SHOCK WAVES IN THE AIR

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(I)Introduction.

Shock wave or *shockwave* in the air is a type of propagating disturbance that moves faster than the local speed of sound in the medium. When an object (or disturbance) moves faster than the information can propagate into the surrounding fluid, then the fluid near the perturbation cannot react or "get out of the way" before the disturbance arrives. In the shock wave the properties of the fluid (density, pressure, temperature, flow velocity, Mach number) change almost instantaneously, nearly discontinuously.[1,2] Shockwaves carries energy and can be found in any medium[1,2] (*solids, liquids, gases and plasmas*). Measurements of the thickness of shock waves in air have values around 200 nm (~ 10^{-5} cm) which is on the same order of magnitude as the mean free path of gas molecules. In **Section 1** are seen **sound waves** in the air.[3,4]In **Section 2** is presented to graduate and undergraduate students of Physics a simple description of **shockwaves** created by bodies (airplanes) moving in the air. In **Section 3,** in our "**LaboDemo(IFUSP)**", are analyzed adiabatic air compressions in tubes and also when shockwaves can be created. In recent review article[1]are mentioned many different kinds of shockwaves observed in Nature, like for instance, supersonic bullet fired by a rifle, supernovae shockwaves, Space return vehicles (Apollo, Space shuttle), bullets, explosions by chemical reactions, ocean waves that break on the shore... In **Section 4** are briefly commented *meteor shockwaves*, when they enter the Earth´s atmosphere,[6] and *Cherenkov radiation*.[7]

(1) Sound waves.

As seen in elementary physics books[3,4] when air is submitted to *soft compressions* along tubes their elementary volumes move along these tubes obeying the *wave equation*,

$$d^2x/dt^2 = -(1/K\rho)(d^2x/dt^2) \qquad \textbf{(1.1),}$$

where K is the air *volumetric compressibility modulus* and ρ its density.

The longitudinal compression waves (**sound waves**) move with velocity $V_s = \sqrt{1/K\rho}$. In usual air conditions $V_s \approx 350$ m/s. The pressure wave p(x,t) in these compressions is given by[3,4]

$$p(x,t) = P \sin[(2\pi/\lambda)(x - ct)] \qquad \textbf{(1.2),}$$

where $\lambda = V_s/f$ is the sound wavelength, f the sound frequency and P the pressure amplitude.[3,4] It is given by $P = (2\pi\rho V_s^2/\lambda)X_o$, where X_o is the maximum displacement amplitude of the wave.

The human ear[3,4] is able to hear sound waves (with $\lambda \sim 35$ cm and $f \sim 10^3$/s) when pressures are in the interval ($P_{atm} \pm 200$) dines/cm^2. Taking into account that $P_{atm} \approx 10^6$ dines/cm^2 we see that

$$X_o = \lambda P/2\pi\rho V_s^2 \sim 10^{-3} \text{ cm},$$

that are very small displacement amplitudes. [3,4]

(2)Airplane Shock Waves.

According to **Section 1** airplane with *subsonic* velocities $V < V_s$, that is, when V < 350 m/s does not create large pressure perturbations in the air. In this way, as seen in **Fig.(2.1a),** these small perturbations, that is, the **sound waves**, would move in front of the airplane.[2] We could say that the airplane send messages to the air in its front to get out of its path.

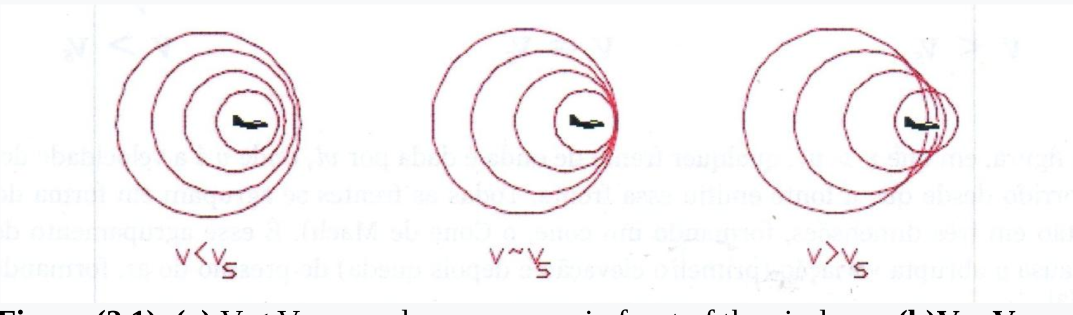

Figure (2.1). (a) $V < V_s$: sound waves move in front of the airplane. **(b)V ~ Vs :**sound and airplane move together. **(c)**V > Vs: airplane moves faster than sound.[4]

When $V > V_s$ (**Fig.(2.1c)**) are created **Shock waves** that are not conventional sound waves; they are formed when a rigid pressure front (like the airplane surface) moves at supersonic speed and pushes on the surrounding air.[3,5]Almost instantaneously[5,8] is created in contact with the airplane surface a very thin layer of air with $\delta \sim 10^{-5}$ cm with very high pressure and temperature. The high pressure air as a function of time is schematically shown in **Figure (2.2).**

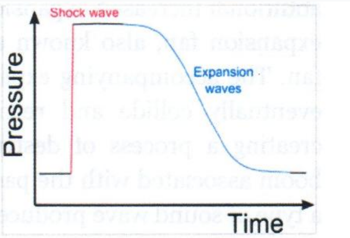

Figure (2.2). Rough temporal description of the pressure created inside the *shockwave*.

This thin layer δ is of the same order of magnitude of the free path of gas molecules.[1,2] In reference to the continuum, this means that the shockwave can be treated as either a line or a plane if the flow field is 2-dimensional or 3-dimensional, respectively. Over longer distances, a shockwave change from a nonlinear wave into a linear one, becoming into a conventional sound wave as it loses energy. The shockwave sound is heard as a loud "crack" or "thump" of a sonic boom created by the supersonic aircrafts.

(2.a) Mach number.

Mach number (M) is defined by $M = V/V_s$. For subsonic velocities we have $M < 1$ and for supersonic $M > 1$. When $V > V_s$ any front of wave is given by $V_s t$ where t is time the since the source issued this front. All these fronts group together in a V(conic) shape, as seen in **Figure (2.3).**[2]

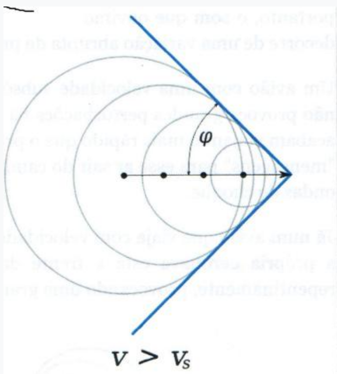

Figure (2.3). Mach Cone and the angle φ.

In **Figure (2.3)** the angle φ is given by sin(φ) = V_s/V. For larger V smaller will be the angle φ. In 3-dimensions all wave fronts group together forming a cone, named **Mach Cone**. This conical shockwave in contact with a plane ground surface becomes a shaped-hyperbola. [1](**Figure (2.4)**)

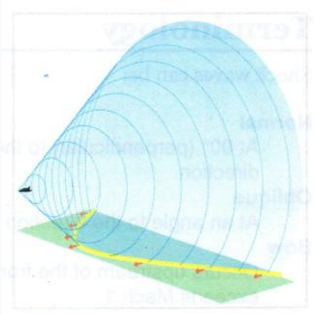

Figure (2.4). Shaped-hyperbola shockwave in contact with plane ground surface.

(3) Shockwaves in Tubes .[5]

Let us investigate conditions to create *shockwaves* by very fast adiabatic air compressions in glass tubes (**Figure (3.1)**).

Figure (3.1). Glass tube, piston and rubber base.

As well known, [3] in adiabatic compressions of ideal gases from volume V_i to V_f their temperatures change from T_i to T_f according to

$$T_f = T_i(V_i/V_f)^{\gamma-1} \qquad \textbf{(3.1),}$$

where $\gamma = C_p/C_v$, where C_p and C_v are the gas specific heats at constant pressure and volume, respectively. In the case of air $\gamma \approx 1.40$.[3]

Our glass tube has length L_i filled with air and closed at one end (**Figure (3.2a))** where we put a swab of cotton (**Fig. (3.2b))**. The diameter of the tube is constant ≈ 7.5 mm.

Figure (3.2a). Glass tube and piston inside.

Figure (3.2b). Detail of the cotton inside the tube.

The ambient temperature is $T_i = (27 + 273)^oK = 300^oK$.

With an adiabatic compression,[3] the final temperature T_f ,when the compression ratio is $r = V_i/V_f$ is given by **Eq.(3.1),** that is:

$$T_f = T_i\, r^{\gamma-1} \qquad \textbf{(3.2).}$$

To burn the cotton is necessary that $T_f = (400 + 273)^oK \approx 673^oK$. Thus, from **Eq.(3.2)** we must have,

$$r = (T_f/T_i)^{(1/0.4)} \approx (673/300)^{2.5} = 2.24^{2.5} \approx 7.5 \qquad \textbf{(3.3).}$$

Therefore, to reach the combustion temperature of the cotton, a *minimum* required compression ratio is r = 7.5.

Our glass tube, has a initial length $L_i \approx 120$ mm **(Figure (3.3a)** and after the compression a final length $L_f \approx 5$ mm **(Figure (3.3b)**. Thus, the compression ratio **r** of the tube is given by:

$$r = (V_i/V_f) = (L_i/L_f) = (120/5) = 24 \qquad \textbf{(3.4).}$$

With this compression ratio, the final T_f temperature would be

$$T_f = T_i\, r^{\gamma-1} = 300(24)^{0,4} \approx 1070\ ^{o}K \approx 796\ ^{o}C \qquad \textbf{(3.5),}$$

which is more than enough to burn the cotton inside the tube, at $T = 673^{o}$, mentioned above. As in the adiabatic compression we have[3]

$$P_f = P_i\,(V_i/V_f)^{\gamma} \qquad \textbf{(3.4),}$$

with $P_i = P_{atm}$ and $\gamma = 1.40$:

$$P_f = P_i\, r^{\gamma} = P_{atm}\, 7{,}5^{1.40} \approx 16{,}8\ P_{atm} \qquad \textbf{(3.5).}$$

Figure (3.4a). At the initial instant of compression L_i = 120 mm.

At the instant $t_i = 0$ we begin to compress the air inside de tube. The total compression will be at $t = t_f$ which is measured with a chronometer (see **Figures (3.4a)** and **(3.4b)**).

Figure (3.4b). At the end of compression, with the cotton burning, L_f = 5 mm.

Due to the air compression in the interval $\Delta t = t_f - t_i$, when the piston covered a distance $\Delta L = L_i - L_f$, the cotton swab caught fire. The compressed air attained the temperature $T_f \approx {}^oK = 796\ {}^oK$ (see **Eq.(3.5)**).

Is the heating due to an adiabatic compression or to shockwave in the tube[5]? To answer this, let us estimate the piston speed V* during the compression. It is given by $V^* \approx \Delta L/\Delta t$ where $\Delta L = (L_i - L_f) \approx 115$ mm and the measured compression time $\Delta t \sim 0.1$s. In this way, the piston speed is given by $V^* \approx (115\text{mm}/0.1\text{s}) \approx 1150\text{mm/s} \approx 1.5$ m/s. So, as piston velocity V* is much smaller than the sound speed V_s = 340m/s, there is **no shock wave** inside the tube, only heating due to *adiabatic compression.*[3]

(4)Meteors and Cherenkov radiation.

Meteors shockwaves are generated when they enter the Earth´s atmosphere.[2,6] In 2013 a meteor (*Tunguska*) entered in Russian territory. It is, up to now, the best documented evidence of the shockwave produced by a meteoroid. Its explosion released an energy equivalent to 100 or more kilotons of TNT, dozens of times powerful than the atomic bomb dropped Hiroshima. The shockwave has produced damages causing, for instance, broken glass in the city of Chelyabinsk and neighboring area. Inflicted more than 1.200 injuries, mainly due to broken glass that fell from windows shattered by the shock wave.

Cherenkov radiation[7] is an electromagnetic radiation emitted when a charged particle, such as an electron, passes through a dielectric medium (such as distilled water) with a speed $\mathbf{V_{el} > c/n}$; **c** is the light velocity in vacuum and **n** the refraction index of the dielectric medium.[3,4]

A classic example of Cherenkov radiation is the characteristic blue glow of an underwater nuclear reactor.[7] This radiation is not the usual *bremsstrahlung*[8] which is, in general, emitted in the medium by a rapidly moving electron. It is a *shockwave* which occurs in a dielectric medium. It involves radiation emitted by the medium under the action of the field created by the electron in motion.[8]

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