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IFUSP/P-990

THE "NAIBEA" ACCELERATOR

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Contribution to the Advanced Accelerator Concepts Workshop, Port Jefferson, Long Island, N.Y., USA, June 14-20, 1992

Junho/1992

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ABSTRACT

The recently developed Nonlinear Amplification of Inverse Bremsstrahlung Electron Acceleration Concept is given adetailed qualitative description here.

INTRODUCTION

In a recent paper[1], M.P. Pato and I have developed the NAIBEA concept of accelerating relativistic charged particle to Gev or Tev[2] energies using lasers as the principal supplier of energy, and an alternating static electric field as a modulator of the transverse motion. The resulting machines were found more than a factor 20 smaller than conventional ones. Generalization of the ideas of Ref.1) to the use of alternating static magnetic field and arbitrary state of polarization of the laser was made by Hussein, Pato and Kerman[3]. In this communication we supply the qualitative considerations of NAIBEA[1].

GENERAL CONSIDERATION

The motivation for seeking alternatives to conventional acceleration concepts is clear. The available electric field usually employed to accelerate particles is about 10 Mev/m. Thus to reach e.g. the CEBAF energy (4 Gev) one needs a tube of about at 400 meters in length (reduced when using superconductivity).

It is clear that to reach higher energies bigger and bigger machines are required and one naturally starts seeking alternatives to the conventional concept.

A laser with a power of say P (W/cm²) supplies an electric field intensity of

$$eE_{laser} = \left[\frac{4\pi e^2}{c} P\right]^{1/2} \qquad (1)$$

If we take for P, say, 10^{16} W/cm² , we obtain

$$eE_{laser} \cong 2.0 \times 10^5 \text{ Mev/m} = 0.2 \text{ Tev/m} \qquad (2)$$

If one were to utilize even as little as 1% of the electric field intensity of the laser, one would end up with an accelerator which is 100 times smaller than the conventional ones. The problem one faces here in that direction of the electric field, to which the particle velocity must couple, is perpendicular to the direction of propagation of both the laser (the Poynting vector) and eventually the particle.

To be able to accelerate the particle along the laser Poynting vector, one must have a very small component of the particle velocity along the laser electric field and be sure to have the particle motion transverse to the laser Poynting vector well contained: oscillatory. The applied alternating static electric or magnetic field is the needed degree of freedom to guarantee a transverse oscillatory motion of the particle which must be within the transverse extention of the laser pulse.

The above considerations allow us to make some useful estimates.

Let me call the wavelength of the laser λ_o. The wave length seen in the particle rest frame is obtained from Doppler shift argument to be

$$\lambda = \lambda_o\left(\frac{1+\beta_o}{1-\beta_o}\right)^{1/2} \quad , \quad \beta_o = \frac{v_o}{c} \qquad (3)$$

where v_o is the initial velocity of the particle. The velocity of the particle in the laboratory is γv_o, $\gamma = \sqrt{1-\beta_o^2}$. Thus the distance in the Lab. traveled by the particle during a time lapse of $\frac{\lambda}{c}$ is

$$\Delta Z = \frac{\lambda}{c}\,\gamma v_o = \lambda_o \frac{\beta_o}{1-\beta_o} \qquad (4)$$

Therefore if λ_o is several microns, ΔZ could be macroscopic (several tens of cm's) if β_o is close enough to unity (relativistic injected particles). The above observations allow the laser accelerator to be macroscopic.

If, for simplicity, we take ΔZ to be $n\,\lambda$, where n is the number of cycles within the laser pulse (in fact, because of the acceleration, $\Delta Z > n\,\lambda$), and call the diameter of the laser d_o (which is not Doppler shifted), then after $n\,\lambda$ encounters with the laser, the particle will have traversed (through the action of the applied alternating static field) the laser n times. The gain in energy, $\Delta\epsilon$, of the particle after traveling a distance of $n\,\lambda$ is then given by

$$\Delta\epsilon \cong e\,E_{laser}\,(n\,d_o)$$

To be able to make sensible comparison with conventional accelerators, it is useful to introduce[4] an effective laser electric field intensity $E_{eff.}$, such that

$$\Delta\varepsilon \equiv e\,E_{eff}(n\lambda) \tag{6}$$

We therefore obtain

$$eE_{eff} = e\,E_{laser}\,\frac{d_o}{\lambda}$$

or

$$e\,E_{eff} = e\,E_{laser}\,\left(\frac{1-\beta_0}{\beta_0}\right)\left(\frac{d_o}{\lambda_0}\right) \tag{7}$$

If we take e.g. $d_o = 1$ mm, $\lambda_0 = 10^{-2}$ mm, $\beta_0 = 0.9999$, then, for the laser of Eq.1), we find

$$e\,E_{eff} = 2\ \text{Gev/m}$$

More realistic calculation, to be described below, gives a smaller value for $e\,E_{eff}$.

Thus, we can say that Eq.(7) supplies an upper limit to the effective laser electric field intensity. Multiplying this intensity by the tube length supplies us with an upper limit to the gain in energy. It is an upper limit since: a) the distance $n\lambda$ is much smaller than the actual distance traveled by the particle during n encounters with the laser, and b) the laser field intensity E_{laser} has its maximum value in the center of the laser beam. It decays in the transversal plane, usually, as $\exp[-|x^2+y^2|/w_o^2]$, where w_o is the spot size of the presumed Gaussian beam[5]. We now turn to a general formulation of NAIBEA[3].

FORMAL DEVELOPMENT

To simplify the presentation, we consider the units of mass in mc^2, the vector potential A in mc/e, the distance x in 1/k, where k is the wave number and time, t in 1/ ; being the frequency. We take the direction of wave propagation and particle, acceleration to be along z. The Hamiltonian of the system, particle + laser + applied field is (note that we are working in the temporal gauge)

$$H = [1 + P_z^2 + (P_x - A_x)^2 + (P_y - A_y)^2]^{1/2} \equiv \gamma \tag{8}$$

in our units.

In Eq.(8) $\vec{A} = \vec{A}^{(0)}(t-z) + \vec{A}_{app}(t,z)$. Then $\vec{E}_{app} = -\dot{\vec{A}}_{app}$ and $\vec{B}_{app} = \vec{\nabla}\times\vec{A}_{app}$ are the external applied fields which add to those of the traveling laser pulse $\vec{A}^{(0)}(t-z)$.

Hamiltons equations follow from Eq.(8), i.e.

$$\dot{P}_z = -\frac{\partial\gamma}{\partial z}\ ,\ \dot{P}_x = 0\ ,\ \dot{P}_y = 0 \tag{9}$$
$$\dot{\gamma} = \frac{\partial\gamma}{\partial t} = \frac{\vec{P}-\vec{A}}{\gamma}\cdot\left[\frac{\partial\vec{A}^{(0)}}{\partial t} + \frac{\partial\vec{A}_{app}}{\partial t}\right]$$

Further reduction of $\dot{P}_z$ gives

$$\dot{P}_z = \frac{\vec{P}-\vec{A}}{\gamma}\cdot\left[\frac{\partial\vec{A}^{(0)}}{\partial t} + \frac{\partial\vec{A}_{app}}{\partial z}\right] \tag{10}$$

Combining $\dot{\gamma}$ and $\dot{P}_z$ we obtain

$$\dot{\gamma} - \dot{P}_z = \frac{1}{\gamma}\left[(\vec{A}\times B_{app})_z + \vec{A}\cdot\vec{E}_{app}\right] \tag{11}$$

which, when integrated yields

$$\gamma = P_z + u\ ;\quad u = u_o + \int_{-\infty}^{t}\frac{[(\vec{A}\times\vec{B}_{app})_z + \vec{A}\cdot\vec{E}_{app}]}{\gamma}\,dt' \tag{12}$$
$$u_o = \frac{1}{\gamma_o}(1+\beta_o)^{-1} = \left[\frac{1-\beta_o}{1+\beta_o}\right]^{1/2}$$

At this point, we remark, as done in Ref. 1), that since $\vec{A}^{(0)}$ will be the dominant field, it is more convenient to use the phase $\varphi = t-z$ as an integration variable. Then, sice $\dot{\varphi} = 1-\dot{z}$ and $\dot{z} = \partial\gamma/\partial P_z = P_z/\gamma$ and from (6), $1-\dot{z} = u/\gamma$, we have $d\varphi/u = dt/\gamma$. Further, since $\dot{u} = (1-\dot{z})du/d\varphi = (u/\gamma)\,du/d\varphi$, we obtain from Eq.(12) the following

$$u^2(\varphi) = u_o^2 + 2\int_{-\infty}^{\varphi}[(\vec{A}\times\vec{B}_{app})_z + \vec{A}\cdot\vec{E}_{app}]\,d\varphi' \tag{13}$$

and from $\gamma^2 = (P_z+u)^2 = 1 + P_z^2 + \vec{A}^2$

$$\gamma(\varphi) = \frac{1+\vec{A}^2+u^2}{2u}\ ;\ P_z = u\frac{dz}{d\varphi} = \frac{1+\vec{A}^2-u^2}{2u} \tag{14}$$

$$z(\varphi) = \int_{-\infty}^{\varphi}\left[\frac{1+\vec{A}^2-u^2}{2u^2}\right]d\varphi' = \int_{-\infty}^{\varphi}\left[\frac{\gamma(\varphi')}{u(\varphi')} - 1\right]d\varphi' \tag{15}$$

The x and y coordinates of the particle are determined from the equations $\dot{P}_x = 0$ and $\dot{P}_y = 0$, which yield for the canonical momenta, $P_x = 0$ and $P_y = 0$, and thus the physical momenta are given by

$$p_x = \gamma \dot{x} = -A_x \quad ; \quad p_y = \gamma \dot{y} = -A_y \tag{16}$$

or

$$x(\varphi) = -\int_{-\infty}^{\varphi} \frac{A_x(\varphi')}{u(\varphi')} d\varphi' \tag{17}$$

$$y(\varphi) = -\int_{-\infty}^{\varphi} \frac{A_y(\varphi')}{u(\varphi')} d\varphi' \tag{18}$$

The set of equations (13-15) constitutes the Generalized NAIBEA equations of Ref.3. These equations are important generalizations of the NAIBEA equations of Ref. 1 in that: 1) the laser could be in any state of polarization, and have any pulse shape, and 2) the applied EM field E_{app} and B_{app} is quite general. Note that since $\vec{A}$ is defined to within an arbitrary constant, we have here full freedom in giving the electron non zero initial value of p_x and/or p_y (Eq. 16). The trajectory parameter, Q, which was introduced in Ref. 1 is here generalized to be a vector in the x-y plane and is defined by the equation

$$\frac{d\vec{Q}}{d\varphi} = -\vec{A} \tag{19}$$

It is a simple matter to show that the second derivative of $\vec{Q}$ can be written as

$$\frac{d^2\vec{Q}}{d\varphi^2} = -\frac{d\vec{A}^{(0)}}{d\varphi} + \frac{\gamma}{u}\vec{E}_{app} + \left[\frac{\gamma}{u} - 1\right]\vec{B}_{app} \tag{20}$$

and the rate of change of γ with respect to φ

$$\frac{d\gamma}{d\varphi} = \frac{\vec{p}\cdot\vec{E}}{u} = -\frac{1}{u}\vec{A}\cdot\vec{E} = \frac{1}{u}\frac{d\vec{Q}}{d\varphi}\cdot\vec{E} \tag{21}$$

Note that Eq.(20) is a nonlinear second-order differential equation for $\vec{Q}$, since u and γ depend on $\frac{d\vec{Q}}{d\varphi}$ and $\frac{d\vec{Q}^2}{d\varphi^2}$. The solution of this equation completely determines the trajectory of the particle.

For γ to increase with φ, $\frac{d\vec{Q}}{d\varphi}\cdot\vec{E}(\varphi)$ must always be positive (notice that $u(\varphi) > 0$)

$$\frac{d\vec{Q}}{d\varphi}\cdot\vec{E} > 0 \tag{22}$$

The above is the condition that guarantees that when $E_i(\varphi_j) = 0$, $p_i(\varphi_j) = \frac{dQ_i}{d\varphi_j}$ is also zero, then $\frac{d\gamma(\varphi_j)}{d\varphi} = 0$ and $\frac{d^2\gamma(\varphi_j)}{d\varphi^2} = 0$. Having such an inflection point in γ at φ_j (instead of a maximum) guarantees that γ keeps increasing for $\varphi > \varphi_j$. If B_{app} is taken to be zero as Ref. 1, then $\vec{p}\cdot\vec{E} = \vec{p}\cdot\vec{E}^{(0)} + \vec{p}\cdot\vec{E}_{app}$. If $\vec{E}$ is taken in the y-direction, then we have $p_y E_y^{(0)} + p_y E_{app} > 0$. since $E_y^{(0)}$ is the dominant field except when passing through zero, the above condition says that we use E_{app} to "fine tune" the sign of p_y so that it is always the same as that of $E_y^{(0)}$. The fundamental role of the applied field is to guarantee the validity of Eq.(22). This can happen even if $E_{app}/E^{(0)}$ or $B_{app}/B^{(0)}$ or both are much smaller than unity. The injection of electrons with a non-zero $p_x(0)$ or $p_y(0)$, albeit very small, is very important to set the machine to work. This is so since in the x-y plane the motion of the electron must be oscillatory in accordance with Eq.(18). The idea behind the NAIBEA acceleration is to optimally determine the applied field so that: a) the transversal motion of the particle is well confined to be within the transversal dimension of the laser beam, and b) the acceleration coefficient u (Eq.(13)) is made as close to zero as possible.

SPECIFIC CHOICES

We turn now to specific choices of the accelerator. We consider a linearly polarized pulse with $\vec{A}$ taken to be along the y direction. We first consider a constant applied electric field, E_{app}, then

$$\vec{A} = \left[A_y^{(0)}(\varphi) - E_{app}t - p_y(0)\right]\hat{j} \tag{23}$$

With the $\vec{A}$ above used in Eqs. (20) and (21) we recover the NAIBEA equation of Ref. 1. Inversions of E_{app} are made at appropriate values of z to assure the validity of Eq.(22), namely $p_y(n\pi) = 0$. This means that the applied field is inverted at φ_j's such that $\left.\frac{dp_y(\varphi)}{d\varphi}\right|_{\varphi_j = \frac{n+1}{2}\pi} = 0$.

We now replace E_{app} by a constant magnetic field along x. Then

$$\vec{A} = \left[A_y^{(0)}(\varphi) - B_{app}z - P_y(0)\right]\hat{j} \qquad (24)$$

The resulting NAIBEA equations are almost identical to those of Ref. 1 except for a change in sign of the second term in Eq.(8) of that reference (with E_{app} replaced by B_{app}). Further, Eq.(13) reduces to

$$u^2 = u_0^2 - 2\,B_{app}Q \qquad (25)$$

The y-component of the momentum is given by (Eq. 18).

$$P_y(\varphi) = P_y(0) - A_y^{(0)}(\varphi) + B_{app}z \qquad (26)$$

The continuous acceleration of the particle results if B_{app} is chosen so that $P_y(n\pi) = 0$. This requires changing the sign of B_{app} at appropriate places ($\varphi \simeq \frac{n+1}{2}\pi$).

We consider the following numerical example. The initial value of γ, $\gamma_0 = 70$, $P = \frac{3}{2}\nu_0^2 10^{15}$ W/cm^2 for $\lambda_0 = 10^{-3}$ cm. The parameter $\nu_0 = 1$ refers to the maximum value of electric field of the pulse in our units. We also take nine cycles within the pulse. The shape of the pulse is taken to be a Gaussian, $A^2 = \exp(-\varphi^2/\Delta^2)$, with $\Delta = 3\pi$. The applied field intensity is taken to be 2.34 teslas which corresponds to $\sim 5\times10^{-5}$ that of the laser. The electrons are injected at an angle of 0.6° with respect to the z-axis ($P_y(0) \approx \frac{0.6}{180}\pi\gamma_0$). We consider, as an example nine changes in the sign of the modulated applied magnetic field. Since we have not self consistently chosen the position of the field reversal we have not actually optimized the decrease of u in Eq.(13). In figure 1 we show the change of γ vs. z obtained by solving Eqs.21,20 and 15. The gain in energy is a factor of 35 over a distance (accelerator length) of seven meters! The accelerator length could be

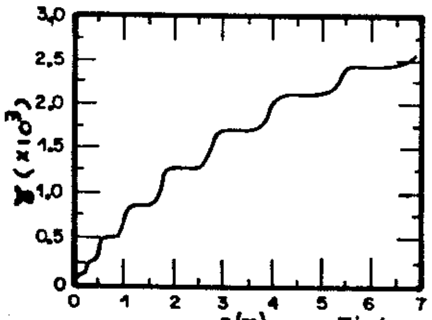

Fig.1

made smaller if full optimization is accomplished. It is interesting to calculate the effective field for this case. From Eqs. 1,5 and 6 we obtain the value $eE_{eff} \cong 150$ Mev/m.

The correspompding trajectory of the particle, confined in the (z,y)-plane is shown in figure 2.

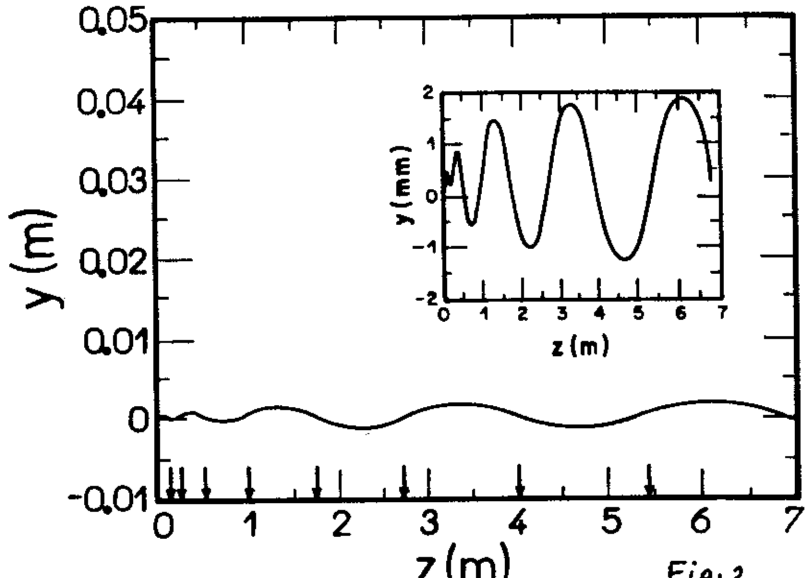

Fig. 2

The arrows indicate the positions where the applied magnetic field is inverted. These positions (in meters) are given in table 1.

1	2	3	4	5	6	7	8	9
0.15	0.24	0.51	1.0	1.75	2.75	4.0	5.4	6.9

Table 1

As a second example, we consider proton acceleration using our concept. We thus take for the initial energy of the protons, the value 0.5 Tev. This corresponds to an initial velocity of $V_0 = 0.999998$ c. The laser power is taken here to be $P = 6.6\times10^{20}$ W/cm^2 with $\lambda_0 = 5\times10^{-3}$ cm. The laser electric field intensity is $eE_{laser} = 49$ Tev/m. T initial injection angle is $\theta_0 = 0.02°$. A Gaussian laser pulse with the above maximum intensity is considered with a width of 3π. The applied magnetic field intensity was taken to be 10 teslas, which is $5.14\times10^{-7}\,E_{laser}$. During the acceleration the sign of the applied field was changed

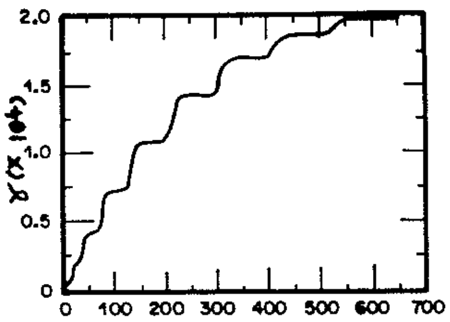

Fig. 3

11 times. The result are shown in figure 3. The proton reaches an energy of 20 Tev, within a length of 3 km. The effective laser electric field intensity is found here to be $eE_{eff} = 6.6$ Gev/m. The trajectory of the proton in the y-z plane is shown in fig.4. The arrows indicate the positions where the last seven inversions of the field are made

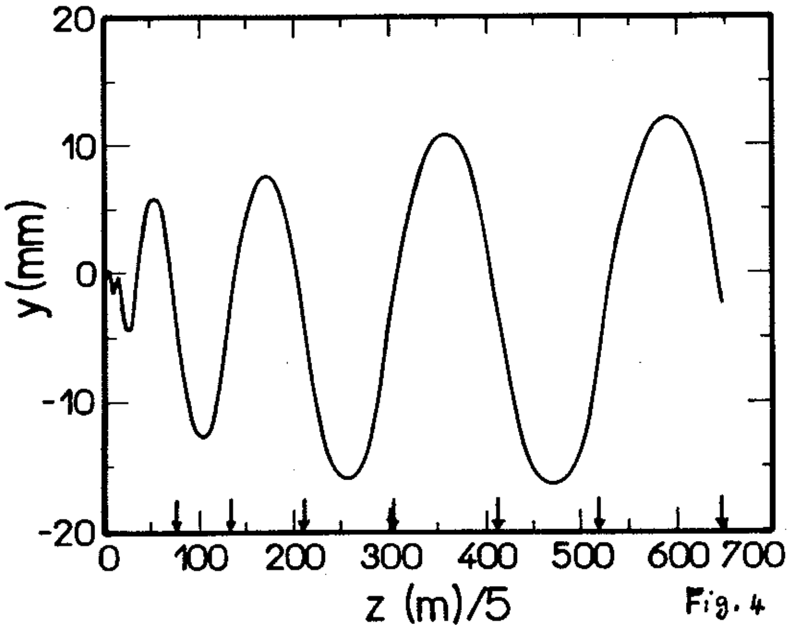

Fig. 4

These positions as well as the first six of them are given in the table 2 below (in km).

1	2	3	4	5	6	7	8	9	10	11
0.031	0.05	0.085	0.18	0.375	0.665	1.045	1.515	2.052	2.63	3.23

Table 2

It is clear from Table 2 that there is ample space along the tube to further fine tune the particle trajectory.

As a final remark we mention that our NAIBEA accelerator can be further optimized by better determining the values of the available fine tune variables: θ_0 , B_{app} , and the inversion positions. The final aim is to have a rather "small", very high energy accelerator with the lowest possible value of B_{app} for a given laser power.

This work was done in collaboration with M.P. Pato and A.K. Kerman. The CNPq-Brasil and IBM do Brasil supplied a partial support.

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