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Abstract: The solutions of the Vlasov equation based on the Walecka model applied to hot and dense nuclear matter are obtained and discussed. It is observed that the effective mass of scalar and vector mesons raises only slightly with temperature for $T < 180MeV$ while the effective nucleon mass decreases. The collective longitudinal modes are obtained and the coupling between them is studied through the energy weighted sum rule. At finite temperature it is found that the zero sound mode merges in the continuum of particle-hole excitations.

1. INTRODUCTION

Heavy-ion collisions at high incident energies offer the opportunity of studying the properties of hot and dense hadronic matter formed in the collision. The properties of mesons are expected to be affected by the presence of hot dense matter. A particularly interesting issue is whether such modifications of mesons properties could be used to diagnose the state of matter produced in high energy heavy-ion collisions. Therefore, the accurate description of hot, dense matter is an important problem in theoretical physics. To construct a theory for heavy-ion collisions at high energies it is important to take into account relativistic dynamics and not just relativistic kinematics. The Walecka model [1] which is known to describe the saturation of nuclear matter and static properties of nuclei, provides us with a covariant framework for the description of hot and dense hadronic matter [2].

Recently we have used a relativistic Vlasov equation based on the Walecka model to study the longitudinal and transverse collective modes at zero temperature [3, 4], and we have obtained results similar to the more demanding calculations based on the one-loop expansion [5, 6, 7]. It has been previously demonstrated, in low energy heavy ion collisions, that the Vlasov equation is a good approximation to the time dependent Hartree-Fock equation [8]. Therefore, the relativistic Vlasov equation, which has been quite used lately to study heavy-ion collisions [9, 10], appears to be an alternative way to study relativistic systems.

The purpose of this work is to use a relativistic Vlasov equation based on the Walecka model to study the random-phase approximation (RPA) collective modes corresponding to small-amplitude oscillations around a stationary state in hot and dense nuclear matter. In section II the ground state of the system is determined and the thermal Vlasov equation is introduced. The discrete longitudinal collective normal modes are studied in section III and the orthogonality, completeness relations and sum rule fulfilled by the RPA modes are given in section IV. Finally, section V contains some conclusions.

2. THE GROUND STATE AND THE VLASOV EQUATION

In the Walecka model the nucleons are coupled to neutral scalar, Φ, and vector, V^μ, meson fields. In a classical approximation the energy of the nuclear system with particles and anti-particles, described respectively by the one-body phase-space distribution functions $n_+(\mathbf{r},\mathbf{p},t)$ and $n_-(\mathbf{r},\mathbf{p},t)$ which give the number of particles and anti-particles at the position $\mathbf{r}$ instant t with momentum $\mathbf{p}$ is

$$E = 4\int \frac{d^3r\,d^3p}{(2\pi)^3}\left(n_+(\mathbf{r},\mathbf{p},t)\varepsilon_+ + n_-(\mathbf{r},\mathbf{p},t)\varepsilon_-\right) + \frac{1}{2}\int d^3r\left(\Pi_\Phi^2 + \nabla\Phi\cdot\nabla\Phi + m_s^2\Phi^2\right)$$

$$+\frac{1}{2}\int d^3r\,[\Pi_{V^i}^2 - 2\Pi_{V^i}\partial_i V^0 + \nabla V^i\cdot\nabla V^i - \partial_j V^i\partial_i V^j + m_v^2(\mathbf{V}^2 - V^{02})], \qquad (2.1)$$

where $\Pi_{\Phi(V^i)}$ is the field canonically conjugated to $\Phi(V^i)$, $m_{s(v)}$ is the mass of the scalar(vector) field, and

$$\epsilon_{\pm}(\mathbf{r},\mathbf{p},t) = \sqrt{(\mathbf{p}\mp g_v\mathbf{V})^2 + (M - g_s\Phi)^2} \pm g_v V^0 ,$$

are the classical effective one-body Hamiltonian for particles (+) and anti-particles (−) since particles and anti-particles have opposite baryonic charge. However, to mantain a notation according to previous works [1, 2] it is convenient to work with the distribution function for particles at the position $\mathbf{r}$, instant t with momentum $\mathbf{p}$ ($f_+(\mathbf{r},\mathbf{p},t) = n_+(\mathbf{r},\mathbf{p},t)$), and the distribution function for anti-particles at the position $\mathbf{r}$, instant t but with momentum $-\mathbf{p}$ ($f_-(\mathbf{r},\mathbf{p},t) = n_-(\mathbf{r},-\mathbf{p},t)$). Equation (2.1) can be rewritten as

$$E = 4\int\frac{d^3r d^3p}{(2\pi)^3}(f_+(\mathbf{r},\mathbf{p},t)h_+ - f_-(\mathbf{r},\mathbf{p},t)h_-) + \frac{1}{2}\int d^3r\,(\Pi_\Phi^2 + \nabla\Phi\cdot\nabla\Phi + m_s^2\Phi^2)$$
$$+\frac{1}{2}\int d^3r\,[\Pi_{V^i}^2 - 2\Pi_{V^i}\partial_i V^0 + \nabla V^i\cdot\nabla V^i - \partial_j V^i\partial_i V^j + m_v^2(\mathbf{V}^2 - V^{02})], \qquad (2.2)$$

where

$$h_{\pm}(\mathbf{r},\mathbf{p},t) = \pm\epsilon_{\pm}(\mathbf{r},\pm\mathbf{p},t) = \pm\sqrt{(\mathbf{p}-g_v\mathbf{V})^2 + (M - g_s\Phi)^2} + g_v V^0 . \qquad (2.3)$$

The parameters of the model are given in ref.[2], i.e., $g_s^2 = 122.88$, $g_v^2 = 169.44$, $m_s = 550MeV$ and $m_v = 783MeV$, and produce a zero-temperature equilibrium at $k_F = 1.30fm^{-1}$, with a binding energy of $15.75MeV$.

In this classical approximation, the Vlasov equation describes the time evolution of the distribution functions and may be written either as

$$\frac{\partial n_\pm}{\partial t} + \{n_\pm, \epsilon_\pm\} = 0 ,$$

or as

$$\frac{\partial f_\pm}{\partial t} + \{f_\pm, h_\pm\} = 0 . \qquad (2.4)$$

The two forms are obviously equivalent but we prefer the second one, eq.(2.4).

In terms of the phase-space distribution functions, $f_\pm$, the nuclear density is given by

$$\rho(\mathbf{r},t) = 4\int\frac{d^3p}{(2\pi)^3}(f_+(\mathbf{r},\mathbf{p},t) - f_-(\mathbf{r},\mathbf{p},t)) , \qquad (2.5a)$$

while the nuclear scalar and current density are expressed by

$$\rho_s(\mathbf{r},t) = 4\int\frac{d^3p}{(2\pi)^3}\frac{M^*}{\epsilon^*}(f_+(\mathbf{r},\mathbf{p},t) + f_-(\mathbf{r},\mathbf{p},t)) , \qquad (2.5b)$$

and

$$\mathbf{j}(\mathbf{r},t) = 4\int\frac{d^3p}{(2\pi)^3}\frac{\mathbf{p}^*}{\epsilon^*}(f_+(\mathbf{r},\mathbf{p},t) + f_-(\mathbf{r},\mathbf{p},t)) \qquad (2.5c)$$

respectively. In eqs.(2.5b and c) $M^* = M - g_s\Phi$, $\mathbf{p}^* = \mathbf{p} - g_v\mathbf{V}$ and $\epsilon^* = \sqrt{\mathbf{p}^{*2} + M^{*2}}$.

The equations describing the time evolution of the fields Φ and V^μ are derived from Hamilton's equations and are

$$\frac{\partial^2\Phi}{\partial t^2} - \nabla^2\Phi + m_s^2\Phi = g_s\rho_s(\mathbf{r},t) , \qquad (2.6a)$$

$$\frac{\partial^2 V^0}{\partial t^2} - \nabla^2 V^0 + m_v^2 V^0 = g_v\rho(\mathbf{r},t) + \frac{\partial}{\partial t}\left(\frac{\partial V^0}{\partial t} + \nabla\cdot\mathbf{V}\right) , \qquad (2.6b)$$

$$\frac{\partial^2 V^i}{\partial t^2} - \nabla^2 V^i + m_v^2 V^i = g_v j^i(\mathbf{r},t) + \frac{\partial}{\partial x^i}\left(\frac{\partial V^0}{\partial t} + \nabla\cdot\mathbf{V}\right) . \qquad (2.6c)$$

It was shown in ref.[3] that if the Vlasov equation is satisfied then we have

$$\partial_\mu j^\mu = 0 , \; \partial_\mu V^\mu = 0 . \qquad (2.7)$$

Therefore, the last term in the right hand side of eqs.(2.6b and c) is eliminated.

The classical entropy of a Fermi gas is given by [11]

$$S = -4\int\frac{d^3r d^3p}{(2\pi)^3}\left(f_+ ln\left(\frac{f_+}{1-f_+}\right) + ln(1-f_+) + f_- ln\left(\frac{f_-}{1-f_-}\right) + ln(1-f_-)\right) , \qquad (2.8)$$

and the thermodynamic potencial is defined as

$$\Omega = E - TS - \mu B , \qquad (2.9)$$

where B is the barionic number:

$$B = \int d^3r\rho(\mathbf{r},t) = 4\int\frac{d^3r d^3p}{(2\pi)^3}(f_+ - f_-) , \qquad (2.10)$$

μ is the chemical potential and T is the temperature.

For a system in equilibrium, the distribution functions should be chosen to make the thermodynamic potencial Ω stationary. We get

$$f_\pm(\mathbf{r},\mathbf{p},t) = \frac{1}{1 + exp[(\epsilon^* \mp \nu)/T]} , \qquad (2.11)$$

with $\nu = \mu - g_v V^0$ being the effective chemical potential. We notice that $f_\pm$ is a function of ϵ^*, and therefore eq.(2.5c) gives $\mathbf{j}(\mathbf{r},t) = 0$. As explained in refs.[1, 2], from the thermodynamic potential we can derive the equations that determine the meson fields (eqs.(2.6)) and all the thermodynamic functions.

In this approach, the ground state of the system (caracterized by the index zero) is determined in the mean-field approximation. Since the nuclear medium is homogeneous, the classical meson fields Φ and V^μ are constants and given by:

$$\Phi_0 = \frac{g_s}{m_s^2}\rho_{0s}(M^*) = \frac{4g_s}{m_s^2}\int\frac{d^3p}{(2\pi)^3}\frac{M^*}{\epsilon}(f_{0+} + f_{0-}) , \qquad (2.12c)$$

$$V_0^0 = \frac{g_v}{m_v^2}\rho_0(M^*) = \frac{4g_v}{m_v^2}\int\frac{d^3p}{(2\pi)^3}(f_{0+} - f_{0-}) , \tag{2.12b}$$

$$\mathbf{V}_0 = 0 , \tag{2.12c}$$

with $\epsilon = \sqrt{\mathbf{p}^2 + M^{*2}}$.

Equations (2.2) and (2.12a) can be recast as

$$\mathcal{E} = \frac{E}{\mathcal{V}} = 4\int\frac{d^3p}{(2\pi)^3}\epsilon(f_{0+} + f_{0-}) + \frac{m_s^2}{2g_s^2}(M - M^*)^2 + \frac{g_v^2}{2m_v^2}(\rho_0(M^*))^2 , \tag{2.13a}$$

$$M^* = M - g_s\Phi_0 = M - \frac{4g_s^2}{m_s^2}\int\frac{d^3p}{(2\pi)^3}\frac{M^*}{\epsilon}(f_{0+} + f_{0-}) , \tag{2.13b}$$

where $\mathcal{V}$ is the volume of the system. Equation (2.13b) is a transcendental self-consistency condition that determines Φ_0. To solve this equation one first chooses T and ν (at low temperature, there may be three solutions for M^* [1, 2]). These values of M^*, ν and T specify f_{0+} and f_{0-} through eq.(2.11) and can then be used to compute $\rho_0(M^*)$ and $\mathcal{E}$ through eqs.(2.12b) and (2.13a). As pointed out in ref.[1], because of the different signs in the expressions for M^* and ρ, there can be a finite shift in the mass of the baryon at zero baryon density due to the presence of baryon-antibaryon pairs.

3. COLLECTIVE MODES

We are now in the position of obtaining the linearized equations of motion that describe the time evolution of small deviations from the ground state. The solutions of the linearized equations give the collective modes of the system since these modes correspond to small oscillations around the equilibrium state. We take for the different fields

$$f_\pm = f_{0\pm} + \delta f_\pm , \tag{3.1a}$$

$$\Phi = \Phi_0 + \delta\Phi , \tag{3.1b}$$

$$V^0 = V_0^0 + \delta V^0 , \tag{3.1c}$$

$$V^i = \delta V^i . \tag{3.1d}$$

In order to study the eigenfrequencies and eigenfunctions of this problem we express the fluctuations of the distribution functions in terms of the generators [12] $S_\pm(\mathbf{r},\mathbf{p},t)$ such that

$$\delta f_\pm = \{S_\pm, f_{0\pm}\} = \{S_\pm, p^2\}\frac{df_{0\pm}}{dp^2} . \tag{3.2}$$

Substituting eq.(3.2) in the linearized Vlasov equation:

$$\frac{\partial\delta f_\pm}{\partial t} + \{\delta f_\pm, h_{0\pm}\} + \{f_{0\pm}, \delta h_\pm\} = 0 , \tag{3.3}$$

we get the following time evolution for the generator

$$\frac{\partial S_\pm}{\partial t} + \{S_\pm, h_{0\pm}\} = \delta h_\pm = \mp g_s\frac{M^*}{\epsilon}\delta\Phi \mp g_v\frac{\mathbf{p}}{\epsilon}\cdot\delta\mathbf{V} + g_v\delta V^0 , \tag{3.4}$$

where

$$h_{0\pm} = \pm\sqrt{p^2 + M^{*2}} + g_v V_0^0 = \pm\epsilon + g_v V_0^0 .$$

When the ansatz for the longitudinal normal modes

$$\begin{pmatrix} S_+(\mathbf{r},\mathbf{p},t) \\ S_-(\mathbf{r},\mathbf{p},t) \\ \delta\Phi \\ \delta V^0 \\ \delta V^i \end{pmatrix} = \begin{pmatrix} S_{+\omega}(\cos\theta, p) \\ S_{-\omega}(\cos\theta, p) \\ \delta\Phi_\omega \\ \delta V_\omega^0 \\ \delta V_\omega\frac{k_i}{k} \end{pmatrix} e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})} , \tag{3.5}$$

is used in the linearized equations of motion for the fields [3] we get

$$i(\omega \mp k\frac{p}{\epsilon}\cos\theta)S_{\pm\omega} = \mp g_s\frac{M^*}{\epsilon}\delta\Phi_\omega + g_v\delta V_\omega^0 \mp g_v\frac{p}{\epsilon}\cos\theta\delta V_\omega , \tag{3.6a}$$

$$\left(\omega^2 - k^2 - m_s^2 - g_s^2\frac{d\rho_{0s}}{dM^*}\right)\delta\Phi_\omega = \frac{i}{\pi^3}g_s kM^*\int d^3p\frac{p}{\epsilon}\cos\theta[S_{+\omega}\frac{df_{0+}}{dp^2} + S_{-\omega}\frac{df_{0-}}{dp^2}] , \tag{3.6b}$$

$$\left(\omega^2 - k^2 - m_v^2\right)\delta V_\omega^0 = \frac{i}{\pi^3}g_v k\int d^3p p\cos\theta[S_{+\omega}\frac{df_{0+}}{dp^2} - S_{-\omega}\frac{df_{0-}}{dp^2}] , \tag{3.6c}$$

$$\left(\omega^2 - k^2 - m_v^2 - \Omega^2\right)\delta V_\omega = \frac{i}{\pi^3}g_v k\int d^3p\frac{p^2}{\epsilon}\cos^2\theta[S_{+\omega}\frac{df_{0+}}{dp^2} + S_{-\omega}\frac{df_{0-}}{dp^2}] . \tag{3.6d}$$

In these equations θ is the angle between $\mathbf{p}$ and $\mathbf{k}$, and

$$\Omega^2 = \frac{2g_v^2}{3\pi^2}\int_0^\infty dp\,(f_{0+} + f_{0-})\frac{p^2}{\epsilon^3}(2p^2 + 3M^{*2}) .$$

The continuity equation (eq.(2.7)) implies

$$\omega\delta V_\omega^0 = k\delta V_\omega . \tag{3.7}$$

Defining the dimensionless quantities

$$Q_{1\omega} = \frac{\delta\Phi_\omega}{M} , \; Q_{2\omega} = \frac{\delta V_\omega^0}{M} , \; Q_{3\omega} = \frac{\delta V_\omega}{M} , \; S_\omega(\cos\theta, p) = \frac{\epsilon}{\pi M}S_\omega(\cos\theta, p)$$

$$G_1 = \frac{g_s M^*}{\pi k} , \tag{3.8a}$$

$$G_2(\epsilon) = \frac{g_v\epsilon}{\pi k} , \tag{3.8b}$$

$$G_3(\epsilon) = G_2(\epsilon) , \tag{3.8c}$$

$$\omega_1^2 = \frac{1}{k^2}\left(k^2 + m_s^2 + g_s^2\frac{d\rho_{0s}}{dM^*}\right) , \tag{3.8d}$$

$$\omega_2^2 = \frac{1}{k^2}(k^2 + m_v^2) , \tag{3.8e}$$

$$\omega_3^2 = \frac{1}{k^2}(k^2 + m_v^2 + \Omega^2)\,, \tag{3.8f}$$

and $\overline{\omega} = \omega/k$, we can rewrite the equations of normal modes as

$$(\overline{\omega} - x)S_{+\omega}(x,p) = iG_1Q_{1\omega} - iG_2(\epsilon)Q_{2\omega} + iG_3(\epsilon)xQ_{3\omega}\,, \tag{3.9a}$$

$$(\overline{\omega} + x)S_{-\omega}(x,p) = -iG_1Q_{1\omega} - iG_2(\epsilon)Q_{2\omega} - iG_3(\epsilon)xQ_{3\omega}\,, \tag{3.9b}$$

$$(\overline{\omega}^2 - \omega_1^2)Q_{1\omega} = \frac{-iG_1}{T}\int_{M^*}^{\infty} d\epsilon \int_{-A}^{A} dx x[S_{+\omega}f_{0+}(1-f_{0+}) + S_{-\omega}f_{0-}(1-f_{0-})]\,, \tag{3.9c}$$

$$(\overline{\omega}^2 - \omega_2^2)Q_{2\omega} = \frac{-i}{T}\int_{M^*}^{\infty} d\epsilon G_2 \int_{-A}^{A} dx x[S_{+\omega}f_{0+}(1-f_{0+}) - S_{-\omega}f_{0-}(1-f_{0-})]\,, \tag{3.9d}$$

$$(\overline{\omega}^2 - \omega_2^3)Q_{3\omega} = \frac{-i}{T}\int_{M^*}^{\infty} d\epsilon G_3 \int_{-A}^{A} dx x^2[S_{+\omega}f_{0+}(1-f_{0+}) + S_{-\omega}f_{0-}(1-f_{0-})]\,, \tag{3.9e}$$

where $A = p/\epsilon$,$x = \frac{p\cos\theta}{\epsilon}$ and eq.(3.7) is now equivalent to

$$\overline{\omega}Q_{2\omega} = Q_{3\omega}\,. \tag{3.10}$$

In writing these equations we have used the relation

$$\frac{df_{0\pm}}{dp^2} = -f_{0\pm}(1-f_{0\pm})\frac{1}{2T\epsilon}\,. \tag{3.11}$$

It is well known that the equations of normal modes may be derived from a Lagrangian formalism. In particular we observe that eqs.(3.9) may be obtained from the following Lagrangian:

$$\begin{aligned}
L = {} & \sum_{j=1}^{3}(-1)^{j+1}(P_j^*\dot{Q}_j + P_j\dot{Q}_j^*) - \frac{i}{T}\int_{M^*}^{\infty} d\epsilon \int_{-A}^{A} dx x(S_+^*\dot{S}_+f_{0+}(1-f_{0+}) - S_-^*\dot{S}_-f_{0-}(1-f_{0-})) \\
& -\frac{1}{T}\int_{M^*}^{\infty} d\epsilon \int_{-A}^{A} dx x^2(S_+^*S_+f_{0+}(1-f_{0+}) + S_-^*S_-f_{0-}(1-f_{0-})) + \sum_{j=1}^{3}(-1)^j(|P_j|^2 + \omega_j^2|Q_j|^2) \\
& -\frac{iG_1Q_1}{T}\int_{M^*}^{\infty} d\epsilon \int_{-A}^{A} dx x(S_+^*(x,\epsilon,\tau)f_{0+}(1-f_{0+}) + S_-^*(x,\epsilon,\tau)f_{0-}(1-f_{0-})) \\
& +\frac{iG_1Q_1^*}{T}\int_{M^*}^{\infty} d\epsilon \int_{-A}^{A} dx x(S_+(x,\epsilon,\tau)f_{0+}(1-f_{0+}) + S_-(x,\epsilon,\tau)f_{0-}(1-f_{0-})) \\
& +\frac{iQ_2}{T}\int_{M^*}^{\infty} d\epsilon G_2 \int_{-A}^{A} dx x(S_+^*(x,\epsilon,\tau)f_{0+}(1-f_{0+}) - S_-^*(x,\epsilon,\tau)f_{0-}(1-f_{0-})) \\
& -\frac{iQ_2^*}{T}\int_{M^*}^{\infty} d\epsilon G_2 \int_{-A}^{A} dx x(S_+(x,\epsilon,\tau)f_{0+}(1-f_{0+}) - S_-(x,\epsilon,\tau)f_{0-}(1-f_{0-})) \\
& -\frac{iQ_3}{T}\int_{M^*}^{\infty} d\epsilon G_3 \int_{-A}^{A} dx x^2(S_+^*(x,\epsilon,\tau)f_{0+}(1-f_{0+}) + S_-^*(x,\epsilon,\tau)f_{0-}(1-f_{0-})) \\
& +\frac{iQ_3^*}{T}\int_{M^*}^{\infty} d\epsilon G_3 \int_{-A}^{A} dx x^2(S_+(x,\epsilon,\tau)f_{0+}(1-f_{0+}) + S_-(x,\epsilon,\tau)f_{0-}(1-f_{0-}))\,.
\end{aligned} \tag{3.12}$$

This Lagrangian is only formal since the time derivative of the time component of the vector field ($\dot{Q}_2$) and its canonically conjugated momentum (P_2), do not appear in a conventional Lagrangian formalism. The Lagrangian of eq.(3.12) was written in this way in order to derive eqs.(3.9).

For $|\overline{\omega}| > 1$ we obtain the discrete solutions of eqs.(3.9) which obey the dispersion relation

$$(\overline{\omega}^2 - \omega_1^2)(\overline{\omega}^2 - \omega_2^2) + I_{G_1}(\overline{\omega}^2 - \omega_2^2) + I_{G_2}(\overline{\omega}^2 - \omega_1^2)(\overline{\omega}^2 - 1) + (\overline{\omega}^2 - 1)(I_{G_2}I_{G_1} - I_{G_1G_2}^2) = 0\,, \tag{3.13}$$

where

$$I_{G_1} = -G_1^2(I_{0+} + I_{0-})\,, \tag{3.14a}$$

$$I_{G_2} = -g_2^2(I_{2+} + I_{2-})\,, \tag{3.14b}$$

$$I_{G_1G_2} = G_1g_2(I_{1+} - I_{1-})\,, \tag{3.14c}$$

$$I_{n\pm} = \frac{1}{T}\int_{M^*}^{\infty} d\epsilon\epsilon^n I_\omega(\epsilon)\, f_{0\pm}(1-f_{0\pm})\,, \tag{3.14c}$$

$$I_\omega(\epsilon) = \pm\int_{-A}^{A} dx\frac{x}{\overline{\omega}\mp x} = -\left(2\frac{p}{\epsilon} + \overline{\omega}ln\left|\frac{\overline{\omega} - p/\epsilon}{\overline{\omega} + p/\epsilon}\right|\right)\,, \tag{3.14d}$$

and $g_2 = G_2/\epsilon$.

In terms of $\omega = \overline{\omega}k$ and in the limit $k \to 0$ eq.(3.13) can be rewritten as

$$(\omega^2 - \mathcal{M}_s^2)(\omega^2 - \mathcal{M}_v^2) = 0\,, \tag{3.15}$$

with solutions

$$\omega_s = \pm\mathcal{M}_s = \pm\left(m_s^2 + g_s^2\frac{d\rho_{0s}}{dM^*}\right)^{1/2}\,, \tag{3.16}$$

$$\omega_v = \pm\mathcal{M}_v = \pm\left(m_v^2 + \Omega^2\right)^{1/2}\,. \tag{3.17}$$

These solutions show that the mesons behave as if they had an effective mass $m_{s(v)}^{eff} = \mathcal{M}_{s(v)}$.

In figure 1 we plot $\mathcal{M}_{s(v)}$ as a function of T for $k = 0$ and zero baryon density. The solution of eq.(2.13b) at this same density is also ploted. As pointed out in ref.[1], the nucleon effective mass decreases as the temperature is raised due to the $N\overline{N}$ pair formation. With respect to the meson effective masses the opposite occurs. Thus, at high temperatures the baryons are essentially massless and the mesons (mainly the scalar one) are very heavy. This rapid decrease of M^* and rapid increase of $\mathcal{M}_s$ and $\mathcal{M}_v$ with increasing temperature resembles a phase transition, and at high temperature and low density the system becomes a dilute gas of baryons in a sea of baryon-antibaryon pairs. The behaviour of the scalar meson effective mass as a function of temperature is different from the behaviour obtained in ref.[13] where the effects of the chiral transition of hot and dense quark matter in physical quantities are analysed. This difference is due to the fact that in ref. [13] the mesons have a quark-antiquark internal structure. Therefore, there is a competitive effect in the temperature dependence of the meson masses between the decrease of the constituent quark masses and the increase of the occupation factors for large values of momentum [14]. On the other hand, in the Walecka model the mesons are phenomenological and have no internal structure. Thus, the only effect present is the increase of the occupation factor ,i.e. the $N\overline{N}$ pair formation, which pushes the meson masses up.

In figure 2 we plot the solutions of eq.(3.13) as a function of k^2 for $T = 25MeV$ and $T = 200MeV$ with $\rho = 0.15fm^{-3}$ (the normal nuclear matter saturation density). We see that similarly to the case $T = 0$ [3] (also included in the figure for comparison), for these values of k, the relation between ω^2 and k^2 remains almost linear even for values of $T > 200MeV$, beyond the phase transition. The only effect of the temperature is the displacement of the curves. Figure 3 shows the dependence of the solutions of eq.(3.13) with the baryon density for $k = 500MeV$ and $T = 25MeV$. The behaviour is also similar to the one obtained at $T = 0$. This is expected since from figure 1 we see that for $T < 100MeV$ there is almost no effect from the temperature in the masses.

We should point out at this moment that the present model is more adequated for temperatures $T < 100MeV$ since for higher temperatures the equation of state will be modified by contributions from thermal pions [2], which are not included in this version of the Walecka model.

4. ORTHOGONALITY COMPLETENESS RELATIONS AND SUM RULE

The discrete solutions of eqs.(3.9), which obey eq.(3.13), always come in pairs $\pm\overline{\omega}_n$, with n standing for s and v, the scalar and vector mesons respectively. These solutions are described by the wave function

$$\Psi_{\pm n} = \begin{pmatrix} Q_{1\pm n} \\ P_{1\pm n} \\ Q_{2\pm n} \\ P_{2\pm n} \\ Q_{3\pm n} \\ P_{3\pm n} \\ S^+_{\pm n}(x,\epsilon) \\ S^-_{\pm n}(x,\epsilon) \end{pmatrix} = \begin{pmatrix} -if(\overline{\omega}_n) \\ \pm\overline{\omega}_n f(\overline{\omega}_n) \\ -i \\ \pm\overline{\omega}_n \\ \mp i\overline{\omega}_n \\ \overline{\omega}_n^2 \\ \frac{1}{\pm\overline{\omega}_n - x}(G_1 f(\overline{\omega}_n) + g_2\epsilon(\pm x\overline{\omega}_n - 1)) \\ \frac{-1}{\pm\overline{\omega}_n + x}(G_1 f(\overline{\omega}_n) + g_2\epsilon(\pm x\overline{\omega}_n + 1)) \end{pmatrix} , \tag{4.1}$$

where

$$f(\overline{\omega}) = \frac{\overline{\omega}^2 - \omega_2^2 + (\overline{\omega}^2 - 1)I_{G_2}}{I_{G_1G_2}} . \tag{4.2}$$

Besides these solutions, eqs.(3.9) also have a continuum of solutions if $|\overline{\omega}| < 1$, which are described by

$$\Psi_\omega = \begin{pmatrix} Q_{1\omega} \\ P_{1\omega} \\ Q_{2\omega} \\ P_{2\omega} \\ Q_{3\omega} \\ P_{3\omega} \\ S_{+\omega}(x,\epsilon) \\ S_{-\omega}(x,\epsilon) \end{pmatrix} = \begin{pmatrix} f(\overline{\omega})Q_{2\omega} + \frac{i\overline{\omega}A_{G_2}}{I_{G_1G_2}} \\ i\overline{\omega}f(\overline{\omega})Q_{2\omega} - \frac{\overline{\omega}^2 A_{G_2}}{I_{G_1G_2}} \\ Q_{2\omega} \\ i\overline{\omega}Q_{2\omega} \\ \overline{\omega}Q_{2\omega} \\ i\overline{\omega}^2 Q_{2\omega} \\ \delta(\overline{\omega} - x) + \frac{i}{\overline{\omega} - x}(G_1Q_{1\omega} + g_2\epsilon(\overline{\omega}x - 1)Q_{2\omega}) \\ \delta(\overline{\omega} + x) + \frac{-i}{\overline{\omega} + x}(G_1Q_{1\omega} + g_2\epsilon(\overline{\omega}x + 1)Q_{2\omega}) \end{pmatrix} , \tag{4.3}$$

with $Q_{2\omega}$ satisfying the equation

$$Q_{2\omega} = -\frac{i\overline{\omega}(A_{G_2}(\overline{\omega}^2 - \omega_1^2 + I_{G_1}) + A_{G_1}I_{G_1G_2})}{(\overline{\omega}^2 - \omega_1^2 + I_{G_1})(\overline{\omega}^2 - \omega_2^2 + (\overline{\omega}^2 - 1)I_{G_2}) - (\overline{\omega}^2 - 1)I^2_{G_1G_2}} , \tag{4.4}$$

and

$$A_{G_1} = G_1(A_{0+} - A_{0-}) , \tag{4.5a}$$

$$A_{G_2} = g_2(A_{1+} + A_{1-}) , \tag{4.5b}$$

$$A_{n\pm} = \frac{1}{T}\int_{M^*}^{\infty} d\epsilon\epsilon^n f_{0\pm}(1 - f_{0\pm})\Theta(p^2/\epsilon^2 - \overline{\omega}^2) , \tag{4.5c}$$

From the Lagrangian eq.(3.12) it can be shown that these solutions are orthogonal and satisfy the following orthogonality relations

$$i\sum_{j=1}^{3}(-1)^{j+1}(P^*_{j\pm n}Q_{j\pm n'} - Q^*_{j\pm n}P_{j\pm n'}) + \frac{1}{T}\int_M^\infty d\epsilon\int_{-A}^{A} dxx[S^{+*}_{\pm n}S^+_{\pm n'}f_{0+}(1 - f_{0+})$$
$$- S^{-*}_{\pm n}S^-_{\pm n'}f_{0-}(1 - f_{0-})] = \pm\eta_n\delta_{nn'} , \tag{4.6a}$$

$$i\sum_{j=1}^{3}(-1)^{j+1}(P^*_{j\omega'}Q_{j\omega} - Q^*_{j\omega'}P_{j\omega}) + \frac{1}{T}\int_M^\infty d\epsilon\int_{-A}^{A} dxx[S^*_{+\omega'}S_{+\omega}f_{0+}(1 - f_{0+})$$
$$- S^*_{-\omega'}S_{-\omega}f_{0-}(1 - f_{0-})] = \overline{\omega}(A_{0+} + A_{0-})\delta(\overline{\omega}' - \overline{\omega}) , \tag{4.6b}$$

$$i\sum_{j=1}^{3}(-1)^{j+1}(P^*_{j\pm n}Q_{j\omega} - Q^*_{j\pm n}P_{j\omega}) + \frac{1}{T}\int_M^\infty d\epsilon\int_{-A}^{A} dxx[S^{+*}_{\pm n}S^+_{\omega}f_{0+}(1 - f_{0+})$$
$$- S^{-*}_{\pm n}S^-_{\omega}f_{0-}(1 - f_{0-})] = 0 , \tag{4.6c}$$

with

$$\eta_n = 2\overline{\omega}_n(f^2(\overline{\omega}_n) - 1 + \overline{\omega}_n^2) + \frac{1}{T}\int_{M^*}^\infty d\epsilon\int_{-A}^{A} dxx$$
$$[|S^+_{+n}|^2 f_{0+}(1 - f_{0+}) - |S^-_{+n}|^2 f_{0-}(1 - f_{0-})] . \tag{4.7}$$

The normal modes form a complete set of solutions [3, 12]. Therefore, it is possible to expand any wave function

$$\Psi_0 = \Psi(\tau=0) = \begin{pmatrix} Q_{01} \\ P_{01} \\ 0 \\ iQ_{03} \\ Q_{03} \\ P_{03} \\ H_+(x,\epsilon) \\ H_-(x,\epsilon) \end{pmatrix} , \tag{4.8}$$

in terms of the normal modes

$$\Psi_0 = \int_{-1}^{1} c(\overline{\omega})\Psi_\omega d\overline{\omega} + \sum_{n=s,v} (c_{+n}\Psi_{+n} + c_{-n}\Psi_{-n}) , \tag{4.9}$$

where [12]

$$c_{\pm n} = \pm\frac{1}{\eta_n}\left(i\sum_{j=1}^{3}(-1)^{j+1}(P^*_{j\pm n}Q_{0j} - Q^*_{j\pm n}P_{0j}) + \frac{1}{T}\int_{M^*}^{\infty} d\epsilon \int_{-A}^{A} dx x[S^{+*}_{\pm n}H_+ f_{0+}(1-f_{0+}) - S^{-*}_{\pm n}H_- f_{0-}(1-f_{0-})]\right) , \tag{4.10}$$

$$c(\overline{\omega}) = \frac{\tilde{c}(\overline{\omega})}{(A_{0+}+A_{0-}) + \pi^2\frac{1}{T}\int_{M^*}^{\infty} d\epsilon[f_{0+}(1-f_{0+})a_+^2(\overline{\omega},\overline{\omega},\epsilon) + f_{0-}(1-f_{0-})a_-^2(\overline{\omega},-\overline{\omega},\epsilon)]\Theta(p^2/\epsilon^2-\overline{\omega}^2)} \tag{4.11a}$$

with

$$\tilde{c}(\overline{\omega}) = \frac{1}{\overline{\omega}}\left(i\sum_{j=1}^{3}(-1)^{j+1}(P^*_{j\omega}Q_{0j} - Q^*_{j\omega}P_{0j}) + \frac{1}{T}\int_{M^*}^{\infty} d\epsilon \int_{-A}^{A} dx x[S^*_{+\omega}H_+ f_{0+}(1-f_{0+}) - S^*_{-\omega}H_- f_{0-}(1-f_{0-})]\right) , \tag{4.11b}$$

and

$$a_\pm(\overline{\omega},x,\epsilon) = iG_1Q_{1\omega} + ig_2Q_{2\omega}\epsilon(x\overline{\omega}\mp 1) . \tag{4.11c}$$

These coefficients satisfy the following energy weighted sum rule (EWSR)

$$m_1 = \int_{-1}^{1} d\overline{\omega}\overline{\omega}^2 c(\overline{\omega})\tilde{c}^*(\overline{\omega}) + \sum_{n=s,v}\overline{\omega}_n\eta_n(|c_{+n}|^2 + |c_{-n}|^2) =$$

$$\frac{1}{T}\int_{M^*}^{\infty} d\epsilon\int_{-A}^{A} dx x^2(|H_+|^2 f_{0+}(1-f_{0+}) + |H_-|^2 f_{0-}(1-f_{0-})) - \sum_{j=1}^{3}(-1)^j(|P_{0j}|^2 + \omega_j^2|Q_{0j}|^2)$$

$$+\frac{iG_1Q_{01}}{T}\int_{M^*}^{\infty} d\epsilon\int_{-A}^{A} dx x(H_+^*(x,\epsilon)f_{0+}(1-f_{0+}) + H_-^*(x,\epsilon)f_{0-}(1-f_{0-}))$$

$$-\frac{iG_1Q^*_{01}}{T}\int_{M^*}^{\infty} d\epsilon\int_{-A}^{A} dx x(H_+(x,\epsilon)f_{0+}(1-f_{0+}) + H_-(x,\epsilon)f_{0-}(1-f_{0-}))$$

$$+\frac{iQ_{03}}{T}\int_{M^*}^{\infty} d\epsilon G_3\int_{-A}^{A} dx x^2(H_+^*(x,\epsilon)f_{0+}(1-f_{0+}) + H_-^*(x,\epsilon)f_{0-}(1-f_{0-}))$$

$$-\frac{iQ^*_{03}}{T}\int_{M^*}^{\infty} d\epsilon G_3\int_{-A}^{A} dx x^2(H_+(x,\epsilon)f_{0+}(1-f_{0+}) + H_-(x,\epsilon)f_{0-}(1-f_{0-})) . \tag{4.12}$$

In order to study the distribution of strength by the discrete modes and the collective modes in the particle-hole continuum we consider the initial condition:

$$\Psi_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} . \tag{4.13}$$

For this condition the strength function, in the interval $0 < \overline{\omega} < 1$, becomes

$$s(\omega) = 2\overline{\omega}^2\tilde{c}^*(\overline{\omega})c(\overline{\omega}) , \tag{4.14}$$

and the fraction exhausted by the discrete modes is

$$F(\overline{\omega}_n) = \frac{2}{m_1}\sum_{n=s,v}\frac{\overline{\omega}_n}{\eta_n}(G_1 f(\overline{\omega}_n)I_{0+} + g_2(\overline{\omega}_n^2-1)I_{1+})^2 , \tag{4.15}$$

with $\overline{\omega}_n$, η_n $f(\overline{\omega}_n)$ and I_{n+} given respectively by eqs.(3.13), (4.7), (4.2) and (3.14c), and from eq.(4.12) we get $m_1 = \frac{2}{3T}\int_{M^*}^{\infty} d\epsilon\frac{p^3}{\epsilon^3}f_{0+}(1-f_{0+})$.

In figure 4 we plot the strength function eq.(4.14) for $k/M = 0.5$, $T = 25MeV$ and $\rho = 0.173fm^{-3}$ (the saturation density at this temperature). This curve is different from the one obtained for $T = 0$ (see fig.5 of ref.[3]). This difference comes from the fact that the zero sound mode, that exist as a discrete mode for $T = 0$, merges in the continuum [15]. This can be proved by doing an expansion of the distribution function in powers of T. For small values of T we can write:

$$f_{0+} = \frac{1}{1+exp\left(\frac{\epsilon-\nu}{T}\right)} \simeq \Theta(\nu-\epsilon) + \frac{\pi^2T^2}{6}\delta'(\nu-\epsilon) , \tag{4.16a}$$

$$f_{0-} = \frac{1}{1+exp\left(\frac{\epsilon+\nu}{T}\right)} \simeq 0 , \tag{4.16b}$$

using this in eq.(3.13) we get for some values of k, just as in the $T = 0$ case, three pairs of solutions. For $T = 0.5MeV$, $\rho = 0.15fm^{-3}$ and $k/M = 1.0$ these solutions are (the particle-hole continuum lies now in the range $|\overline{\omega}| < p_\nu/\nu = 0.4544$):

zero sound mode $\overline{\omega}_z = 0.4565$

scalar mode $\overline{\omega}_s = 1.2121$

vector mode $\overline{\omega}_v = 1.3708$.

The percentages of the EWSR exhausted by these modes and the continuum are: 15.04% (zero sound), 2.44% (scalar), 18.40% (vector) and 64.12% (continuum). However, for the same values of the constants eq.(3.13) without any approximation has only the last two discrete solutions (scalar and vector) located exactly at the same points and the percentages of the EWSR are: 2.44% (scalar), 18.40% (vector) and 79.16% (continuum). In figure 5 the strength function in both cases (Fermi function and expansion for small values of T) are plotted as a function of $\tilde{\omega}$. From this figure and from the percentages of the EWSR it is clear that the zero sound mode merges in the continuum for $T \neq 0$ when the full Fermi function is used. In this figure the position of the zero-sound mode is indicated by an arrow, and we can see that its position coincides with the maximum of the exact strength function.

The percentages of the EWSR exhausted by the continuum and discrete modes for $T = 25MeV$, $k/M = 0.5$, $\rho = 0.173 fm^{-3}$ and the initial condition eq.(4.13) are plotted, as a function of k/M, in figure 6. We see that, despite the temperature, the scalar mode remains unexcited by this initial condition just as in the case $T = 0$ [3]. Even for $T = 200MeV$ the percentages of The EWSR exhausted by the scalar mode by this initial condition is always smaller than 10%. Comparing figure 6 with figure 4 of ref.[3] it is seen clearly that the strength exhausted by the zero-sound is contained in the strength function of the continuum.

For the initial condition which favors the scalar mode:

$$\Psi_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \tag{4.17}$$

the percentages exhausted by the modes are plotted in figure 7. From both initial conditions we learn that the system behaves essentially as if the scalar mode and the continuum were decoupled, but there is a strong coupling between the continuum and the vector field, and between the scalar and vector fields according to the initial condition considered. For the initial condition that favors the vector mode ($Q_{03} = 1$ and the other fields zero) both continuum and scalar modes are coupled to the vector mode as can be seen in figure 8.

5. CONCLUSIONS

In this work we have studied the properties of hot and dense nuclear matter using a thermal relativistic Vlasov equation based on the Walecka model. We have obtained that the masses of the discrete RPA collective normal modes, corresponding to the meson fields, increase with the temperature due to the $N\overline{N}$ pair formation, but temperature effects are still relatively small up to

$T = 150MeV$. The same small dependence of the collective modes on temperature is also well know from non-relativistic nuclear physics [16].

The dependence of the meson collective modes with transfered momentum fo $k < 1GeV$ ($\omega^2_{s(v)}$ depends linearly on k^2) does not change when temperature is introduced and for $T < 100MeV$ the mass of these collective modes as a function of k is only slightly affected.

An interesting effect of temperature is the fact that the zero-sound mode, also identified in refs.[3-7] for $T = 0$, merges in the continuum of single-particle excitations at finite temperature. Therefore, the strength distribution function of the modes in the continuum contains also the strength exhausted by the zero-sound.

We have observed that at finite temperature the scalar mode is not dynamically coupled to the continuum but the vector mode couples strongly both to the scalar mode and to the continuum. This same behaviour was observed at $T = 0$.

As a concluding remark, it would be interesting to explore further the properties of collective modes in a model that includes the contributions from thermal pions. However, in a rather different context, the same small temperature dependence was obtained in ref.[17] for the mass of the ρ meson in a hot pion gas with the standard $\pi - \rho$ dynamics.

ACKNOWLEDGMENTS

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FIGURE CAPTIONS

Fig.1- Self-consistent nucleon mass (full line), scalar meson effective mass (dashed line) and vector meson effective mass (dotted line) in units of their free masses as a function of temperature at vanishing baryon density and vanishing transfered momentum.

Fig.2- The dispersion relation for the scalar and vector modes at normal nuclear saturation density for $T = 0$ (dotted line), $T = 25MeV$ (full line) and $T = 200MeV$ (dashed line). The frequency ω_n and wave vector k are in units of M. The top line for each T corresponds to the vector mode and the botton line to the scalar mode.

Fig.3- The dispersion relation for the scalar (full line) and vector (dashed line) modes as a function of the density for $T = 25MeV$ and $k = 500MeV$. The frequency ω_n is in units of M.

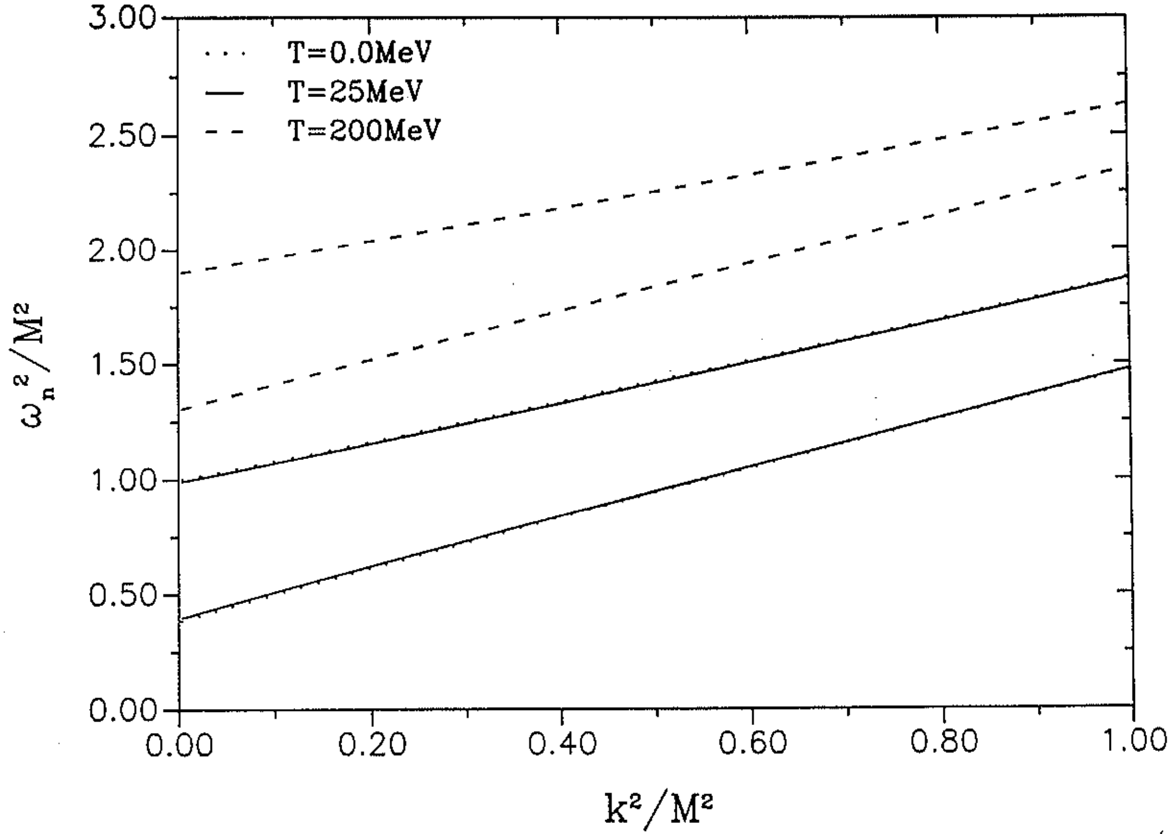
Fig.4- Strenght function, as a function of $\overline{\omega}$ for the initial condition eq.(4.13), for $T = 25MeV$, $\rho = 0.173fm^{-3}$, and $k/M = 0.5$.

Fig.5- Exact strenght function (full line) and with the expansion for small values of T (dashed line) as a function of $\overline{\omega}$ for $T = 0.5MeV$, $\rho = 0.15fm^{-3}$, and $k/M = 1.0$. The arrow indicates the position of the zero-sound mode.

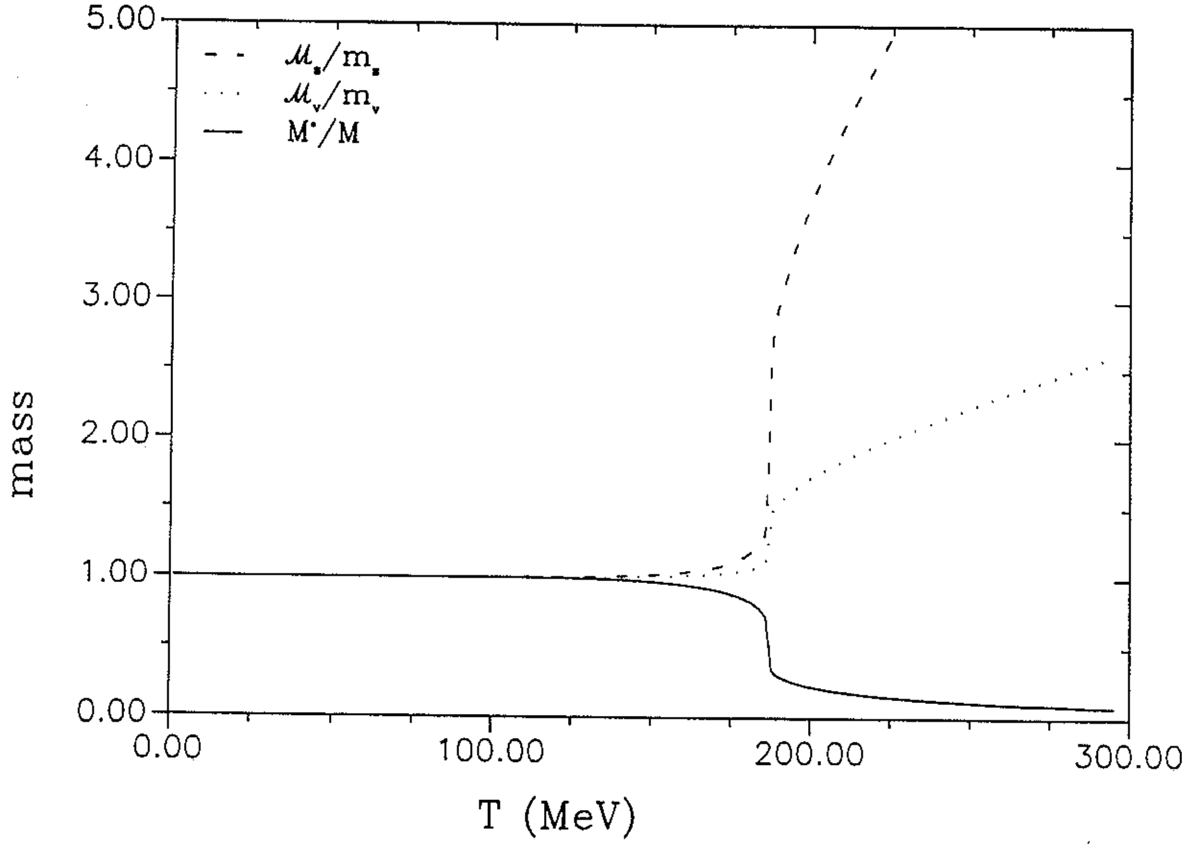
Fig.6- Percentages of the EWSR exhausted by the continuum (full line) and discrete modes as a function of k, for the initial condition eq.(4.13), $T = 25MeV$ and for $\rho = 0.173fm^{-3}$. The dashed and dotted lines represent the scalar and vector modes respectively.

Fig.7- Same as fig.6 for the initial condition eq.(4.17).

Fig.8- Same as fig.6 for the initial condition $Q_{03} = 1.0$.



3.00
2.50
2.00
1.50
1.00
0.50
0.00
ω_n^2/M^2
0.00
0.20
0.40
0.60
0.80
1.00
k^2/M^2
T=0.0MeV
T=25MeV
T=200MeV



5.00
4.00
3.00
2.00
1.00
0.00
mass
0.00
100.00
200.00
300.00
T (MeV)
$\mathcal{M}_s/m_s$
$\mathcal{M}_v/m_v$
M^*/M

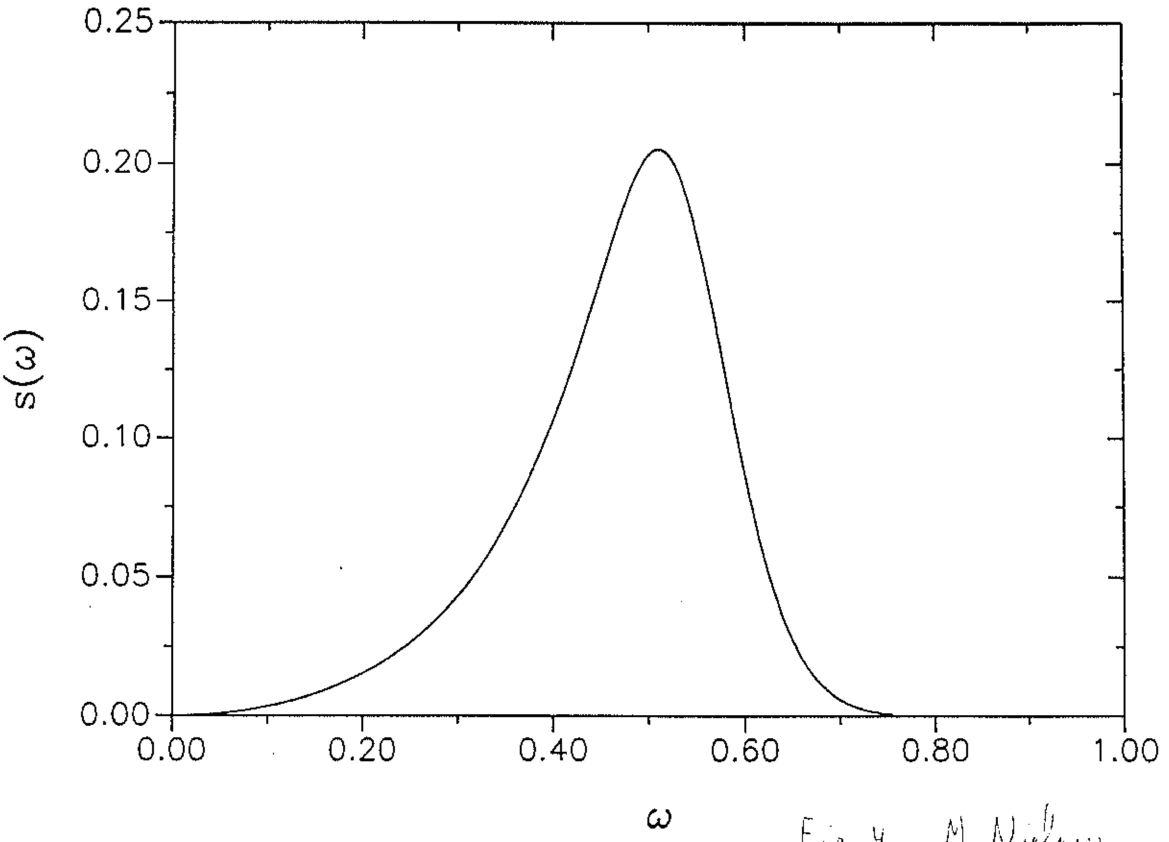


Fig 4 M Nielsen "Coll... Modes in Hot ..."

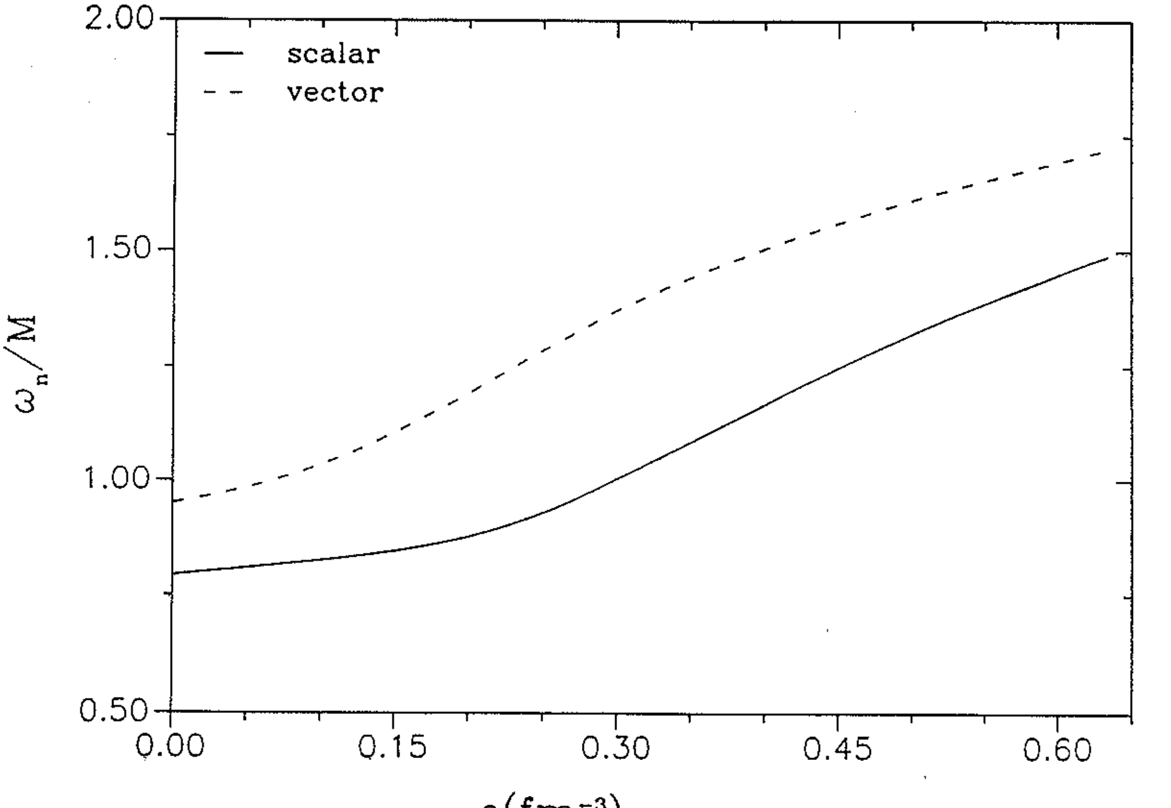


Fig 3 M Nielsen "Coll... Modes in Hot ..."

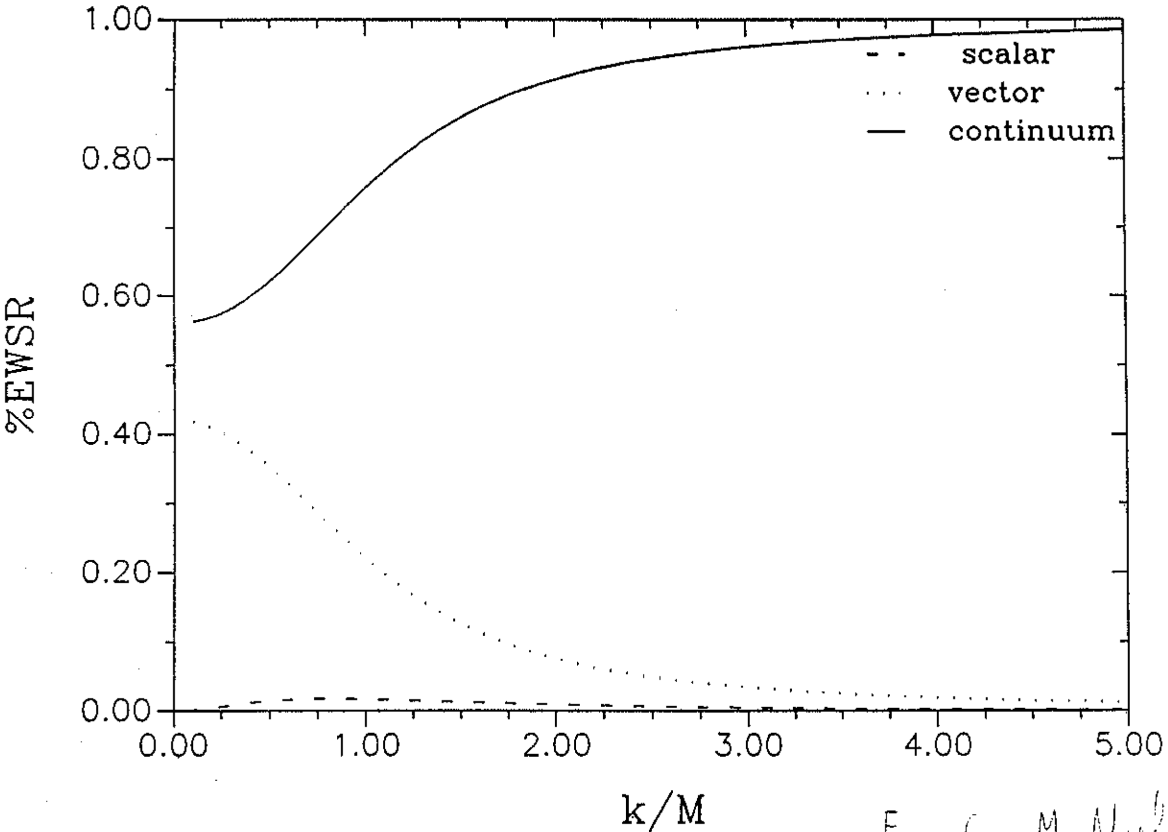


Fig 6

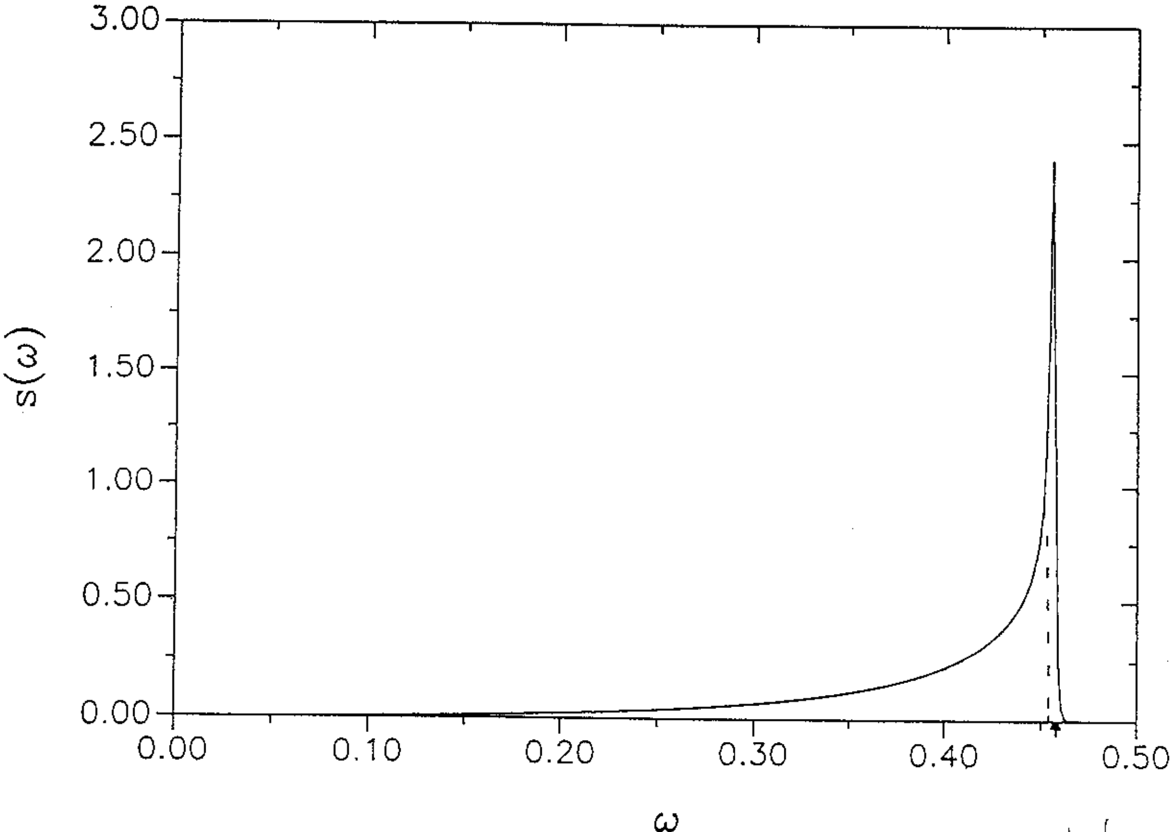
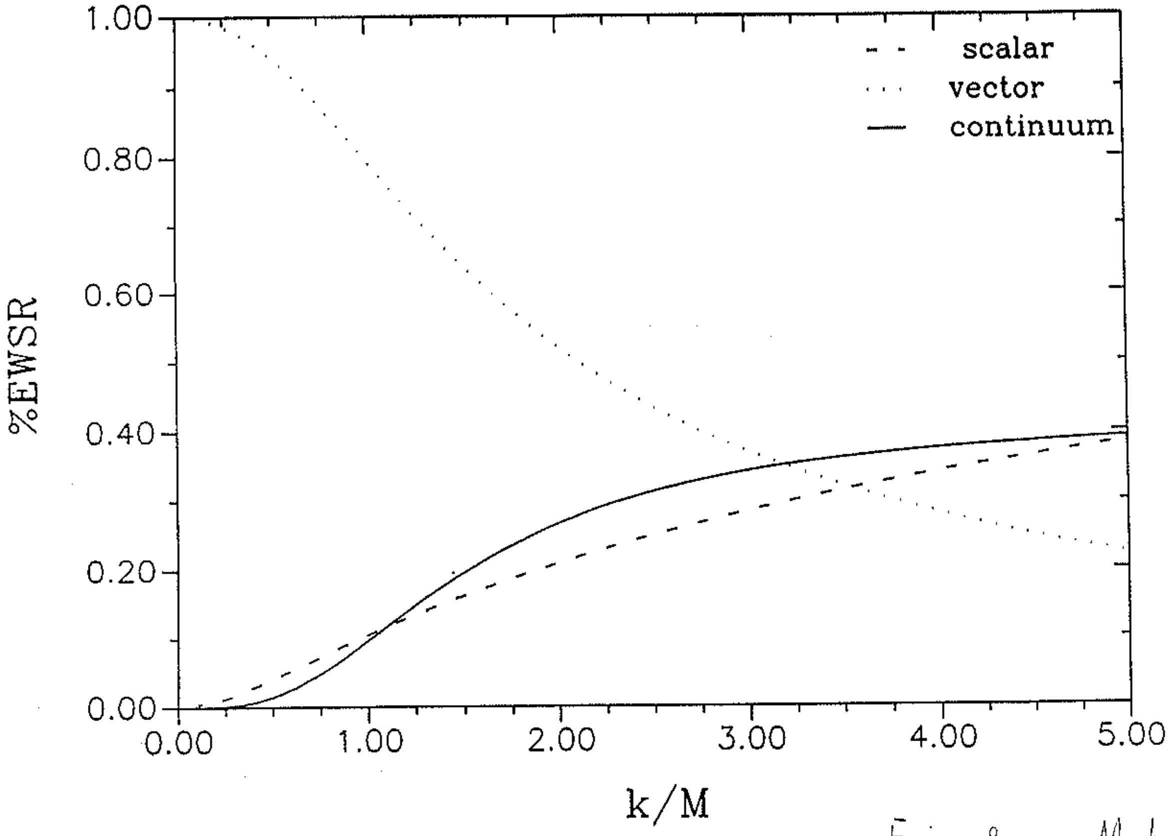


Fig 5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k/M
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vector
continuum
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4.00
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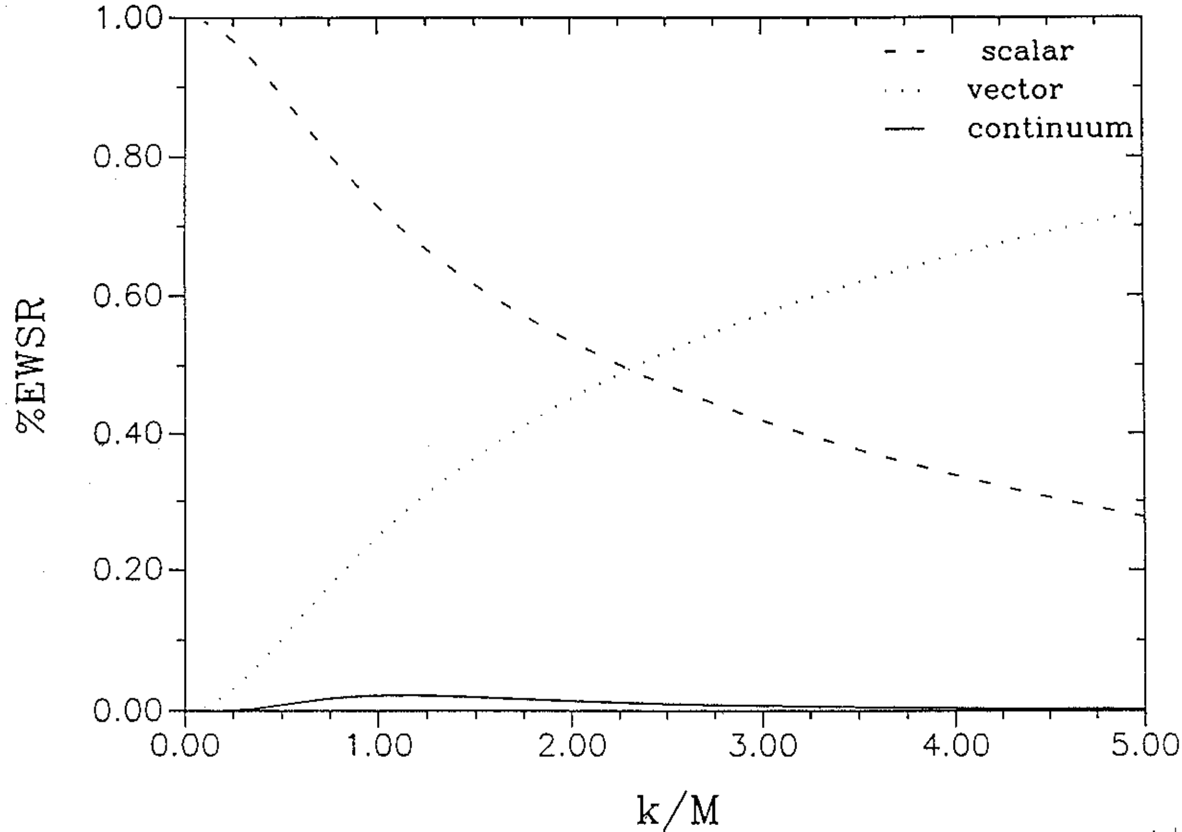
Fig 8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k/M
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