (2.38) and obtain

$$\begin{aligned}\mathcal{A}_N^{nm} &= (s + N - 3)(s + N - 4)\cdots(s+1)\,[\mu\Delta(-\rho)]^s \\ &\quad [-\pi\Delta(\rho^{-1})]^m\,[-\pi\Delta(\rho)]^n\prod_{j=1}^{N}(-\pi)\Delta(\frac{1}{2}(\beta_j^2 - k_j^2)) \quad ,\end{aligned} \tag{2.41}$$

therefore, redefining the screening operators, T_{k_j} and μ as before, we have:

$$\mathcal{A}_N^{nm} = \frac{\partial^{N-3}}{\partial \mu}\mu^{s+N-3} \tag{2.42}$$

which is a remarkable result; however, it is valid only in the kinematic region already mentioned. In order to extend for general k, we can use the same technique as used in [20]. Notice that the amplitude (2.41) factorizes as in the case without screening charges.

3- Supersymmetric Correlators

In a recent paper[19] we have calculated the 3- and 4-point NS correlations functions using DHK formulation[17] of super Liouville theory coupled to superconformal matter on the sphere without screening charges. The total action S is given by the sum of the super Liouville action S_{SL} and the matter piece S_M,

$$\begin{aligned}S_{SL} &= \frac{1}{4\pi}\int d^2\mathbf{z}\hat{E}\left(\frac{1}{2}\hat{D}_\alpha\Phi_{SL}\hat{D}^\alpha\Phi_{SL} - Q\hat{Y}\Phi_{SL} - 4i\mu e^{\alpha_+\Phi_{SL}}\right) \quad , \\ S_M &= \frac{1}{4\pi}\int d^2\mathbf{z}\hat{E}(\frac{1}{2}\hat{D}_\alpha\Phi_M\hat{D}^\alpha\Phi_M + 2i\alpha_0\hat{Y}\Phi_M) \quad ,\end{aligned} \tag{3.1}$$

where Φ_{SL}, Φ_M are super Liouville and matter superfields respectively. The central charge of the matter sector is $c = \frac{3}{2}\hat{c}$, $(\hat{c} = 1 - 8\alpha_0^2)$. Analogously to the bosonic case the parameters Q and $\alpha_\pm$ are given by (compare with (2.2))

$$Q = 2\sqrt{1 + \alpha_0^2} \quad , \quad \alpha_\pm = -\frac{Q}{2} \pm \frac{1}{2}\sqrt{Q^2 - 4} = -\frac{Q}{2} \pm |\alpha_0| \quad , \quad \alpha_+\alpha_- = 1 \quad . \tag{3.2}$$

We shall call $\bar{\Psi}_{NS}$ the gravitationally dressed primary superfields, whose form is given by $\bar{\Psi}_{NS}(\mathbf{z}_i, k_i) = d^2\mathbf{z}\hat{E}e^{ik\Phi_M(\mathbf{z})}e^{\beta(k)\Phi_{SL}(\mathbf{z})}$, where

$$\beta(k) = -\frac{Q}{2} + |k - \alpha_0| \quad . \tag{3.3}$$

The calculation of the three-point function of the primary superfield $\tilde{\Psi}_{NS}$, involves the expression:

$$\left\langle \prod_{i=1}^{3} \int \tilde{\Psi}_{NS}(\mathbf{z}_i, k_i) \right\rangle \equiv \int [\mathcal{D}_{\hat{E}}\Phi_{SL}][\mathcal{D}_{\hat{E}}\Phi_M] \prod_{i=1}^{3} \tilde{\Psi}_{NS}(\mathbf{z}_i, k_i) e^{-S} \quad . \tag{3.4}$$

We closely follow the method already used in the bosonic case. After integrating over the bosonic zero modes we get

$$\left\langle \prod_{i=1}^{3} \int \tilde{\Psi}_{NS}(\mathbf{z}_i, k_i) \right\rangle \equiv 2\pi\delta \left(\sum_{i=1}^{3} k_i - 2\alpha_0 \right) \mathcal{A}_3(k_1, k_2, k_3) \quad ,$$

$$\mathcal{A}(k_1, k_2, k_3) = \Gamma(-s)(\frac{-\pi}{2})^3(\frac{i\mu}{\pi})^s \left\langle \int \prod_{i=1}^{3} d^2\tilde{\mathbf{z}}_i e^{ik_i\Phi_M(\tilde{\mathbf{z}}_i)} e^{\beta_i\Phi_{SL}(\tilde{\mathbf{z}}_i)} \left(\int d^2\mathbf{z} e^{\alpha_+\Phi_{SL}(\mathbf{z})} \right)^s \right\rangle_0 \tag{3.5}$$

where $\langle \cdots \rangle_0$ denotes again the expectation value evaluated in the free theory ($\mu = 0$) and we have absorbed the factor $[\alpha_+(-\pi/2)^3]^{-1}$ into the normalization of the path integral. the parameter s is defined as in the bosonic case (see (2.6)).

For s non-negative integer, after fixing the $\widehat{SL}_2$ gauge,[30] $\tilde{z}_1 = 0\,,\ \tilde{z}_2 = 1\,,\ \tilde{z}_3 = \infty\,,\ \tilde{\theta}_2 = \tilde{\theta}_3 = 0\,,\ \tilde{\theta}_1 = \theta$, in components ($\Phi_{SL} = \phi + \theta\psi + \bar{\theta}\bar{\psi}$) (the integral above is the supersymmetric generalization of (B.9) of Ref.[16]) we have

$$\mathcal{A}(k_1, k_2, k_3) = \Gamma(-s)(\frac{-\pi}{2})^3(\frac{i\alpha_+^2\mu}{\pi})^s \beta_1^2$$
$$\times \int \prod_{i=1}^{s} d^2 z_i \prod_{i=1}^{s} |z_i|^{-2\alpha+\beta_1} |1 - z_i|^{-2\alpha+\beta_2} \prod_{i<j}^{s} |z_i - z_j|^{-2\alpha_+^2} \langle \overline{\psi}\psi(0)\overline{\psi}\psi(z_1) \cdots \overline{\psi}\psi(z_s) \rangle_0 \quad . \tag{3.6}$$

Observe that this is non-vanishing only for s odd ($s = 2l+1$). One may evaluate $\langle \overline{\psi} \cdots \overline{\psi} \rangle_0$ and $\langle \psi \cdots \psi \rangle_0$ independently, since the rest of the integrand is symmetric, one may write the result in a simple form by relabelling coordinates:

$$\mathcal{A}_3(k_1, k_2, k_3) = \Gamma(-s)(\frac{-\pi}{2})^3 \frac{1}{\alpha_+^2} (\frac{i\alpha_+^2\mu}{\pi})^s \alpha^2 (-1)^{\frac{s+1}{2}} s!!$$
$$\times \int \prod_{i=1}^{s} d^2 z_i \prod_{i=1}^{s} |z_i|^{2\alpha} |1 - z_i|^{2\beta} \prod_{i<j}^{s} |z_i - z_j|^{4\rho} \prod_{i=1}^{\frac{s-1}{2}} |z_{2i-1} - z_{2i}|^{-2} |z_s|^{-2}$$

Redefining the variables as $z_s \equiv w\,,\quad z_{2i-1} \equiv \zeta_i$ and $z_{2i} \equiv \eta_i$ we have:

$$\mathcal{A}(k_1, k_2, k_3) = -i\frac{-\pi}{2}^3 \Gamma(-s)\Gamma(s+1) \frac{1}{\alpha_+^2} \left(\frac{\alpha_+^2\mu}{\pi} \right)^s I^l(\alpha, \beta; \rho) \quad , \tag{3.7}$$

where

$$I^l(\alpha, \beta; \rho) = \frac{1}{2^l l!} \alpha^2 \int d^2 w \prod_{i=1}^{l} d^2\zeta_i d^2\eta_i |w|^{2\alpha-2} |1 - w|^{2\beta} \prod_{i=1}^{l} |w - \zeta_i|^{4\rho} |w - \eta_i|^{4\rho}$$
$$\times \prod_{i=1}^{l} |\zeta_i|^{2\alpha} |1 - \zeta_i|^{2\beta} |1 - \eta_i|^{2\beta} \prod_{i,j}^{l} |\zeta_i - \eta_j|^{4\rho} \prod_{i<j}^{l} |\zeta_i - \zeta_j|^{4\rho} |\eta_i - \eta_j|^{4\rho} \prod_{i=1}^{l} |\zeta_i - \eta_i|^{-2} \quad , \tag{3.8}$$

and α, β, ρ are defined as before. In ref.[19] we calculated I^l in detail by using the symmetries $I^l(\alpha, \beta; \rho) = I^l(\beta, \alpha; \rho)$, $I^l(\alpha, \beta; \rho) = I^l(-1 - \alpha - \beta - 4l\rho, \beta; \rho)$ and looking at its large α behavior we obtained:

$$I^l(\alpha, \beta; \rho) = -\frac{\pi^{2l+1}}{2^{2l}} \left[\Delta\left(\frac{1}{2} - \rho \right) \right]^{2l+1} \prod_{i=1}^{l} \Delta(2i\rho) \prod_{i=1}^{l} \Delta\left(\frac{1}{2} + (2i+1)\rho \right)$$
$$\times \prod_{i=0}^{l} \Delta(1 + \alpha + 2i\rho)\Delta(1 + \beta + 2i\rho)\Delta(-\alpha - \beta + (2i - 4l)\rho)$$
$$\times \prod_{i=1}^{l} \Delta(\frac{1}{2} + \alpha + (2i-1)\rho)\Delta(\frac{1}{2} + \beta + (2i-1)\rho)\Delta(-\frac{1}{2} - \alpha - \beta + (2i - 4l - 1)\rho) \tag{3.9}$$

We can choose, $k_1, k_3 \geq \alpha_0, k_2 \leq \alpha_0$. We proceed now as in the bosonic case, obtaining for the parameter β,

$$\beta = \begin{cases} \rho(1 - s) & (\alpha_0 > 0) \\ -\frac{1}{2} - \rho s & (\alpha_0 < 0) \end{cases} \quad . \tag{3.10}$$

Now we are ready to write down the amplitude. For $\alpha_0 < 0$ we have the non-trivial amplitude:

$$\mathcal{A}(k_1, k_2, k_3) = (\frac{-i\pi}{2})^3 \left[\frac{\mu}{2}\Delta\left(\frac{1}{2} - \rho \right) \right]^s \Delta\left(\frac{1}{2} - \frac{s}{2} \right) \Delta\left(1 + \alpha - (s-1)\rho\right) \Delta\left(\frac{1}{2} - \alpha + \rho \right)$$
$$= \left[\frac{\mu}{2}\Delta\left(\frac{1}{2} - \rho \right) \right]^s \prod_{j=1}^{3} (-\frac{i\pi}{2}) \Delta\left(\frac{1}{2}[1 + \beta_j^2 - k_j^2] \right) \tag{3.11}$$

In the case $\hat{c} = 1$, we obtain for the external legs, renormalization factors of the form $\Delta(1 - |k_i|)$, which should be compared to the bosonic case (2.30); it permits as well comparison to super matrix model as well, whenever those are available.

By redefining the cosmological constant and the primary superfield $\tilde{\Psi}_{NS}$

$$\mu \to \frac{2}{\Delta\left(\frac{1}{2} - \rho\right)}\mu \quad , \quad \tilde{\Psi}_{NS}(k_j) \to \frac{1}{(-\frac{i}{2}\pi)\Delta\left(\frac{1}{2}[1 + \beta_j^2 - k_j^2]\right)} \tilde{\Psi}_{NS}(k_j) \quad , \tag{3.12}$$

we get

$$\mathcal{A}_3(k_1, k_2, k_3) = \mu^s \quad . \tag{3.13}$$

As in the bosonic case we have a remarkably simple result. The only differences with respect to the bosonic case are in the details of the renormalization factors. Compare (3.12) with (2.11). Note that the singular point at the renormalization of the cosmological constant is $\rho = -1$ in the bosonic case, which corresponds to $c = 1$, and $\rho = -\frac{1}{2}$ in the supersymmetric case, corresponding to $\hat{c} = 1$ or $c = 3/2$.

We shall now generalize the above result to the case which includes screening charges in the supermatter sector. We consider n charges $e^{id_+\Phi_M}$ and m charges $e^{id_-\Phi_M}$, where $d_\pm$ are solutions of the equation $\frac{1}{2}d(d-2\alpha_0) = \frac{1}{2}$. After integrating over the matter and Liouville zero modes we get

$$\left\langle \prod_{i=1}^{3} \int \tilde{\Psi}_{NS}(\tilde{\mathbf{z}}_i, k_i) \prod_{i=1}^{n} \int \frac{d^2\mathbf{t}_i}{n!} e^{id_+\Phi_M(\mathbf{t}_i)} \prod_{i=1}^{m} \int \frac{d^2\mathbf{r}_i}{m!} e^{id_-\Phi_M(\mathbf{r}_i)} \right\rangle$$

$$\equiv 2\pi\delta\left(\sum_{i=1}^{3} k_i + nd_+ + md_- - 2\alpha_0\right) \mathcal{A}_3^{nm}(k_1,k_2,k_3)$$

$$\mathcal{A}_3^{nm}(k_1,k_2,k_3) = \Gamma(-s)(\frac{-\pi}{2})^3(\frac{i\mu}{\pi})^s \left\langle \prod_{i=1}^{n} \int \frac{d^2\mathbf{t}_i}{n!} e^{id_+\Phi_M(\mathbf{t}_i)} \prod_{i=1}^{m} \int \frac{d^2\mathbf{r}_i}{m!} e^{id_-\Phi_M(\mathbf{r}_i)} \right.$$
$$\left. \times \int \prod_{i=1}^{3} d^2\tilde{\mathbf{z}}_i e^{ik_i\Phi_M(\tilde{\mathbf{z}}_i)} e^{\beta_i\Phi_{SL}(\tilde{\mathbf{z}}_i)} \left(\int d^2\mathbf{z} e^{\alpha_+\Phi_{SL}(\mathbf{z})} \right)^s \right\rangle_0, \tag{3.14}$$

Integrating over the Grasmann variables and fixing the $\widehat{SL(2)}$ symmetry as before ($\tilde{z}_1 = 0$, $\tilde{z}_2 = 1$, $\tilde{z}_3 = \infty$, $\tilde{\theta}_1 = \theta$, $\tilde{\theta}_2 = \tilde{\theta}_3 = 0$) we obtain (using $d_+d_- = -\alpha_+\alpha_- = -1$)

$$\mathcal{A}_3^{nm}(k_1,k_2,k_3) = \Gamma(-s)\left(\frac{-\pi}{2}\right)^3 \left(\frac{i\mu\alpha_+^2}{\pi}\right)^s \frac{(-d_+^2)^n}{n!} \frac{(-d_-^2)^m}{m!}$$
$$\times \prod_{i=1}^{n} \int d^2t_i |t_i|^{-2d_+k_1} |1-t_i|^{-d_+k_2} \prod_{i<j}^{n} |t_i - t_j|^{2d_+^2}$$
$$\times \prod_{i=1}^{m} \int d^2r_i |r_i|^{-2d_-k_1} |1-r_i|^{-2d_-k_2} \prod_{i<j}^{m} |r_i - r_j|^{2d_-^2} \prod_{i=1}^{n}\prod_{j=1}^{m} |t_i - r_j|^{-2}$$
$$\times \prod_{i=1}^{s} \int d^2z_i |z_i|^{-2\alpha_+\beta_1} |1-z_i|^{-2\alpha_+\beta_2} \prod_{i<j}^{s} |z_i - z_j|^{-2\alpha_+^2}$$
$$\times \left\langle (\beta_1^2 \overline{\psi}\psi(0) - k_1^2\overline{\xi}\xi(0)) \prod_{i=1}^{n} \overline{\xi}\xi(t_i) \prod_{i=1}^{m} \overline{\xi}\xi(r_i) \prod_{i=1}^{s} \overline{\psi}\psi(z_i) \right\rangle_0 . \tag{3.15}$$

Since the vacuum expectation value of an odd number of $\overline{\psi}\psi$ (or $\overline{\xi}\xi$) operators is zero we have only two non-trivial cases: in the first case $n+m =$ odd, $s =$ even and in the

second one $n+m =$ even, $s =$ odd. Thus we have (see also [21] for comparison)

$$\mathcal{A}_3^{nm}(k_1,k_2,k_3) = \Gamma(-s)\left(\frac{-\pi}{2}\right)^3 \left(\frac{i\mu\alpha_+^2}{\pi}\right)^s \frac{(-d_+^2)^n}{n!} \frac{(-d_-^2)^m}{m!}$$
$$\times \begin{cases} I_M^{nm}(\tilde{\alpha},\tilde{\beta};\overline{\rho}) \times I_G^s(\alpha,\beta;\rho), & n+m = \text{even}, s = \text{odd} \\ J_M^{nm}(\tilde{\alpha},\tilde{\beta};\overline{\rho}) \times J_G^s(\alpha,\beta;\rho), & n+m = \text{odd}, s = \text{even} \end{cases} \tag{3.16}$$

where

$$I_M^{nm}(\tilde{\alpha},\tilde{\beta};\overline{\rho}) = \prod_{i=1}^{n} \int d^2t_i |t_i|^{2\tilde{\alpha}} |1-t_i|^{2\tilde{\beta}} \prod_{i<j}^{n} |t_i - t_j|^{2\overline{\rho}}$$
$$\times \prod_{i=1}^{m} \int d^2r_i |r_i|^{2\tilde{\alpha}'} |1-r_i|^{2\tilde{\beta}'} \prod_{i<j}^{m} |r_i - r_j|^{2\overline{\rho}'} \prod_{i=1}^{n}\prod_{j=1}^{m} |t_i - r_j|^{-2}$$
$$\times \left\langle \prod_{i=1}^{n} \overline{\xi}\xi(t_i) \prod_{i=1}^{m} \overline{\xi}\xi(r_i) \right\rangle_0 , \tag{3.17}$$

$$I_G^s(\alpha,\beta;\rho) = \alpha^2 \int \prod_{i=1}^{s} d^2z_i \prod_{i=1}^{s} |z_i|^{2\alpha} |1-z_i|^{2\beta} \prod_{i<j}^{s} |z_i - z_j|^{4\rho} \left\langle \overline{\psi}\psi(0) \prod_{i=1}^{s} \overline{\psi}\psi(z_i) \right\rangle_0 \tag{3.18}$$

with $\overline{\rho} = d_+^2, \overline{\rho}' = d_-^2$ and $\alpha,\beta,\rho,\tilde{\alpha},\tilde{\alpha}',\tilde{\beta},\tilde{\beta}'$ defined as before. Note that I_M^{nm} is the supersymmetric generalization of (B.10) of the second reference in [16]. The integral J_M^{nm} differs from I_M^{nm} by the introduction of a factor $\overline{\xi}\xi(0)$ and J_G^s can be obtained from I_G^s by dropping $\overline{\psi}\psi(0)$. Henceforth we assume, for simplicity, $n+m =$ even, $s =$ odd. We will work out explicitly only the case n,m even. However, the final result for the amplitude does not depend on which case we choose. In ref. [24] we have calculated I_M^{nm} for n and

m even and we get:

$$
\begin{aligned}
I_M^{nm}(\tilde{\alpha},\tilde{\beta};\overline{\rho}) &= (-)^{\frac{n+m}{2}}\frac{\pi^{n+m}}{2^{n+m}}n!m!\left(-\frac{\overline{\rho}}{2}\right)^{-2nm}\left[\Delta\left(\frac{1}{2}-\frac{\overline{\rho}}{2}\right)\right]^n\left[\Delta\left(\frac{1}{2}-\frac{\overline{\rho}'}{2}\right)\right]^m \\
&\times\prod_1^{\frac{n}{2}}\Delta(i\overline{\rho})\Delta\left(\frac{1}{2}+\overline{\rho}\left(i-\frac{1}{2}\right)\right)\prod_1^{\frac{m}{2}}\Delta(i\overline{\rho}'-\frac{n}{2})\Delta\left(\frac{1}{2}-\frac{n}{2}-\overline{\rho}'\left(i-\frac{1}{2}\right)\right) \\
&\times\prod_{i=0}^{\frac{n}{2}-1}\Delta(1+\tilde{\alpha}+i\overline{\rho})\Delta(1+\tilde{\beta}+i\overline{\rho})\Delta(m-\tilde{\alpha}-\tilde{\beta}+\overline{\rho}(i-n+1)) \\
&\times\prod_{i=1}^{\frac{n}{2}}\Delta(\frac{1}{2}+\tilde{\alpha}+(i-\frac{1}{2})\overline{\rho})\Delta(\frac{1}{2}+\tilde{\beta}+(i-\frac{1}{2})\overline{\rho})\Delta(-\frac{1}{2}-\tilde{\alpha}+m-\tilde{\beta}+\overline{\rho}(i-n+\frac{1}{2})) \\
&\times\prod_{i=0}^{\frac{m}{2}-1}\Delta(1+\tilde{\alpha}'-\frac{n}{2}+i\overline{\rho}')\Delta(1-\frac{n}{2}+\tilde{\beta}'+i\overline{\rho}')\Delta(\frac{n}{2}-\tilde{\alpha}'-\tilde{\beta}'+\overline{\rho}'(i-m+1)) \\
&\times\prod_{i=1}^{\frac{m}{2}}\Delta(\frac{1}{2}-\frac{n}{2}+\tilde{\alpha}'+(i-\frac{1}{2})\overline{\rho}')\Delta(\frac{1}{2}-\frac{n}{2}+\tilde{\beta}'+(i-\frac{1}{2})\overline{\rho}')\Delta(-\frac{1}{2}+\frac{n}{2}-\tilde{\alpha}'-\tilde{\beta}'+\overline{\rho}'(i-m+\frac{1}{2}))
\end{aligned}
\tag{3.19}
$$

In the case where $s = 2l+1$ the gravitational contribution to $\mathcal{A}_3^{nm}(k_1,k_2,k_3)$, i.e, I_G^s is just the same as in the case without screening charges, thus from the last section we have the supersymmetric generalization of (B.9) of ref.[16]:

$$
\begin{aligned}
I_G^s &= (-)^{\frac{s-1}{2}}\frac{\pi^s}{2^{s-1}}s!\left[\Delta\left(\frac{1}{2}-\rho\right)\right]^s\prod_{i=1}^{\frac{s-1}{2}}\Delta(2i\rho)\prod_{i=0}^{\frac{s-1}{2}}\Delta(\frac{1}{2}+(2i+1)\rho) \\
&\times\prod_{i=0}^{\frac{s-1}{2}}\Delta(1+\tilde{\alpha}+2i\tilde{\rho})\Delta(1+\tilde{\beta}+2i\overline{\rho})\Delta(-\tilde{\alpha}-\tilde{\beta}+2\overline{\rho}(i-s+1)) \\
&\times\prod_{i=0}^{\frac{s-1}{2}}\Delta(\frac{1}{2}+\tilde{\alpha}+(2i-1)\overline{\rho})\Delta(\frac{1}{2}+\tilde{\beta}+(2i-1)\overline{\rho})\Delta(-\frac{1}{2}-\tilde{\alpha}-\tilde{\beta}+\overline{\rho}(2i-2s+1))
\end{aligned}
\tag{3.20}
$$

To obtain $\mathcal{A}_3^{nm}(k_1,k_2,k_3)$ (see (3.16)) we have to calculate $I^{nm}\times I_G^s$. Using the same kinematics as in the case without screening charges, and considering that $\alpha_0<0$ it is easy

to deduce:

$$
\begin{aligned}
&\tilde{\alpha} = \alpha-2\rho \quad , \quad \tilde{\alpha}' = -1+\frac{\rho^{-1}\alpha}{2} \\
&\overline{\rho} = -2\rho \quad , \quad \overline{\rho}' = -\frac{\rho^{-1}}{2} \\
&\beta = -\frac{1}{2}-\frac{m}{2}-(n+s)\rho \\
&\tilde{\beta} = -\beta-1 = (n+1)\rho+\frac{m}{2}-\frac{1}{2} \\
&\tilde{\beta}' = \frac{(n+s)}{2}+\frac{\rho^{-1}}{4}(m-1) \quad .
\end{aligned}
\tag{3.21}
$$

Substituting in (3.19) and (3.20) and using (3.16) we obtain a very involved expression (see also [26,21]):

$$
\begin{aligned}
&\mathcal{A}_3^{nm}(k_1,k_2,k_3)\\
&=\Gamma(-s)\left(\frac{-\pi}{2}\right)^3\left(\frac{i\mu}{\pi}\right)^s\alpha_+^{2(s-1)}(2\rho)^{n-m}(-)^{\frac{n+m+s-1}{2}}\frac{\pi^{s+n+m}}{2^{m+n+s-1}}\rho^{-2mn}s!\\
&\times\left[\Delta(\frac{1}{2}+\rho)\right]^n\left[\Delta(\frac{1}{2}+\frac{\rho^{-1}}{4})\right]^m\\
&\times\prod_{i=1}^{\frac{n}{2}}\Delta(-2i\rho)\Delta(\frac{1}{2}+\rho(1-2i))\prod_{i=1}^{\frac{m}{2}}\Delta(-\frac{n}{2}-\frac{i\rho^{-1}}{2})\Delta\left(\frac{1}{2}-\frac{n}{2}+\left(\frac{1}{4}-\frac{i}{2}\right)\rho^{-1}\right)\\
&\times\prod_{i=0}^{\frac{n}{2}-1}\Delta\left(\frac{1}{2}+\frac{m}{2}+\rho(n+s-2i)\right)\prod_{i=1}^{\frac{n}{2}}\Delta\left(\frac{m}{2}+\rho(n+s+1-2i)\right)\\
&\times\prod_{i=0}^{\frac{m}{2}-1}\Delta\left(1+\frac{s}{2}-\frac{\rho^{-1}}{2}(i+\frac{1}{2}-\frac{n}{2})\right)\prod_{i=1}^{\frac{m}{2}}\Delta\left(\frac{1}{2}+\frac{s}{2}-\frac{\rho^{-1}}{2}(i-\frac{m}{2})\right)\\
&\times\left[\Delta(\frac{1}{2}-\rho)\right]^s\prod_{i=1}^{\frac{s-1}{2}}\Delta(2i\rho)\prod_{i=0}^{\frac{s-1}{2}}\Delta\left(\frac{1}{2}+(2i+1)\rho\right)\\
&\times\prod_{i=0}^{\frac{s-1}{2}}\Delta(\frac{1}{2}-\frac{m}{2}+(2i-n-s)\rho)\prod_{i=1}^{\frac{s-1}{2}}\Delta(-\frac{m}{2}+(2i-1-n-s)\rho)\\
&\times\prod_{i=1}^{\frac{n}{2}}\Delta(\frac{1}{2}+\alpha-\rho(2i+1))\Delta(\frac{1}{2}+\frac{m}{2}-\alpha-\rho(s-n-2+2i))\\
&\times\prod_{i=0}^{\frac{s-1}{2}}\Delta(\frac{m}{2}+\frac{1}{2}-\alpha+(2i-s+n+2)\rho)\prod_{i=1}^{\frac{s-1}{2}}\Delta(\frac{1}{2}+\alpha+(2i-1)\rho)\\
&\times\prod_{i=1}^{\frac{n}{2}}\Delta(\frac{m}{2}-\alpha-\rho(2i-n+s-1))\Delta(1+\alpha-2i\rho)\\
&\times\prod_{i=0}^{\frac{s-1}{2}}\Delta(1+\alpha+2i\rho)\prod_{i=1}^{\frac{s-1}{2}}\Delta(\frac{m}{2}-\alpha+(2i-s+n+1)\rho)\\
&\times\prod_{i=1}^{\frac{m}{2}}\Delta(\frac{1}{2}-\frac{s}{2}-\frac{\rho^{-1}}{2}(i+\alpha-\frac{m}{2}))\Delta(-\frac{n}{2}-\frac{\rho^{-1}}{2}(i-1-\alpha))\\
&\times\prod_{i=1}^{\frac{m}{2}}\Delta(-\frac{n}{2}-\frac{1}{2}-\frac{\rho^{-1}}{2}(i-\alpha-\frac{1}{2}))\Delta(1-\frac{s}{2}-\frac{\rho^{-1}}{2}(i-\frac{(m+1)}{2}+\alpha))\quad .
\end{aligned}
\tag{3.22}
$$

In order to obtain a simple expression for the amplitude we have to combine in each term the matter and the gravitational parts as in the bosonic case. The calculation is more

complicate now, but we finally get

$$
\begin{aligned}
\mathcal{A}_3^{nm}(k_1,k_2,k_3)&=\left(-\frac{\pi}{2}\right)^3\left[\frac{\mu}{2}\Delta(\frac{1}{2}-\rho)\right]^s\left[-\frac{i\pi}{2}\Delta(\frac{1}{2}+\frac{\rho^{-1}}{4})\right]^m\left[-\frac{i\pi}{2}\Delta(\frac{1}{2}+\rho)\right]^n\\
&\times\Delta\left(\rho-\alpha+\frac{1}{2}\right)\Delta\left(\frac{1}{2}-\frac{n+s}{2}-\frac{m\rho^{-1}}{4}\right)\Delta(1-\frac{m}{2}+\alpha+(s-n-1)\rho)\\
&=\left[\frac{\mu}{2}\Delta\left(\frac{1}{2}-\rho\right)\right]^s\left[-\frac{i\pi}{2}\Delta\left(\frac{1}{2}+\frac{\rho^{-1}}{4}\right)\right]^m\left[-\frac{i\pi}{2}\Delta\left(\frac{1}{2}+\rho\right)\right]^n\\
&\times\prod_{i=1}^{3}\left(-\frac{i\pi}{2}\right)\Delta\left(\frac{1}{2}+\frac{1}{2}(\beta_i^2-k_i^2)\right)\quad .
\end{aligned}
\tag{3.23}
$$

Therefore after redefining the cosmological constant, the NS operators and the screening charges

$$
e^{id_+\Phi_M(t_i)}\to\left[-\frac{i\pi}{2}\Delta\left(\frac{1}{2}+\rho\right)\right]^{-1}e^{id_+\Phi_M(t_i)}
\tag{3.24a}
$$

$$
e^{id_-\Phi_M(t_i)}\to\left[-\frac{i\pi}{2}\Delta\left(\frac{1}{2}+\frac{\rho^{-1}}{4}\right)\right]^{-1}e^{id_-\Phi_M(t_i)}
\tag{3.24b}
$$

$$
\Psi_{NS}\to\left[-\frac{i\pi}{2}\Delta\left(\frac{1}{2}+\frac{1}{2}(\beta_i^2-k_i^2)\right)\right]^{-1}\Psi_{NS}
\tag{3.24c}
$$

$$
\mu\to\left[\frac{1}{2}\Delta\left(\frac{1}{2}-\rho\right)\right]^{-1}\mu\quad .
\tag{3.24d}
$$

we obtain the very simple result:

$$
\mathcal{A}_3^{nm}(k_1,k_2,k_3)=\mu^s\quad .
\tag{3.25}
$$

In view of the complexity of (3.22), the simplicity of the result is remarkable.

As in the bosonic case we can calculate ratios of correlation functions either using (3.23) or (3.25). We obtain in a generic kinematic region:

$$
R=\frac{\langle\Psi_{NS}(k_{r_1r_1'})\Psi_{NS}(k_{r_2r_2'})\Psi_{NS}(k_{r_3r_3'})\rangle^2\mathcal{Z}}{\prod_{j=1}^{3}\langle\Psi_{NS}(k_{r_jr_j'})\Psi_{NS}(k_{r_jr_j'})\rangle}
\tag{3.26}
$$

$$
R=(2\alpha_+)^3\frac{\prod_{i=1}^{3}|r_id_-+r_i'd_+|}{(2\rho-1)(2\rho+1)}\quad .
\tag{3.27}
$$

Compare with (2.27).

The above result agrees with other results[26,21] simultaneously obtained in the literature. Although the the continuations to non integer values of s used in [26] and [21] are not the same and, in principle, do not correspond to the procedure used here, the physical results seem to be independent of such details.

We now show that it is possible to obtain a simple result for the most general case of a N-point amplitude with an arbitrary number of screening charges ($\mathcal{A}_N^{nm}$). In that general case to compute the amplitudes we have to calculate the following integral

$$\mathcal{A}_N^{nm}(k_1,\cdots,k_N) = \frac{\Gamma(-s)}{-\alpha_+}\left(\frac{i\mu}{\pi}\right)^s \left\langle \prod_{i=1}^{N}\int d^2\tilde{\mathbf{z}}_i e^{ik_i\Phi_M(\tilde{\mathbf{z}}_i)+\beta_i\Phi_{SL}(\tilde{\mathbf{z}}_i)} \right. \\ \left. \times\prod_{i=1}^{n}\int d^2\mathbf{t}_i e^{id_+\Phi_M(\tilde{\mathbf{t}}_i)}\prod_{j=1}^{m}\int d^2\mathbf{r}_j e^{id_-\Phi_M(\tilde{\mathbf{r}}_j)}\prod_{j=1}^{s}\int d^2\mathbf{z}_j e^{\alpha_+\Phi_{SL}(\tilde{\mathbf{z}}_i)}\right\rangle_0 \tag{3.28}$$

where $s=-\frac{1}{\alpha_+}(\sum_{i=1}^{N}\beta_i+Q)$ and $\sum_{i=1}^{N}k_i+nd_++md_-=2\alpha_0$. After fixing the $\widehat{SL_2}$ symmetry as before and integrating over the Grassmann variables the amplitude becomes

$$\mathcal{A}_N^{nm}=\Gamma(-s)\left(-\frac{\pi}{2}\right)^3\left(\frac{i\mu\alpha_+^2}{\pi}\right)^s(-d_+^2)^n(-d_-^2)^m \\ \times\prod_{j=4}^{N}\int d^2\tilde{z}_j\prod_{i=1}^{n}\int d^2t_i\prod_{i=1}^{m}\int d^2r_i\prod_{i=1}^{s}d^2w_i|w_i|^{-2\alpha+\beta_1}|1-w_i|^{-2\alpha+\beta_2} \\ \times\prod_{i<j}|w_i-w_j|^{-2\alpha_+^2}\prod_{i=1}^{s}\prod_{j=4}^{N}|w_i-\tilde{z}_j|^{-2\alpha+\beta_j} \\ \prod_{j=4}^{N}|\tilde{z}_j|^{2(k_1k_j-\beta_1\beta_j)}|1-\tilde{z}_j|^{2(k_2k_j-\beta_2\beta_j)}\prod_{j<l=4}^{N}|\tilde{z}_j-\tilde{z}_l|^{2(k_jk_l-\beta_j\beta_l)} \\ \times\prod_{i=1}^{n}|t_i|^{2k_2d_+}|1-t_i|^{2k_2d_+}\prod_{i<j}^{n}|t_i-t_j|^{2d_+^2}\prod_{i=1}^{n}\prod_{j=1}^{m}|t_i-r_j|^{-2} \\ \times\prod_{i=1}^{m}|r_i|^{2k_jd_-}|1-r_i|^{2k_2d_-}\prod_{i<j}^{m}|r_i-r_j|^{2d_-^2}\prod_{i=1}^{n}\prod_{j=4}^{N}|t_i-\tilde{z}_j|^{2d_+k_j}\prod_{i=1}^{m}\prod_{j=4}^{N}|r_i-\tilde{z}_j|^{2d_-k_j} \\ \times\left\langle(\beta_1^2\overline{\psi}\psi(0)-k_1^2\overline{\xi}\xi(0))\prod_{j=4}^{N}(\beta_j^2\overline{\psi}\psi(\tilde{z}_j)-k_j^2\overline{\xi}\xi(\tilde{z}_j))\prod_{i=1}^{n}\overline{\xi}\xi(t_i)\prod_{i=1}^{m}\overline{\xi}\xi(r_i)\prod_{i=1}^{s}\overline{\psi}\psi(w_i)\right\rangle_0 \tag{3.29}$$

Now we have several terms which give non-trivial amplitudes, in the following we assume $m+n$ and $N+s$ even; thus we have:

$$\mathcal{A}_N^{nm}=\Gamma(-s)\left(-\frac{\pi}{2}\right)^3\left(\frac{i\mu\alpha_+^2}{\pi}\right)^s(-d_+^2)^n(-d_-^2)^m\left(\prod_{j=4}^{N}\beta_j^2\right)(\alpha_+)^{-2} \\ \times\alpha^2\prod_{j=4}^{N}\int d^2\tilde{z}_j\prod_{i=1}^{n}\int d^2t_i\prod_{i=1}^{m}\int d^2r_i\prod_{i=1}^{s}d^2w_i|w_i|^{2\alpha}|1-w_i|^{2\beta} \\ \times\prod_{i<j}|w_i-w_j|^{4\rho}\prod_{i=1}^{s}\prod_{j=4}^{N}|w_i-\tilde{z}_j|^{2p_j}\prod_{j=4}^{N}|\tilde{z}_j|^{2\alpha_j}|1-\tilde{z}_j|^{2\beta_j}\prod_{j<l=4}^{N}|\tilde{z}_j-\tilde{z}_l|^{2\rho_{jl}}$$

$$\times\prod_{i=1}^{n}|t_i|^{2\tilde{\alpha}}|1-t_i|^{2\tilde{\beta}}\prod_{i<j}^{n}|t_i-t_j|^{2\tilde{\rho}}\prod_{i=1}^{n}\prod_{j=1}^{m}|t_i-r_j|^{-2} \\ \times\prod_{i=1}^{m}|r_i|^{2\tilde{\alpha}'}|1-r_i|^{2\tilde{\beta}'}\prod_{i<j}^{m}|r_i-r_j|^{2\tilde{\rho}'}\times\prod_{i=1}^{n}\prod_{j=4}^{N}|t_i-\tilde{z}_j|^{2\tilde{\alpha}_j}\prod_{i=1}^{m}\prod_{j=4}^{N}|r_i-\tilde{z}_j|^{2\tilde{\alpha}'_j} \\ \times\left\langle\prod_{i=1}^{n}\overline{\xi}\xi(t_i)\prod_{i=1}^{m}\overline{\xi}\xi(r_i)\right\rangle_0\left\langle\overline{\psi}\psi(0)\prod_{j=1}^{s}\overline{\psi}\psi(w_i)\prod_{j=4}^{N}\overline{\psi}\psi(\tilde{z}_j)\right\rangle_0 \quad . \tag{3.30}$$

The definitions of the kinematics parameters are defined as in the bosonic case

We shall use the kinematics: $k_1,k_2,\cdots,k_{N-1}\geq\alpha_0$, $k_N<\alpha_0\leq 0$ in order to eliminate all parameters in terms of α,β,ρ and $p_j\,(4\leq j\leq N-1)$.

The symmetries:

$$\mathcal{A}_N^{nm}(\alpha,\beta,\rho,p_1,p_2,\cdots,p_{N-1})=\mathcal{A}_N^{nm}(\beta,\alpha,\rho,p_1,p_2,\cdots,p_{N-1}) \tag{3.31}$$

$$\mathcal{A}_N^{nm}(\alpha,\beta,\rho,p_1\cdots p_{N-1})=\mathcal{A}_N^{nm}(-\alpha-\beta+\frac{(m-1)}{2}-P+\rho(N+n-s-1),\beta,\rho,p_1\cdots p_{N-1})$$

with $P=\sum_{j=4}^{N-1}p_j$ and the large-α behaviour: $\mathcal{A}_N^{nm}(\alpha\to\infty)\sim\alpha^{1-m+2\beta+2\rho(s-N-n+3)+2P}$ motivate us to write down the ansatz:

$$\mathcal{A}_N^{nm}=f_N^{nm}(\rho,p_1,\cdots,p_{N-1})\Delta(\frac{1}{2}+\rho-\alpha)\Delta(\frac{1}{2}+\rho-\beta)\Delta(1-\frac{m}{2}+P+\alpha+\beta+\rho(2+s-n-N)) \tag{3.32}$$

Taking the limit $k_i\to 0$ $(3\leq i\leq N-1)$, which implies $p_j\to 2\rho\,(4\leq j\leq N-1)$, we can determine $f_N^{nm}(\rho,p_1,\cdots p_{N-1})$ using:

$$\mathcal{A}_N^{nm}(\alpha,\beta,\rho,k_i\to 0)=\left(-\frac{i\pi}{2}\right)^{N-3}\frac{\partial^{N-3}}{\partial_\mu}\mathcal{A}_3^{nm}(k_1,k_2,k_N) \tag{3.33}$$

and the result for $\mathcal{A}_3^{nm}$ (see (3.23)). We arrive at the result

$$\mathcal{A}_N^{nm}=(s+N-3)(s+N-4)\cdots(s+1)\left[\mu\Delta(\frac{1}{2}-\rho)\right]^s \\ \times\left[-\frac{i\pi}{2}\Delta(\frac{1}{2}+\rho)\right]^n\left[-\frac{i\pi}{2}\Delta(\frac{1}{2}+\frac{\rho^{-1}}{4})\right]^m\prod_{i=1}^{N}\left(-\frac{i\pi}{2}\right)^N\Delta(\frac{1}{2}(1+\beta_i^2-k_i^2)) \tag{3.34}$$

Redefining Ψ_{NS} , μ and the screening charges we have our final result:

$$\mathcal{A}_N^{nm}=\frac{\partial}{\partial\mu}^{N-3}\mu^{s+N-3} \quad , \tag{3.35}$$

which has the same functional form as the bosonic amplitude (2.42). As in that case the above result is correct only in the kinematic region used to calculate it and has to be continued outside this region.

4- Conclusion

In the first part of this paper we have generalized previous results[9] for the N-point tachyon correlator in Liouville theory coupled to conformal matter (on the sphere) with $c \leq 1$ to the case which includes screening charges in the matter sector. The results might be useful in understanding the issue of fusion rules in the calculation of the 3-point correlator (see discussion in [20]).

In the second part we have obtained the N-point NS-correlators in super Liouville coupled to $\hat{c} \leq 1$ matter (also on the sphere), including screening charges thus generalizing the results of [21,26] to the limit case $\hat{c} = 1$, $(N > 3)$, and the results of [20], obtained for $s = 0$, to any value of s.

We have obtained, explicit formulae for the corresponding 2D-integrals involved and the final form of the amplitudes factorizes in the N-external legs factors, confirming the results obtained in [20] (for $s = 0$) through a detailed analysis of the pole structure of the integrals.

In our calculations it was possible to see the singularity in the renormalization of the cosmological constant ($\mu \rightarrow \tilde{\mu}/\Delta(\frac{1}{2} - \rho)$) at the point $\hat{c} = 1$ ($\rho = -1/2$). This is similar to the bosonic case where $\mu \rightarrow \tilde{\mu}/\Delta(-\rho)$ and the $c = 1$, ($\rho = -1$) point is also singular. The final (renormalized) N-point amplitude has the same form of the bosonic one. The similarity with the bosonic case has been found before in the discrete approach [20,25].

Finally we should stress that our results must be continued to other kinematic regions, this is likely to be very similar to the bosonic case (without s.c.) worked out in [20].

As a further development it would be interesting to carry out analogous calculations in the case with $N = 2$ supersymmetry and see, among other aspects, whether the barrier at $c = 3$ indeed disappears. The inclusion of the Ramond sector (for $s \neq 0$) is also of interest.

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